Measuring Barriers to Firm Growth
The Importance of International Trade*

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Abstract

A growing literature relies on the firm size distribution to measure barriers to firm growth, usually under the closed economy assumption. We question how international trade matters. To do so, we develop a model of trade with an endogenous productivity choice ("innovation"). Trade and innovation costs differ across countries. We derive a closed form, endogenous steady state size distribution of firms with a Pareto tail. We then calibrate the model to a set of European countries. The model can account for between 54 and 87% of the differences in manufacturing value added per worker. The least productive countries are Italy and Spain, where firms are relatively smaller. In Italy, large innovation costs (33% larger than Germany's) prevent firms from growing. In Spain, the problem is trade costs (43% larger than Germany's). A closed economy model would (i) not identify the biggest problem in Spain; (ii) greatly underestimate the innovation costs in Italy (12% larger than Germany's); and (iii) greatly over-predict the effects of a change in innovation costs on productivity by between 40 and 75%, and on welfare by between 32 and 65%. We conclude that trade is important for measuring barriers to firm growth.

1 Introduction

The large availability of firm level data is driving researchers to use the distribution of firms to infer differences in cost structures across countries. By affecting firm behavior, differences in unobservable barriers to firm growth lead to observable differences in firm size distributions.

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Economists can infer the underlying barriers from the observed distributions. Restuccia and Rogerson (2008) do this for the United States, while Hsieh and Klenow (2009) compare costs across the United States, China, and India. A drawback of these papers is that economies are closed, in spite of the importance of international trade costs in firm behavior.

We develop an open economy framework to study the differences in firm size distributions in European countries. Trade affects the size of firms via two main channels. First, lower trade costs increase the demand faced by exporters, and therefore their size. Second, the higher demand increase incentives to invest in the adoption of new technologies. We refer to this as innovation. It has been well documented that exporters innovate more than non exporters. Moreover, our model reveals that the possibility of exporting in the future affects the incentives to grow for non exporters. Correspondingly, we introduce both trade and innovation costs and calibrate them to match key targets in the data.

Our findings reveal two main drawbacks of the closed economy assumption. First, when an economy is exceptionally good at exporting (as is Italy), the closed economy makes it look less distorted than it is. Under the open economy, we estimate Italian innovation costs 33 percent larger than German. Under the closed economy, this drops to only 12 percent. Low export costs (8% lower than German) mask huge domestic distortions. Only an open economy model can unmask them.

Second, even if the estimates of innovation costs in the open and closed economies are similar, the counterfactual experiments drastically over-predict the impact of changes in domestic distortions. For instance, our estimates for the Spanish innovation costs are roughly the same under the open and closed economy assumptions. This is because trade costs are so high (43 percent larger than German) that the closed economy model is not too far off (of course, it fails to indentify the main problem). But the impact of changes in innovation costs are way off. The closed economy predicts reductions in productivity from an increase in innovation costs 43 percent larger than the open economy.

Aggregate productivity in the closed economy depends only on the productivity of individual domestic firms. An increase in innovation costs makes all worse off, producing big aggregate changes. In the open economy aggregate productivity depends on the productivity of domestic firms and the productivity of the foreign firms producing imports. An increase in innovation costs only affect domestic firms, thus producing a smaller changes.

Our model is a continuous time model version of Melitz (2003) as Impulitti et. at. (*forth-*

coming), except that process innovation is endogenous. Any firm may innovate, i.e., invest to increase its productivity, by incurring a convex cost, as in Rubini (2011) and Atkeson and Burstein (2010). Since more efficient firms sell to larger markets and hire more workers, innovation endogenizes the size distribution of firms. Additionally, firms can become exporters by incurring a sunk cost. In equilibrium, firms grow by innovating and become exporters after reaching a productivity threshold.

We fully characterize these decisions. Exporters grow at a constant rate. This is consistent with Gibrat's law (firm growth rates are independent of firm size). Non exporters grow slower than exporters, consistent with Bernard and Jensen (1999) and Bernard (2004). Their rate increases (continuously) as they approach the export threshold. At the export threshold, their growth rate is the same as exporter's. Non exporters thus invest to export: the closer they are to becoming exporters, the faster they want to get there, thus increasing innovation. This channel creates a neat link between non-exporting firms and trade costs. Actually, we show that if export costs drop, non exporters increase their growth rates.

The fact that non exporters react to changes in trade policy can help understand an open puzzle: does exporting increase productivity, or do highly productive firms become exporters? Bernard and Jensen (1999) find the latter is more likely, since firm productivity increases prior to becoming exporters. We argue that firms increase their productivity *because* they are about to export. Also, this implies that the behavior of non exporters is not exogenous to trade policy, a very common assumption to measure gains from trade (see, for example, Van Biesebroeck (2005) and De Loecker (2007)).

We then characterize the distribution of firms. The upper tail of the distribution of firms follows a Pareto distribution, following Zipf's law; and further, the upper tail is related to the growth rate of exporters: the larger this growth rate, the flatter the slope. Luttmer (2007, 2010) and Acemoglu and Cao (2010) derive similar conclusions for the closed economy case.

Overall, the model performs reasonably well. It can capture between 54 and 87 percent of the differences in manufacturing value added per worker in the data. The countries with the lowest value added per worker, both in the model and in the data, are Italy and Spain, also the countries with the smallest average firm size. Tybout (2000) notes that countries with distributions of firms biased towards small firms have lower GDP per capita.

Our model provides a direct link between average firm's growth and average size: the smaller the firm's growth the smaller the average size. In turn, smaller average size translates into lower output per worker (our measure of productivity). Thus, output per worker in Italy and Spain is 19 percent and 24 percent lower than in Germany, respectively. In Italy, the problem is large costs of innovation (33 percent larger than in Germany). In fact, accounting for differences in innovation costs reduces the productivity gap to just 4 percent. In Spain, differences in trade and innovation costs account for 10 percentage points of the productivity gap. Seventy percent of this comes from trade costs, which are 43 percent larger than Germany's. The remaining 14 percentage points are accounted for by size. Germany is almost three times larger than Spain in terms of employment. This, plus the large trade costs, prevents innovation and firm growth in Spain.

If we shut down trade, the estimates change dramatically for Italy, with innovation costs only 12 percent larger than Germany's. The estimates are much closer in Spain, which is reasonable since we find that Spain is relatively closed. However, even in Spain, the closed economy over-predicts the effect of changing innovation costs on productivity: a marginal change in innovation costs reduces productivity by 43 percent more in the closed economy. This is even worse in France, where the excess change is of 73 percent.

The fact that we solve most of the model analytically makes the task of identifying the differences across countries relatively easy. We directly pin down parameters to match the targeted moments. For instance, there is a direct (unique) mapping between the slope of the tail of the distribution and the growth rates of firms. Hence, the slopes tell the firm growth rates in equilibrium. Further, the full characterization of the firm's dynamics provides a direct mapping between growth rates and each of the frictions. Thus, we reverse-engineer the innovation cost that generate such growth rates. The only stage in which we must rely on numerical solutions is when solving for the equilibrium wages. But this just involves finding the solution of a standard non-linear system of equations.

Lastly, we evaluate the performance of the model along a series of non targeted dimensions, including expenditures in R&D, wages, and value added per worker. We compare the share of innovation employees in the model with R&D personnel in the EFIGE dataset. The model performs well in all countries except the U.K. This is to be expected: Griffith et al. (2006) points out that most U.K. firms perform their R&D activities abroad, mainly in the U.S. We then compare wages. The model captures the differences among countries successfully. The comparison of manufacturing value added is similar. The model accounts for between 54 and 87 percent of the differences in value added per worker.

We also compare our estimates to the literature. As in Waugh (2010), our Spanish trade costs the largest of the sample, although our magnitudes are different. Waugh's Spanish trade costs are 18 percent larger than France's, while ours are 43% larger.[1] We compare our innovation costs with the Doing Business Report Ranking by the World Bank. Both rankings are similar, with the exception of U.K. We find innovation costs are slightly larger than Germany's, while they are lower according to the report.

Our work is related to Kambourov (2009), who analyzes the interaction between trade and domestic barriers. While our analysis remains abstract as to what the domestic barriers are, Kambourov explicitly introduces labor adjustment costs. It is also related to Bhattacharya et al. (2011), that use a model with endogenous innovation to identify resource misallocation in the data in a closed economy framework.

The rest of the paper is organized as follows. Section 3.1 describes the data we use in the calibration of the model. Section 2 introduces the model. Section 3 calibrates the model. Section 4 shows the results. Section 5 evaluates the model against non targeted dimensions. Section 6 concludes.

2 The Model

The model builds on Melitz (2003). Time is continuous. There are J countries that produce a continuum of differentiated goods that can be traded. Each good can only be produced in one country. There is an infinitively lived representative consumer that derives utility from consuming as many goods as possible. There are incumbent firms each period that make production, innovation, and exporting decisions. Firms die each period with an exogenous probability δ. There is a pool of potential entrants that can enter by paying an entry cost κ_e.

The preferences of the consumer in country j are given by the following utility function,

[1] Waugh (2010) does not estimate trade costs for Germany, so we reference to France.

for $i, j = 1, 2, i \neq j$:

$$U_j\left(q_i(\omega, t)\right) = \int_0^\infty e^{-\rho t} \ln Q_j(t) dt$$
$$Q_j(t) = \left[\int_{\Omega_i(t)} q_{jj}(\omega, t)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\Omega_i(t)} q_{ij}(\omega, t)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

where ω is the name of the good consumed, Ω_j is the set of goods produced in country j and $q_{ij}(\omega)$ is quantity of good ω produced in country i and consumed in j. $\sigma > 1$ is the elasticity of substitution between goods.

Each instant, there is a continuum of incumbent firms that produce the goods. Firms are owned by the domestic consumer. Each firm is a monopolist producing each good. Given a productivity level z and labor services n, the firm producing good ω has access to the following technology:

$$y(\omega; z, n) = z^{\frac{1}{\sigma-1}} n$$

Note that there is a preference parameter in the technology. [2]. This is simply a normalization that simplifies the algebra. The counterpart is that changing σ would change both preferences and technologies, so a change in this parameter would be hard to interpret.

A firm can make innovation expenses to increase its productivity level z. We choose a functional form for the innovation cost that guarantees that in equilibrium Gibrat's law emerges for exporters (large firms in equilibrium). That is, in equilibrium, the exporter growth rate is independent of firm size. Increasing productivity by $\dot{z}$ costs, in labor units,

$$c_j(z, \dot{z}) = \frac{\kappa_{Ij} z}{2} \left(\frac{\dot{z}}{z}\right)^2$$

To increase productivity by a certain proportion, a firm must incur a cost proportional to that proportion squared. Additionally, if a very productive firm wants to increase its productivity by 10%, it must incur a cost that is greater than what a low productivity firm would need to incur to increase its productivity by 10%. κ_{Ij} determines how costly innovation is, and it may differ across countries.

[2]This is standard, see for instance Atkeson and Burstein (2010)

A firm can export by incurring a sunk export cost equal to κ_{xj} units of labor, and it may depend on the country. Once a firm becomes an exporter, it remains an exporter until it dies, without the need of paying additional export costs.

There is a large pool of potential entrants that can enter anytime by incurring an entry cost equal to κ_e units of labor. After paying the entry cost, entrants start producing with productivity $z = 1$.

Exports are subject to iceberg trade costs. Transport depletes a proportion τ of the good. So if a consumer consumes an amount q of a good, the exporter in country j exporting to country i must ship an amount $(1 + \tau_{x,ji})q$.

The labor market clearing condition closes the model. Let $M_j(t)$ be the measure of entrants in country j at time t and L_j the total number of workers. The labor market clearing condition is

$$L_j = \int_{\Omega_i(t)} \left[n_j(\omega, t) + c_j(z(\omega, t), \dot{z}(\omega, t)) + \kappa_{xj}\mathcal{I}(\omega, t) \right] d\omega + M_j(t)\kappa_e \tag{1}$$

where $\bar{c}(\omega, t)$ is the labor demand for innovation of firm ω at time t, and $\mathcal{I}$ is the indicator function, which equals 1 if a firm producing good ω becomes an exporter in t, 0 otherwise.

There are labor taxes τ_{lj} and profit taxes $\tau_{\pi j}$[3], with profits rebated lump sum to domestic consumers.

We close the model with a trade balance condition. The exact specification of the trade balance depends on the assumption of who trade with whom. In the next section, we clarify how trade balance works.

2.1 Small Open Economy

In the quantitative section, we treat each country as a small open economy. Therefore, this section describes the monopolistically competitive equilibrium for a small open economy. Alternatively, the equations that characterize the equilibrium are simple enough to admit several characterizations, including a world with J asymmetric countries. The reason why we choose to model it as a world with small open is twofold. First, we do not have data to calibrate all the countries in the world (we have data for at most seven). Second, we

[3]We assume innovation expenses cannot be deducted from profits for tax purposes.

believe each individual country is small enough, so changes at the domestic level will have negligible effects in other countries. We also describe the steady state, and therefore we drop the argument t.

Let w_j be the wage rate in country j. Let $p_j(\omega)$ be the price of good ω produced in country j. Since in equilibrium a producer will charge the same price no matter the market in which it sells, we do not introduce additional notation for country of destination. for prices. This price is set by the monopolist to maximize profits subject to the demand for its product. This demand function comes from the consumer maximization problem. Consumers choose how much to consume of each good taking each price as given. Each instant, consumers solve

$$\begin{aligned}
&\max \ln Q_j \\
&s.t. \\
&Q_j = \left[\int_{\Omega_j} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\Omega^*} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \\
&\int_{\Omega} p(\omega)q(\omega)d\omega + (1+\tau_x^*)\int_{\Omega^*} p(\omega)q(\omega)d\omega = 1 + \int_{\Omega} \pi(\omega)d\omega + R
\end{aligned}$$

The last line is the budget constraint. $\pi(\omega)$ is profits of a firm ω. R is tax revenue. A * denotes rest of the world variables. Let the right hand side be equal to I (for income). The demand of a particular good is

$$q(\omega; p, P, I, \tau_x^*) = \begin{cases} p^{-\sigma} P^{\sigma-1} I \text{ if } \omega \in \Omega \\ ((1+\tau_x^*)p)^{-\sigma} P^{\sigma-1} I \text{ if } \omega \in \Omega^* \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

P_j is the Dixit-Stiglitz aggregate price in country j,

$$P_j = \left[\int_{\Omega_j} p_j(\omega)^{1-\sigma} d\omega + (1+\tau_x^*)^{1-\sigma} \int_{\Omega^*} p^*(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \tag{3}$$

Firms solve two kinds of problems, a static problem and a dynamic problem. The static problem is how much to produce and the price given their current productivity, and the dynamic is how much to innovate and, for non exporters, whether to become exporters.

The static problem depends on whether the customer is domestic or foreign. For domestic customers, this problem is, given $z(\omega)$,

$$\max_{p,q,n}(1-\tau_{\pi j})(pq - w_j(1+\tau_{lj})n)$$

$$s.t.$$

$$q = z(\omega)^{\frac{1}{\sigma-1}} n = p^{-\sigma} P_j^{\sigma-1} I_j$$

If the customer is foreign, the problem is

$$\max_{p,q,n}(1-\tau_{\pi j})((1+\tau_{xj})pq - w_j(1+\tau_{lj})n)$$

$$s.t.$$

$$q = \frac{z(\omega)^{\frac{1}{\sigma-1}} n}{(1+\tau_{xj})} = (1+\tau_{xj}p)^{-\sigma} P_j^{\sigma-1} I_j$$

The solution to these problems is the mark-up rule

$$p(\omega) = \frac{\sigma}{\sigma-1} \frac{1}{z(\omega)^{\frac{1}{\sigma-1}}}$$

Let $\pi_d(P, I, z)$ be the variable profits for a non exporter (profits before paying innovation or exporting costs). These are

$$\pi_{dj}(z(\omega), P_j, I_j) = \sigma^{-1} I_j P_j^{\sigma-1} z(\omega) = \pi_{dj} z(\omega) \tag{4}$$

and for exporters

$$\pi_{xj}(z(\omega), P_j, I_j, \tau_{xj}) = \pi_d(z(\omega), P_j, I_j) + (1+\tau_{xj})^{1-\sigma} \pi_d(z(\omega), P^*, I^*) = \pi_{xj} z(\omega) \tag{5}$$

Next we describe the dynamic problem of the firms. Before we do that, note that, as in Dixit and Stiglitz (1977), we can drop out the name of the good ω, since all that matters for profits is z. This saves on notation. Firms decide how much to innovate each period, and non exporters choose whether to become exporters. We start by solving the problem of exporters. Their Hamilton-Jacobi-Bellman equation is

$$(\rho+\delta)V_{xj}(z) = \max_{\dot{z}}(1-\tau_{\pi j})\pi_{xj}z - \frac{w_j(1+\tau_{lj})\kappa_{Ij}z}{2}\left(\frac{\dot{z}}{z}\right)^2 + V'_{xj}(z)\dot{z} \tag{6}$$

For non exporters, the dynamic problem consists on when to become exporters and how much to innovate[4]. Their problem is a stopping time problem. They need to choose when to become exporters, and how much to grow while being non exporters. Let z_{xj} be the optimal size at which firms choose to become exporters. The problem of non exporters is, for $z \in [1, z_{xj}]$

$$(\rho+\delta)V_{dj}(z) = \max_{\dot{z}}(1-\tau_{\pi j})\pi_{dj}z - \frac{w_j(1+\tau_{lj})\kappa_{Ij}z}{2}\left(\frac{\dot{z}}{z}\right)^2 + V'_{dj}(z)\dot{z} \tag{7}$$

$$s.t.$$

$$V'_{dj}(z_{xj}) = V'_{xj}(z_{xj}) \tag{8}$$

$$V_{dj}(z_{xj}) = V_{xj}(z_{xj}) - w_j(1+\tau_{lj})\kappa_{xj} \tag{9}$$

Equation (8) is the smooth pasting condition. It imposes that the change in value at the point of switch in status is equal before and after switching. Equation (9) imposes that the value of the firm must be the same before and after switching.

New firms enter the economy whenever their expected profits exceed the entry cost. That is, in equilibrium, the free entry condition is

$$w_j(1+\tau_{lj})\kappa_e = V_{dj}(1) \tag{10}$$

2.2 Characterizing the Steady State

To solve the exporter problem, we guess and verify that $V_x(z)$ is homogeneous of degree 1. The solution is the productivity of exporters grows at a constant rate, and is therefore

[4]It is straightforward to show that a non exporter will always choose to become an exporter if it survives long enough. Simply calculate its value given that it never exports and show that, for a sufficiently large z, the value of becoming an exporter exceeds the value of continuing as a non exporter.

independent of firm size. Thus, Gibrat's law holds. This rate of growth is

$$\begin{aligned} g_{xj} =& (\rho+\delta)\left(1-\sqrt{1-h_{xj}}\right) \\ h_{xj} =& \frac{2\pi_{xj}}{(\rho+\delta)^2\kappa_{Ij}} \end{aligned}$$

The rate of growth is increasing in exporter profits and decreasing in innovation costs.

The closed form solution for this value function is

$$V_{xj}(z) = w_j(1+\tau_{lj})\kappa_{Ij}g_{xj}z \tag{11}$$

The first order condition to the non exporter problem is

$$g_{dj} = \frac{V'_{dj}(z)}{w_j(1+\tau_{lj})\kappa_{Ij}}$$

Introducing the solution in the Bellman equation

$$(\rho+\delta)V_{dj}(z) = \left[(1-\tau_{\pi j})\pi_{dj} + \frac{V'_{dj}(z)^2}{2w_j(1+\tau_{lj})\kappa_{Ij}}\right]z, \quad \forall z \in [1, z_{xj}] \tag{12}$$

Equation (12) defines a first order differential equation that pins down the non exporter value function. From the first order condition, this pins down also the non exporter growth rate. The border condition is given by the value matching condition and the smooth pasting condition. Together, these imply the following

$$\begin{aligned} g_{dj}(z_{xj}) &= g_{xj} \\ z_{xj} &= \frac{(\rho+\delta)\kappa_{xj}}{(\rho+\delta)\kappa_{Ij}g_{xj} - \frac{(1-\tau_{\pi j})\pi_{dj}}{w_j(1+\tau_{l,j})} - \frac{\kappa_{Ij}}{2}g_{xj}^2} \end{aligned}$$

Equation 12 is a first order differential equation that cannot be solved in closed form. However, the next proposition shows that it is strictly increasing in z and depends negatively on trade costs.

Proposition 1 *The non exporter growth rate is (i) increasing in z, (ii) decreasing in τ_{xj} and κ_{xj} and (iii) weakly smaller than the exporter growth rate.*

Proof: We omit the country subindices for the proof. Notice that (i) and the previously derived condition that $g_d(z_x) = g_x$ implies (iii). So we only need to prove (i) and (ii). To see (ii), first notice that $g_d(z) = \frac{V_d'(z)}{w(1+\tau_l)\kappa_I}$ from the first order condition. Thus, we can rewrite equation (12) as

$$\kappa_I g_d(z) = \sqrt{w(1+\tau_l)\kappa_I}\sqrt{(\rho+\delta)V_d(z) - (1-\tau_\pi)\pi_d} \tag{13}$$

The proof works by showing $\partial V_d(z)/\partial \tau_x < 0$ and $\partial V_d(z)/\partial \kappa_x < 0$. Using equation (13), this implies $\partial g_d(z)/\partial \tau_x < 0$ and $\partial g_d(z)/\partial \kappa_x < 0$. Write the value function in its time dependent form:

$$\begin{aligned} V_d(z) = \max_{T(z),g(t)} & \int_0^{T(z)} e^{-(\rho+\delta)t}\left[\pi_d z(t) - \frac{\kappa_I}{2} z(t) g(t)^2\right] dt + \\ & \int_{T(z)}^{\infty} e^{-(\rho+\delta)t}\left[\pi_x z(t) - \frac{\kappa_I}{2} z(t) g(t)^2\right] dt - e^{-(\rho+\delta)T(z)}\kappa_x \\ s.t. & \\ & \dot{z}(t) = z(t)g(t), z(0) = z \end{aligned}$$

Using the envelope theorem shows the result (notice that $\partial \pi_x/\partial \tau_x < 0$).

To see point (i), insert the first order condition into the Bellman equation for non exporters to obtain

$$(\rho+\delta)w(1+\tau_l)\kappa_I g_d(z) = \left[(1-\tau_\pi)\pi_d + w(1+\tau_l)\frac{\kappa_I}{2} g_d(z)^2\right] z$$

Differentiating both sides and rearranging,

$$\frac{g_d(z)}{(\rho+\delta)g_d(z) - \frac{(1-\tau_\pi)\pi_d}{w(1+\tau_l)\kappa_I} - \frac{g_d^2(z)}{2}} g_d'(z) = \frac{1}{z} \tag{14}$$

We show $g_d'(z) > 0$ by showing the denominator in the left hand side is positive. This denominator is a polynomial, and as such can be written as a function of its roots:

$$(\rho+\delta)g_d(z) - \frac{(1-\tau_\pi)\pi_d}{w(1+\tau_l)\kappa_I} - \frac{g_d^2(z)}{2} = (g_d(z) - g_1)(g_2 - g_d(z))$$

where

$$
\begin{aligned}
g_1 &= (\rho+\delta)(1-\sqrt{1-h}) \\
g_2 &= (\rho+\delta)(1+\sqrt{1-h}) \\
h &= \frac{2\pi_d}{(\rho+\delta)^2\kappa_I}
\end{aligned}
$$

This holds if $g_1 < g_d(z) < g_2$. To see this, notice that if π_d was replaced by π_x, g_1 would be equal to g_x. In fact, g_1 is the growth rate of a firm that expects to make profits π_d forever, or, in other words, if $\tau_x(\kappa_x) \to \infty$. Since we showed already $\partial g_d(z)/\partial\tau_x < 0 \Rightarrow g_1 < g_d(z)$ for all $z \in [1, z_x]$. Also notice that $g_2 > g_x > g_d(z)$. This implies that the denominator in the left hand side of equation (14) is positive, and thus $g_d'(z) > 0$. $\square$

A problem with the solution of equation (12) is that it has no closed form solution. We need this to derive the distribution of firms. We work around this by solving it numerically and then approximating the solution by the following functional form:

$$
g_{dj}(z) = (a_j + b_j z + c_j z^2 + d_j z^3)^{-1} \tag{15}
$$

where $a, b, c,$ and d are parameters to be determined in equilibrium. This functional form allows for a closed form distribution of firms in equilibrium. In the quantitative section, we show that the fit is very good.

We next describe the steady state distribution. The details of its characterization are in Appendix B.

$$
\mu_j(z) = \begin{cases} M_j \exp\left[\delta(b_j(1-z) + c_j/2(1-z^2) + d_j/3(1-z^3))\right] z^{-a_j\delta}, \text{ if } z < z_{xj} \\ A_j z^{-\frac{\delta}{g_{xj}}}, \text{ if } z > z_{xj} \end{cases}
$$

where $A_j = z_{xj}^{(\delta/g_{xj} - a_j\delta)} \exp(b_j(1-z_{xj}) + c_j/2(1-z_{xj}^2) + d_j/3(1-z_{xj}^3))$.

It is straightforward to see that this satisfies Zipf's law. This law is that the upper tail of the distribution of firms according to employees (or sales) follows a Pareto distribution. The upper tail is completely populated by exporters. The distribution of exporters is Pareto in z. Since employees (and sales) are linearly proportional to z, this satisfies Zipf's law.

Given this distribution, we solve for the equilibrium in each country by solving a system of three equations and three unknowns. The unknowns are π_{dj}, M_j and w_j. The equations are free entry (10), labor market clearing (1), and trade balance:

$$\int_{z_{xj}}^{\infty} (1+\tau_{xj}) p_j(z) q_{j,*}(z) \mu_j(dz) = \int_{z_x^*}^{\infty} (1+\tau_x^*) p^*(z) q_{*,j}(z) \mu_j(dz) \tag{16}$$

It is convenient to rewrite labor market clearing and trade balance in terms of the unknowns. First derive the following relations

$$\begin{aligned} q_j(z) &= (\sigma-1) w_j (1+\tau_{lj}) \pi_{dj} z \\ q_{j,*}(z) &= (\sigma-1)(1+\tau_x^*)^{1-\sigma} (w_j(1+\tau_{lj}))^{-\sigma} (w^*(1+\tau_l^*))^{(\sigma-1)} \pi_d^* z \\ q_{*,j}(z) &= (\sigma-1)(1+\tau_{xj})^{1-\sigma} (w^*(1+\tau_l^*))^{-\sigma} (w_j(1+\tau_{lj}))^{(\sigma-1)} \pi_{dj} z \end{aligned}$$

This determines as well labor used in production. For domestically sold goods, $n_j(z) = q_j(z)/z^{1/(\sigma-1)}$. For exported goods, $n_{j,*}(z) = q_{j,*}(z)/z^{1/(\sigma-1)}$ and $n_{*,j}(z) = q_{*,j}(z)/z^{1/(\sigma-1)}$.

Next, define $\hat{\mu}_j = \mu_j/M_j$ ($\mu(z)$ is linear in M). Labor market clearing is

$$\frac{L_j}{M_j} = \int_1^{\infty} n_j(z)\hat{\mu}_j(dz) + (1+\tau_{xj}) \int_{z_{xj}}^{\infty} n_{j,*}(z)\hat{\mu}_j(dz) + \int_1^{z_{xj}} \frac{z\kappa_{Ij}}{2} g_{dj}^2(z)\hat{\mu}_j dz + \hat{\mu}_j(z_{xj}) + \kappa_e$$

and trade balance is

$$(\pi_{xj} - \pi_{dj}) \int_{z_{xj}}^{\infty} z\mu_j(dz) = (\pi_x^* - \pi_d^*) \int_{z_x^*}^{\infty} z\mu^*(dz)$$

Given w_j, we pin down prices $p(z)$. With π_{dj}, we pin down the quantities $q(z)$, the labor used in production per firm, their innovation rates and the distribution of firms up to a scalar M_j.

Finally, we derive a measure of productivity similar to Atkeson and Burstein (2010). This is output per production workers, where output is defined as the CES aggregate of each individual good as defined in the preference specification. That is,

$$Q_j = \left[\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\Omega^*} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

Given this definition of output, we show in Appendix C that the following holds

$$Q_j = Z_j N_{pj}$$

where N_{pj} is labor used in production and Z_j is a constant, which is our measure of productivity. This is

$$Z_j^{\sigma-1} = \int_1^\infty z\mu_j(dz) + (1+\tau_x^*)^{1-\sigma} \left(\frac{w^*}{w_j}\right)^{1-\sigma} \int_{z_x^*}^\infty z\mu^*(dz) \tag{17}$$

Notice that this includes two terms. $\int_1^\infty z\mu_j(dz)$ is a measure of the average productivity of the domestic firms, and $\int_{z_x^*}^\infty z\mu^*(dz)$ is a measure of the average productivity of imports.

3 Data and Calibration

3.1 Data

We use the *European Firms In a Global Economy* (EFIGE) database, which contains detailed manufacturing firm level information in seven European countries: Austria, France, Germany, Hungary, Italy, Spain, and U.K. We do not include Austria and Hungary in the analysis, since these samples are too small. A policy report for the Bruegel Institute, Rubini et al. (2012) performs a similar analysis to this paper including all seven countries.

The database contains around 15,000 firms. We exclude firms that do not export but maintain some kind of international activity, such as importing, being part of a multinational, or investing abroad since these activities are not modeled in this paper.[5]

We first document large differences in employee-size distributions across the European countries. France, Germany, and U.K. have relatively larger firms than Italy and Spain. The latter countries have the lowest productivity in the sample according to several definitions of productivity[6], an observation that is consistent with Tybout (2000), who surveyed the literature studying firm size distributions and noted that countries with relatively smaller firms have lower GDP per capita.

[5]Figures 1 and 2 would hardly change by including these firms. Figure 3 would, if it includes firms that belong to a multinational organization.

[6]For example, figure 11 shows how these countries compare in manufacturing value added per worker.

Figure 1 shows these distributions. It includes firms with more than 30 employees, and excludes firms with more than 10,000 employees. The x-axis plots the log of employees, and the y-axis the log of the share of firms with more than x employees. The slope of this figure shows the “speed” at which the mass of given sizes decreases. That is, a steeper slope implies relatively higher number of small firms. The difference is robust to a number of control variables and to a one digit level industry (unfortunately, higher digit levels implies very few firms in some industries). Also, the estimation is robust to different minimum employee thresholds.

One determinant that is highly relevant is export status. Figure 2 shows the distribution excluding non exporters. At first sight, the picture looks the same as figure 1. In contrast, figure 3 shows only non exporters. Here we can appreciate important differences. While Italy still has the steepest distribution, Spain now is mixed with France and U.K. Germany has the flattest distribution. This suggests that trade costs are important in accounting for the difference in distributions.

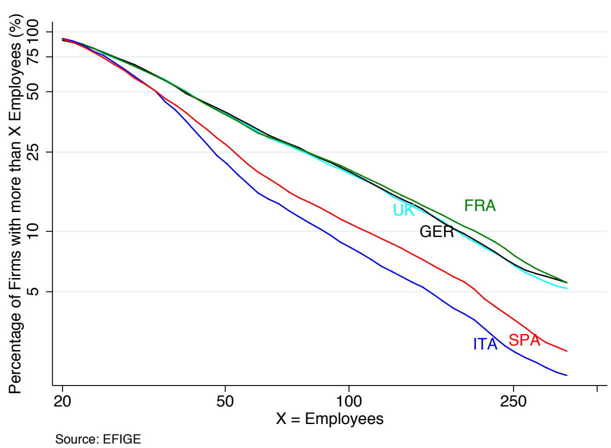


Figure 1: Firm Size Distributions

3.2 Calibration

We set the total number of countries equal to 5 to represent France, Germany, Italy, Spain and U.K. This leaves out Austria and Hungary, also in the database, on the basis of there

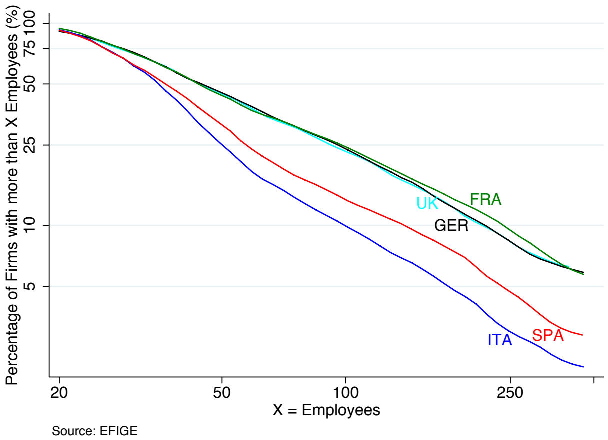


Figure 2: Exporters' Distributions

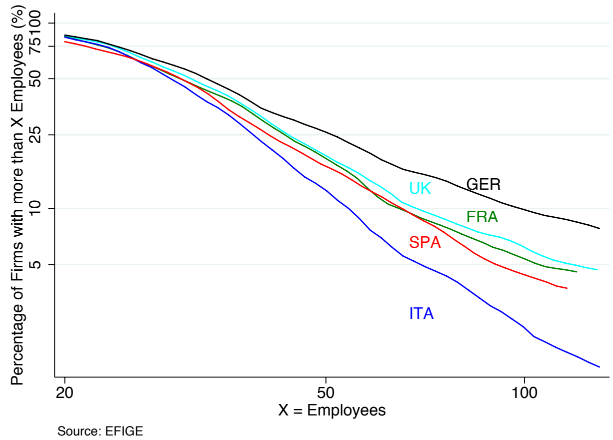


Figure 3: Non-exporters' Distributions

being less than 1 million workers in those countries in the database. The numeraire is the wage rate in Germany, which we set to 1. We set $\rho = 0.04, \delta = 0.06$ and $\sigma = 5$, following Atkeson and Burstein (2010). We obtain labor taxes from McDaniel (2007) using the 2007-2009 average and profit taxes from the *Doing Business* report for 2012 (the only year with data).

A problem is how to calibrate the rest of the world. As it turns out, if we normalize the iceberg trade cost for one country, we do not need to calibrate the rest of the world. Thus, our calibrated iceberg costs are subject to the normalization that we choose. We normalize $\tau_{x,GER} = 0$. This does not affect the ratio $\frac{1+\tau_{xj}}{1+\tau_{x,GER}}$. We next describe this normalization.

Intuitively, the argument is as follows. We use independent targets to pin down π_{dj} and π_{xj}. These contain information about the demand for goods produced in country j in the rest of the world. However, this depends also on the iceberg export costs in country j. To pin these down, we normalize this cost in Germany, which determines the foreign demand in Germany. Given the foreign demand in Germany we identify the demand in the remaining countries, and use this to pin down the iceberg costs.

Start with trade balance. We can simplify equation (16) to

$$(\pi_{xj} - \pi_{dj}) \int_{z_{xj}}^{\infty} z\mu_j(dz) = \frac{\pi_{dj}}{((1+\tau_{lj})w_j)^{1-\sigma}}(1+\tau_x^*)^{1-\sigma}((1+\tau_l^*)w^*)^{1-\sigma} \int_{z_x^*}^{\infty} z\mu^*(dz)$$

Let $X^* = (1+\tau_x^*)^{1-\sigma}((1+\tau_l^*)w^*)^{1-\sigma} \int_{z_x^*}^{\infty} z\mu^*(dz)$. This determines the supply of good from the rest of the world. The assumption of small open economy implies that we can take this as a constant. Next divide this equation by the analogous for Germany:

$$\frac{(\pi_{xj}-\pi_{dj})\int_{z_{xj}}^{\infty} z\mu_j(dz)}{(\pi_{x,GER}-\pi_{d,GER})\int_{z_{x,GER}}^{\infty} z\mu_{GER}(dz)} = \frac{\frac{\pi_{dj}}{((1+\tau_{lj})w_j)^{1-\sigma}}}{\frac{\pi_{d,GER}}{((1+\tau_{l,GER}))^{1-\sigma}}} \tag{18}$$

Given $\pi_{dj}, \pi_{xj}, \pi_{d,GER}$ and $\pi_{x,GER}$ (we explain later how we identify these), we can determine $z_{xj}, z_{x,GER}$ and $\mu_j(z), \mu_{GER}(z)$, and therefore the only unknown is w_j.

To determine the iceberg costs in each country, we first need the demand from the rest of the world. The equation that determines export profits is

$$\pi_{xj}-\pi_{dj} = (1+\tau_{xj})^{1-\sigma}(w_j(1+\tau_{lj}))^{1-\sigma}\frac{\pi_d^*}{(w^*(1+\tau_l^*))^{1-\sigma}}$$

Again, divide by the same equation for Germany:

$$\frac{\pi_{xj}-\pi_{dj}}{\pi_{x,GER}-\pi_{d,GER}} = \frac{(1+\tau_{xj})^{1-\sigma}(w_j(1+\tau_{lj}))^{1-\sigma}}{(1+\tau_{x,GER})^{1-\sigma}(1+\tau_{lj})^{1-\sigma}} \tag{19}$$

Normalizing $\tau_{x,GER}=0$, this equation determines the iceberg cost in all countries relative to Germany.

We use the EFIGE database to calibrate the size of each economy L_i, the innovation cost κ_{Ii}, the fixed export cost κ_{xi} and the variable export cost τ_{xi}. These are four parameters per country. We use four targets from the EFIGE database. We clean the database by eliminating firms that do not export but have some foreign operations, such as importing and investing abroad. The targets are

- The number of workers in each country
- The slope of the distribution of exporters. We calculate the slope by focusing on firms with more than 29 employees since we are mostly interested in the upper tail.
- The share of firms that export
- The value of exports relative to the value of production. This is problematic since firms do not report their sales. They do report the number of employees and the share of

output exported. Our measure of trade volume in country i is the sum of employees in country i times the export ratio divided by the sum of employees in country i.

Table 1: Calibration Targets and Relevant Parameter

Country	Employment	Exp Vol	Exp Firms	Slope	Profit Tax	Labor Tax	
France	2,903,820	27%	71%	1.11	8%	10%	
Germany	5,739,365	20%	65%	1.16	19%	10%	
Italy	3,555,052	33%	77%	1.42	23%	14%	
Spain	2,010,424	21%	68%	1.27	1%	9%	
U.K.	3,768,663	26%	73%	1.06	23%	15%	
Parameter	L	τ_x	κ_x	κ_I	τ_π	τ_l	δ

The calibration strategy is especially clean and direct. It works as follows. From the data, we know the exporter growth rates g_x. These generate the slope of the upper tail as in the data. These growth rates pin down the exporter profits in each country π_x. We then calculate what the export threshold z_x and the non exporter profit π_d should be to match the share of exporting firms and the ratio of exports to total sales in each country. We set the parameters κ_{Ii}, κ_{xi} and τ_{xi} consistent with these equilibrium variables. The details are as follows:

1. We first obtain π_{xi}. The slope of the distribution of exporters in the data identifies the exporter growth rate, given the death rate. This relationship is $slope_i = 1 - \frac{\delta}{g_{xi}}$. Knowing g_{xi} we also know π_{xi}.

2. Given g_{xi}, we identify κ_{Ii}, κ_{xi} and π_{di} with the free entry condition, the share of exporters, and the export volume. To do so, we find κ_{xi} and κ_{Ii} as a function of g_{di}, and then solve a non linear equation in g_{di}.

3. The last remaining parameter is τ_{xj}. Given π_{xi}, π_{di}, equation (18) determines w_j. We use this in equation (19) to determine the iceberg costs.

Table 1 shows the calibration targets and the parameter that is most affected by each target.

We should mention that a key step in this calibration is the approximation of the non exporter growth rates. In Appendix D we show the values for the fitted parameters and the goodness of fit. The goodness of fit essentially plots the numerical solution together with the approximation for the growth rates. We also plot the numerically obtained value function and the one derived from our approximation. Figures 14 through 18 show that, for most cases, the approximation is indistinguishable from the numerical solution.

4 Results

Table 2 shows the values for the calibration of the key parameters and the implied exporter growth rates. We normalize costs so that they equal 1 in Germany.

Table 2: Growth Rates and Parameter Values

Country	g_x	κ_I	κ_x	$1+\tau_x$
France	2.84%	1.08	1.48	1.00
Germany	2.78%	1.00	1.00	1.00
Italy	2.48%	1.33	1.67	0.92
Spain	2.64%	1.07	1.02	1.43
U.K.	2.91%	1.02	1.24	0.92

Consider Italy and Spain, the countries with the flattest distributions. This flattness is consistent with the low rates of growth we identify. Italy's low growth rates are mainly because innovation is expensive: 33% more expensive than in Germany. Spain, on the other hand, has a hard time exporting: it costs 43% more than in Germany.

We can learn also from the behavior of the remaining countries. Notice that France has higher costs than Germany, particularly higher sunk export costs, and still their exporters grow faster. This is intuitive. The larger sunk export costs acts as a barrier to entry in the export market, which reduces competition, and so insiders enjoy larger profits and thus innovate more, growing faster. U.K. exporters grow fast because the sunk export costs are larger, and the variable trade costs are lower than in Germany. This more than compensates a slightly larger cost of innovation, resulting in a larger growth rate.

Before moving any further, we try to understand the magnitude of these estimates. We focus on innovation costs and iceberg costs. Innovation costs have to do with how easy it is for firms to grow. Our estimates suggest that it is much costlier to grow in Italy than in any other country in the sample. The World Bank estimates how easy it is to do business in each country, based on a number of costs such as dealing with construction permits, registering property, getting credit, and enforcing contracts. Based on these (and more) categories, they prepare a general ranking. Table 3 reports the ranking of each country in the sample, together with our estimate for innovation costs.

Table 3: Doing Business Report Ranking(2011)

Country	Ease of Doing Business Rank	κ_I
France	26	1.08
Germany	19	1.00
Italy	83	1.33
Spain	45	1.07
United Kingdom	6	1.02

Source: http://www.doingbusiness.org

The order of both the World Bank ranking and our estimates are quite similar. The only exception is the U.K., where it is easier to do business than in Germany according to the report, but not according to our estimates of innovation costs.

Next we turn to iceberg costs. Waugh (2010) estimates trade costs for many countries. His list includes all the countries we have except Germany. Table 4 shows his estimates of trade costs relative to France for the four countries we have in common, and ours. In both estimates Spanish trade costs are the highest, although our differences are larger than his.

Table 4: Waugh's Trade Cost Estimates

Country	Waugh's	$1+\tau_x$
UK	0.85	0.92
Italy	1.04	0.92
France	1.00	1.00
Spain	1.18	1.43

Source: Waugh (2010)

Another characteristic that may contribute to Spain's large export costs is the fact that their customers are farther away than the other countries', and this naturally requires larger transport costs. The EFIGE dataset includes information on where the products are shipped. Table 5 reports the share of exports within Europe, to North America, and to South and Central America. The farthest place is South and Central America, and the country exporting the most to South and Central America is Spain.

Table 5: Export Destination

Country	Europe	North America	South and Central America
France	76%	6%	2%
Germany	81%	6%	2%
Italy	79%	6%	3%
Spain	73%	4%	8%
UK	67%	12%	1%

Source: EFIGE

Finally, the World Bank reports measures of export costs. This measures are of reference when we compare countries that choose similar export methods. This is the case of Italy and Spain, two Peninsulas, with a natural advantage in maritime exports. The World Bank reports two big differences that increase export costs in Spain relative to Italy: (i) Spanish exports require 50% more paperwork than Italian (an average of 4 documents in Italy vs. 6 in Spain); and (ii) Spanish goods take 50% more days from the time they leave the factory until they reach the port of departure (2.6 in Italy vs. 4 in Spain). This last point may well be due to geography. While both countries are Peninsulas, Italy is a relatively thin area of land surrounded by water, while Spain is not so thin. This translates into many more ports in Italy than Spain: 212 vs. 105. These are the total number of ports, but only the largest are used to export goods. In Spain there is only one large port in Barcelona. Italy has five major ports, in Genoa, La Spezia, Livorno, Venice, and Napoli.[7]

4.1 Italy and Spain

Finally, we turn our attention to the countries that look different: Italy and Spain. Our exercise reveals that an Italian worker produces, on average, 19% less than a German worker. Spanish workers produce 24% less than German workers. How much of this difference is due

[7] http://www.worldportsource.com.

to trade costs, and how much due to innovation costs? To answer, we replace the different costs in Germany with the costs in Italy and Spain and measure the productivities relative to Germany. We do this for trade costs (both sunk and iceberg costs), and innovation costs. We report the results in table 6.

We divide the type of distorsions into observable and unobservable. Observable are those that we calibrate directly from the data, including employees and taxes. Unobservable are trade and innovation costs.

In Italy, observable distorsions account for 40% of the differences with Germany, and unobservable for 60%. Of these, by far the only important one is innovation costs. Accounting for differences in innovation costs, the difference in productivity is only 4%. Adding variable trade costs increases this difference, because these are lower in Italy. Accounting for changes in sunk export costs has a very small negative effect. The reason why reducing this cost reduces productivity is that when there are more exporters, firms are relatively larger, demand more resources, and this reduces profits for the firms that were already exporting. Consequently, these reduce their innovation, reducing productivity.[8]

In Spain, observable distorsions account for 58% of the difference with Germany. The remaining 42% is accounted for by unobservable distortions. In Spain's case, trade distortions are the most important ones, accounting for almost 7 percentage points of the difference. Innovation costs are also important, accounting for 4 percentage points.

4.2 Open vs. Closed Economies

In this section, we compare our results with a closed economy model. This is the type of model most of the literature focuses on, so our results can provide guidance as to how the predictions of such models would be affected by adding international trade. Examples of such models are Luttmer (2007), Luttmer (2010) and Acemoglu and Cao (2010), who develop closed economy models in which firms decide how much to grow by making innovation investments.

In the closed economy, there is only one type of firm, and their maximization problem is similar to the problem of exporters. Their static profits are given by $\pi_j(z) = \pi_j z$, and their value function is $V_j(z) = \kappa_{Ij} g_j z$, where $g_j = (\rho + \delta)\left(1 - \sqrt{1 - h_j}\right)$ where $h_j = 2\pi_j/((\rho +$

[8]The equilibrium in this economy is not Pareto efficient (the planner would innovate more), so it should not come as a surprise that reducing a cost increases productivity.

Table 6: Accounting for Productivity Differences with Germany

Parameter	Italy	Spain
Productivity difference with Germany	**18.83%**	**23.98%**
Difference accounted for by (Pct. Points)		
κ_x	-0.53	-0.06
τ_x	-1.79	6.75
κ_I	14.85	3.83
$\tau_x + \kappa_x$	-2.86	6.74
$\kappa_x + \kappa_I$	13.03	3.74
$\tau_x + \kappa_I$	11.38	9.98
$\kappa_x + \tau_x + \kappa_I$	11.30	9.96
L, t_c, t_w	7.53	14.02

$\delta)^2\kappa_{Ij})$. The free entry condition pins down the rate of growth of firms in the economy by setting $\kappa_e = \kappa_{Ij} g_j = V_j(1)$.

The distribution of firms is given by $\mu_j(z) = M_j z^{-\delta/g_j}$. We calibrate the model so that the slope of the distribution matches the data. To be consistent with the open economy model, we use the observed slope for exporters, as opposed to the entire sample. We set $\delta = 0.06$ as in the open economy model. Thus, the slopes of the distributions pin down the firm growth rate g_j, and then free entry determines the innovation costs κ_{Ij}. Table 7 shows the results, and compares them to the open economy model.

Table 7: Estimates of Innovation Costs in Open and Closed Economies

Country	Closed Economy	Open Economy
France	0.98	1.07
Germany	1.00	1.00
Italy	1.12	1.33
Spain	1.05	1.07
U.K	0.95	1.02

The closed economy model does a fairly good job at estimating differences in costs with Germany in all cases except Italy, where the percentage difference with Germany is 21 percentage points larger under the open economy model. Thus, in Italy, working with an open economy framework is important.

On the other extreme, in Spain, the innovation costs are similar under both frameworks. Interestingly, they are actually smaller under the closed economy assumption. Naturally, one would expected that innovation costs would capture part of the large trade costs in the closed economy, making innovation costs larger, not smaller. The reason for this lies on the interaction between trade and country size. Spanish firms grow less than German firms. In the closed economy, this would happen even with the same innovation costs, because size matters in the same way as it matters in Krugman (1980), and Spain is much smaller than Germany (by a factor of three). Now imagine opening these economies just a little. Spain would gain more than Germany (the relative size of the market increases more, given the same trade costs), and so do the incentives to grow. Thus, to account for the low firm growth rates in Spain, one needs to increase innovation costs.

Given the similarity of the open and closed economy assumptions, a natural question is, suppose a researcher is only interested in the innovation costs in Spain, can she abstract from the open economy?

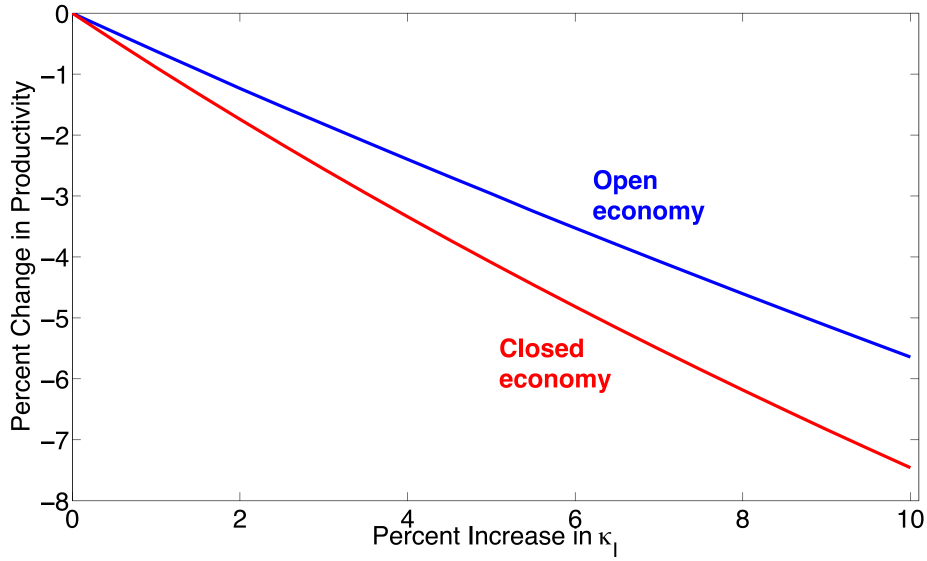
Figure 4 compares the effect of increasing innovation costs in Spain under the closed and open economy assumption. Increasing innovation costs by 10 percent reduces productivity by 7.5 percent in the closed economy, and by 5.6 percent in the open economy. Thus, the predictions of the closed economy model are more than 30% larger than the ones of the open economy model.

The reason why productivity in the closed economy model reacts more than in the open economy is that innovation costs only affect domestic firms. These are all the firms that make up for productivity in the closed economy model, but not in the open economy model. Recall the definition of productivity in equation (17). This measure includes both domestic firms and foreign firms. The change in innovation cost affects only domestic firms. In the closed economy model, equation (17) is

$$Z_i^{\sigma-1} = \int_1^{\infty} z d\mu_i(z)$$

so that innovation costs affect all firms within this measure. Figure 5 shows a decomposition of this increase in productivity for the open economy. Clearly, the increase in the productivity

Figure 4: Productivity Under an Increase in Innovation Costs



of domestic firms drive the increase in aggregate productivity. The productivity of foreign firms stays almost constant.[9]

This finding extends to welfare: a closed economy model overpredicts the consequences of changing innovation costs on welfare relative to the closed economy model. We use the aggregate consumption good C as a measure of welfare. For country j, this is

$$C_j = \left[\sum_{i=1}^{5} \int_{z_{xi}}^{\infty} q_{ij}(z)^{\frac{\sigma-1}{\sigma}} + \int_{1}^{z_{xj}} q_{jj}(z)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

From the derivation of productivity, $C_j = Z_j L_{pj}$, where L_{pj} is labor used in production. Table 8 shows the change in welfare in the open and closed economy models, with the decomposition between productivity and labor.

Table 9 shows the elasticity of productivity with respect to a change in innovation costs for each country, and compares the closed and open economy predictions. The closed economy estimates an elasticity between 43 and 73 percent larger than the open economy. These

[9]The small decrease is because the wages in the domestic economy decreased, which implies that foreign exports are relatively more expensive.

Figure 5: Decomposing the Effect of Reducing κ_I in Spain

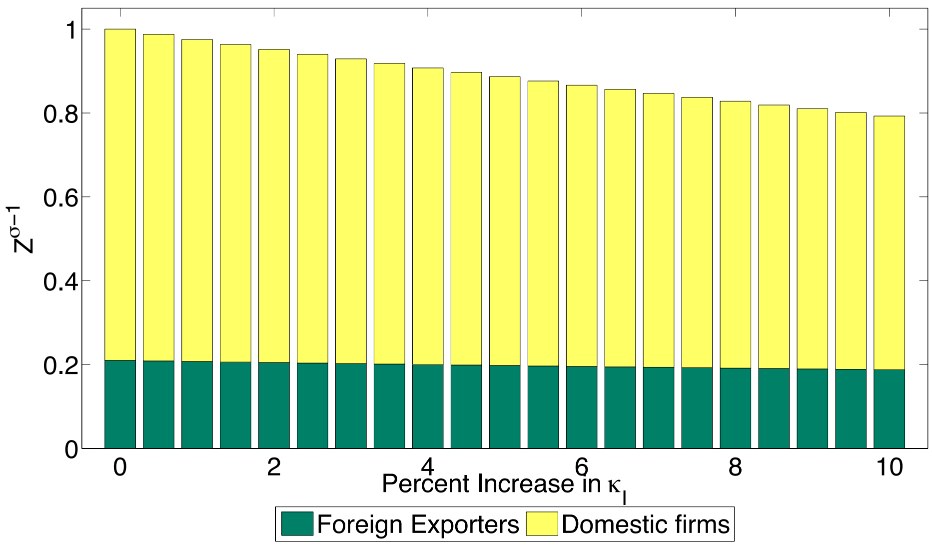


Table 8: Welfare effects of increasing κ_I by 10% in Spain

	Productivity	Production Labor	Welfare
Open economy	-5.6%	-21.1%	-25.5%
Closed economy	-7.4%	-25.6%	-31.1%

differences are similar for welfare, as we show in Table 10.

4.3 Changing Iceberg Trade Costs

The tractability of our model allows us to characterize very precisely the reaction of firms to a change in trade costs, both exporters and non exporters. Proposition 1 states that a reduction in trade costs would trigger an increase in the growth rate of non exporters, in addition to the increase by exporters. While the proposition is for partial equilibrium, we show in this section that it extends to general equilibrium. Figures 8 through 10 show the behavior of non exporter growth rates, before and after a 10 percent reduction in iceberg trade costs. The x-axis shows the productivity of the firm, and the y-axis the growth rate. The numbers in the x-axis show the export thresholds and the numbers in the y-axis the

Table 9: Percentage Change in Productivity per Percentage Change in Innovation Costs

Country	Open Economy	Closed Economy	Ratio Closed to Open
France	-0.62	-1.07	1.73
Germany	-0.62	-0.91	1.47
Italy	-0.48	-0.69	1.44
Spain	-0.63	-0.90	1.43
U.K.	-0.64	-1.01	1.58

Table 10: Percentage Change in Welfare per Percentage Change in Innovation Costs

Country	Open Economy	Closed Economy	Ratio Closed to Open
France	-3.11	-5.13	1.65
Germany	-3.09	-4.37	1.42
Italy	-2.50	-3.29	1.32
Spain	-3.13	-4.31	1.38
U.K.	-3.25	-4.83	1.49

exporter growth rates.

As proposition 1 suggests, the rate of growth is increasing in z, and equals the growth rate of exporters at the switching threshold z_x. A reduction in trade costs increases the growth rate for everyone, exporters and non exporters. It also increases the ratio of exporting firms by reducing the threshold z_x. This holds for every country.

Figure 6: French Growth Rates

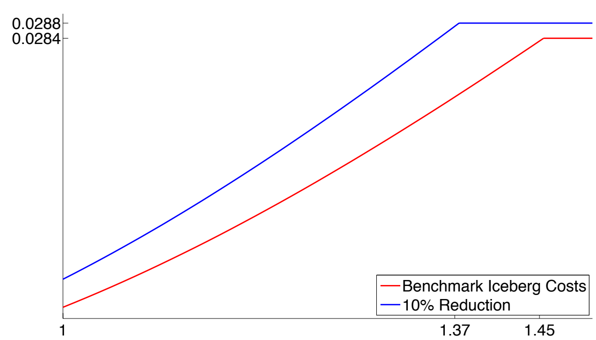


Figure 7: German Growth Rates

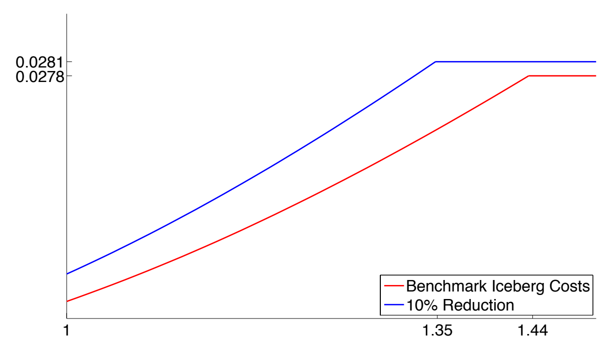


Figure 8: Italian Growth Rates

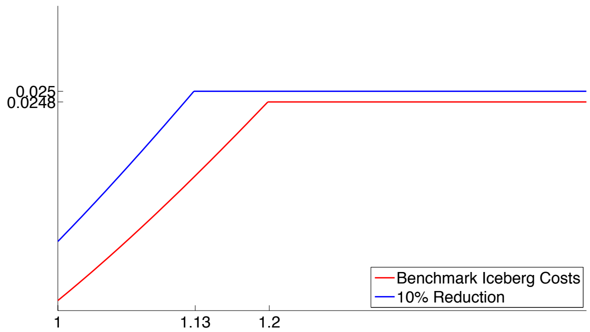


Figure 9: Spanish Growth Rates

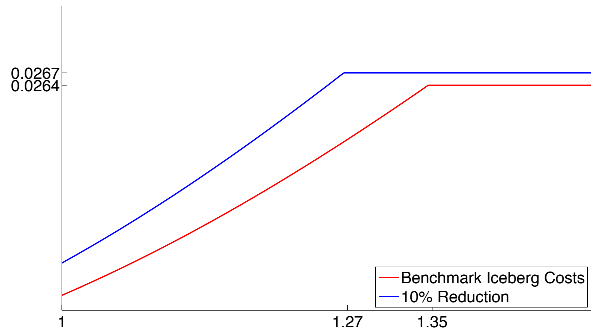
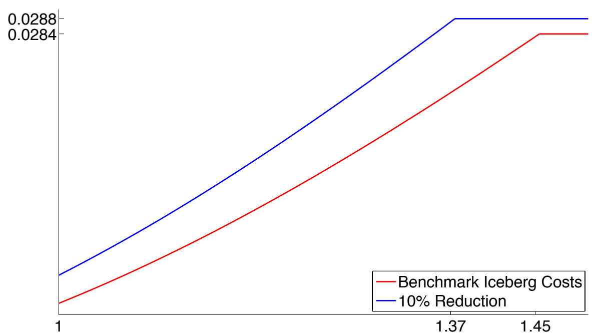


Figure 10: UK Growth Rates



The fact that non exporter behavior changes in response to a change in trade costs has, to the best of our knowledge, not been highlighted so far. This is an important contribution to the empirical trade literature. Many papers study the gains from trade by observing the behavior of exporters relative to a control group of non exporters after a trade liberalization episode. Our results show that, if the non exporter used as a control has the potential of becoming an exporter in the future, its productivity should increase, and as such their behavior cannot be taken as exogenous. This implies that the gains from trade are larger than what these papers find.

Next we consider aggregate effects of changing trade costs. Atkeson and Burstein (2010) show that adding innovation into a model of trade does not change considerably the gains from reducing iceberg trade costs. In this section we confirm their results using our model. We do so in the same way as they do. We simulate a small reduction in trade costs in our model, and then report the percentage change in productivity per percentage change in iceberg costs. We compare these numbers with those obtained with a model with no innovation, in which all firms export. In this economy, we can obtain the change in productivity in closed form solution. We recalibrate this economy so that the trade volumes are as in the benchmark model. We report these numbers in Table 11.

Table 11: Percentage Change in Productivity per Percentage Change in Iceberg Costs

Country	Benchmark Model	No innovation, all firms export
France	-0.30	-0.27
Germany	-0.22	-0.20
Italy	-0.24	-0.33
Spain	-0.22	-0.27
U.K.	-0.25	-0.21

The gains in the model with no innovation are only the direct gains, that is, the gains from exporting being more efficient (less is lost in transit). The fact that in the model with innovation and an extensive margin of exporters the gains are the same implies that the indirect effects cancel each other. That is, any gain in innovation by exporters will be offset by a corresponding reduction in innovation by non exporters and a reduction in the measure of entrants in the economy.

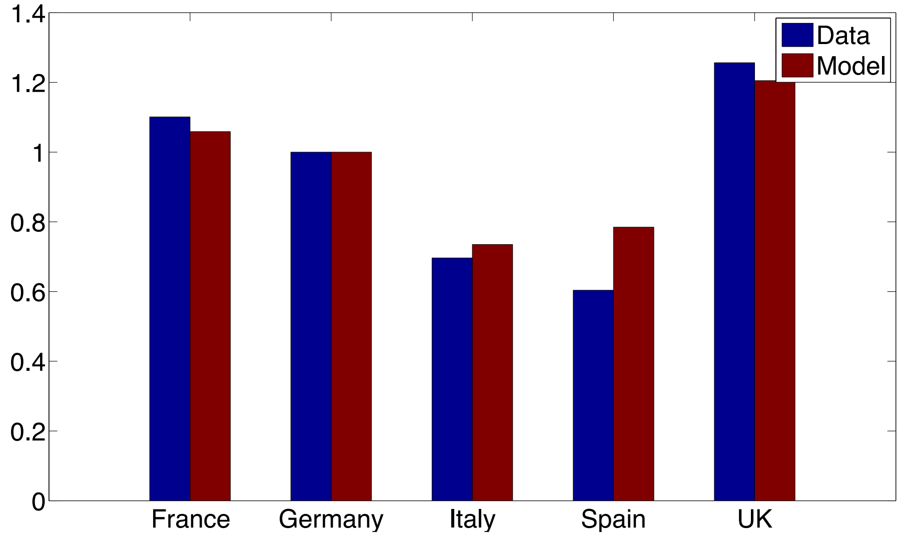
5 The Model Along Non Targeted Dimensions

This section explores the fit of the model along dimensions that were not targeted in the calibration. We compare innovation rates, wage rates and value added per worker in the model with their data counterparts.

5.1 Value Added Per Worker: Model vs. Data

Value added per worker in the data is from Eurostat, averaging years 2004 through 2010. The model performs exceptionally well. Figure 11 shows this comparison. The model can account for a large fraction of the differences in value added per worker in the manufacturing sector in these economies. Table 12 compares the value added per worker in the data with the model.

Figure 11: Manufacturing Value Added per Worker (Germany = 1)



The model can account for between 54 and 87 percent of the differences in value added per worker. It is worth mentioning that the accounting of the closed economy model would be very similar, except that the closed economy would put the blame entirely on innovation cost differences, ignoring the effect of trade barriers.

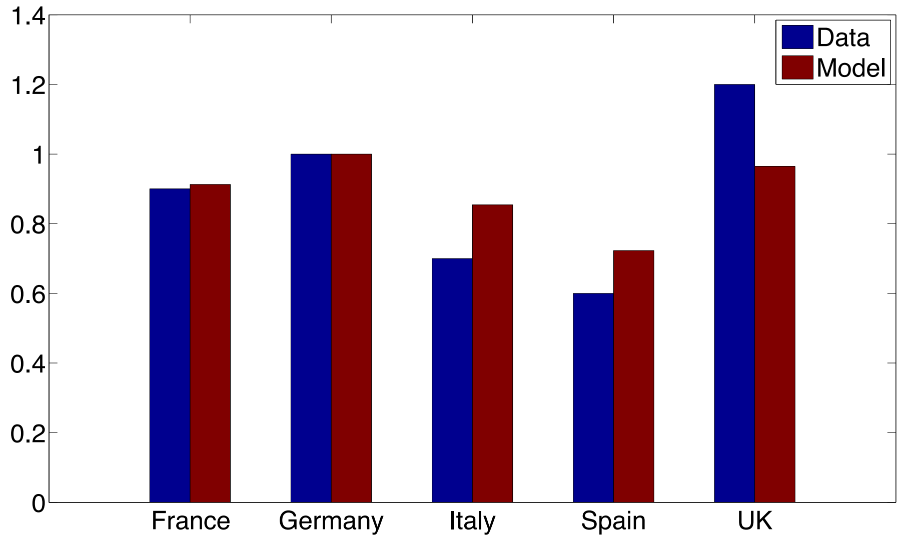
Table 12: Manufacturing Value Added per Worker Relative to Germany

Country	Data	Model	Model accounting
France	1.10	1.06	58%
Italy	0.70	0.74	87%
Spain	0.60	0.78	54%
U.K.	1.26	1.21	80%

5.2 Wages: Model vs. Data

Next we turn to comparing wages in the model and the data. We use data from Eurostat, for the year 2006, since the year 2008 did not include data for U.K. We compare the wages in the model with the mean hourly wage relative to Germany in the data. Figure 12 shows this comparison. The model does a good job at generating the wage differences we observe

Figure 12: Mean Hourly Wages (Germany = 1)



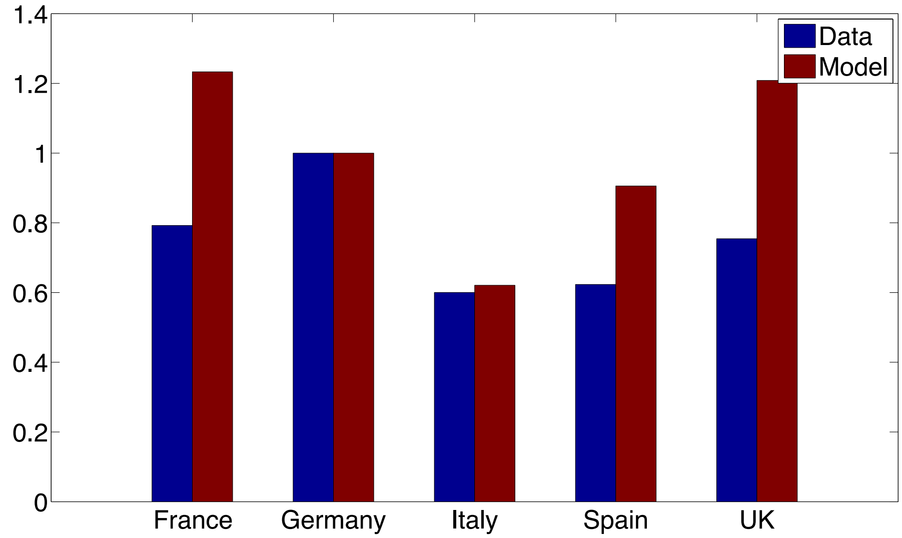
in the data.

5.3 Innovation: Model vs. Data

Identifying innovation rates in the data is challenging, since it is an abstract concept and as such not well defined. In the model, it is any expense that increases productivity. Probably the most obvious expense of this sort in the data is R&D. The EFIGE database has information on the fraction of employees devoted to R&D in each firm. Thus, we use this number to compare with innovation employees in the model. However, since R&D is only part of innovation, we cannot compare the two numbers directly. Instead, we assume that the share of R&D to total innovation is constant in all firms, normalize everything so that the data and model are the same in Germany, and compare the relative levels for the other countries. There is a problem of missing data: some firms do not report their R&D employees. We eliminate these observations for our comparison.

The model performs particularly well for Italy. In Spain, France and U.K. it predicts too many employees will go into R&D.

Figure 13: Share of Employees in R&D (Germany = 1)



6 Conclusion

The large availability of firm level data allows economists to analyze the dstribution of firms in a country and derive conclusions based on its shape. Typically, models that focus on these distributions work under the assumption of closed economy. We have argued that this abstraction is very costly in countries that are open, such as European countries, both qualitatively and quantitatively.

In particular, we find that when analyzing the distribution of firms in Europe, a model of closed economy will wrongly conclude that innovation costs are lower in Italy than what the open economy model would conclude. Also, the closed economy model would predict that changes that affect domestic innovation costs have too much of an effect on domestic macro aggregates, such as productivity. This is because in the open economy model, only some of the firms that count for domestic macro aggregates are affected by the change in innovation costs: the domestic firms.

Finally, we deliver a key message for the empirical estimates on gains from trade. Many trade econometricians estimate the effects of a change in trade policy by comparing the performance of exporters versus non exporters, under the assumption that non exporters are not affected by the change in policy. We find that their behaviour is not exogenous: both exporters and non exporters react to a change in trade costs.

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Appendix A The Problem of Non Exporters

The value function for the non exporters is

$$(\rho+\delta)V_d(z) = \max_{g_d}\{\Pi_d z - \frac{\kappa_I}{2}g_d^2 z + V_d'(z)g_d z\}, \quad \forall z \in [1, z_x]$$

Subject to

$$V_d(z_x) = \kappa_I g_x z_x - \kappa_x \quad \text{Value matching}$$

$$V_d'(z_x) = \kappa_I g_x \qquad \text{Smooth pasting}$$

The first order condition yields

$$g_d = \frac{V_d'(z)}{\kappa_I}$$

Notice first that by comparing the smooth pasting condition and the first order condition, it follows that $g_d(z_x) = g_x$. That is, the instant non exporters become exporters, their growth rate was that of exporters'. Introducing the solution in the Bellman equation

$$(\rho+\delta)V(z) = [\Pi_d + \frac{1}{2\kappa_I}V_z(z)^2]z, \quad \forall z \in [1, z_x]$$

Rearranging, we obtain the differential equation that, given z_x, solves for the non exporter growth rate

$$V_d'(z) = \sqrt{2\kappa_I\left((\rho+\delta)\frac{V_d(z)}{z} - \pi_d\right)} \tag{20}$$

where $g_d(z) = \frac{V_d'(z)}{\kappa_I}$. To solve this differential equation, in addition to z_x, we need an initial condition, which we derived before: $g_d(z_x) = g_x$.

Equation (20) cannot be solved in closed form. This closed form would be useful to obtain later the distribution of non exporters, and solve the general equilibrium model. Therefore, we approximate this solution with the following parameterization:

$$g_d(z) \approx (a + bz + cz^2 + dz^3)^{-1}$$

The approximation works as follows. We first obtain a numerical solution to equation (20), then compute numerically the non exporter growth rates, and then pin down the parameters a, b, c, and d to minimise the distance between the numerical and the approximated solution. We use a Matlab built in function (*ode45*) for the numerical solution of the differential equation, and another Matlab built in function (*fit*) for the approximation. We show the fitted values and the goodness of fit in Appendix D.

Appendix B Deriving the Endogenous Distribution of Firms

Define $\mathcal{Z} = [z_1, z_2]$

$$\hat{\mu}(t + dt, \mathcal{Z}) = \int_{\mathcal{Z}} \hat{\mu}(t, z - \dot{z}dt)e^{-\delta dt}dz$$

Taking limits as $z_1 \to z_2 \to z$

$$\hat{\mu}(t + dt, z) = \hat{\mu}(t, z - \dot{z}dt)e^{-\delta dt}$$

For small dt, the following holds:

$$\begin{aligned}
\hat{\mu}(t + dt, z) &\approx \hat{\mu}(t, z) + \hat{\mu}_1(t, z)dt \\
\hat{\mu}(t, z - \dot{z}dt) &\approx \hat{\mu}(t, z) - \hat{\mu}_2(t, z)\dot{z}dt \\
e^{-\delta dt} &\approx (1 - \delta dt)
\end{aligned}$$

Thus,

$$\hat{\mu}(t, z) + \hat{\mu}_1(t, z)dt = \hat{\mu}(t, z) - \hat{\mu}_2(t, z)\dot{z}dt - \delta dt \left(\hat{\mu}(t, z) + \hat{\mu}_2(t, z)\dot{z}dt\right)$$

Note that in steady state $\hat{\mu}_1(t, z) = 0$. Putting all together,

$$\hat{\mu}(t, z) = \hat{\mu}(t, z) - \hat{\mu}_2(t, z)\dot{z}dt - \delta dt \left(\hat{\mu}(t, z) + \hat{\mu}_2(t, z)\dot{z}dt\right)$$

Eliminating all the terms with dt elevated to a power larger than 1,

$$\hat{\mu}(t,z) = \hat{\mu}(t,z) - \hat{\mu}_2(t,z)\dot{z}dt - \delta\hat{\mu}(t,z)dt$$

Cancelling terms and dividing by dt,

$$\delta\hat{\mu}(t,z) = -\hat{\mu}_2(t,z)\dot{z}$$

Define the steady state distribution as $\mu(z) = \hat{\mu}(t,z)$ for all t. For non exporters, the distribution is

$$\delta\mu(z) = -\mu'(z)g_d(z)z$$

To solve, use the border condition $\mu(1) = M$. For exporters

$$\delta\mu(z) = -\mu'(z)g_x z$$

To solve, use the border condition $\mu(z_x) = \mu_d(z_x)$, where $\mu_d(z_x)$ is the measure of non exporters that reach the export threshold.

The solution to these distributions works as follows. Start with the exporter distribution. The differential equation can be written as

$$-\frac{\mu'(z)}{\mu(z)} = \frac{\delta}{g(z)z} \tag{21}$$

where $g(z) = g_x$ for exporters and $g_d(z)$ for non exporters. For exporters, integrating on both sides,

$$\log(\mu(z)) = \log(z^{-\delta/g_x}) + C_x$$

where C_x is the constant of integration, and is determined using the border condition. Taking exponentials yields the distribution of exporters.

For non exporters, we can only integrate both sides of (21) given our guess for the growth rates. The equation becomes

$$-\frac{\mu'(z)}{\mu(z)} = \delta(a/z + b + cz + dz^2)$$

Integrating on both sides,

$$\log(\mu(z)) = \delta(a\log(z) + bz + \frac{cz^2}{2} + \frac{dz^3}{3}) + C_d$$

where C_d is the constant of integration and is determined using the border condition. Taking exponentials yields the distribution of non exporters.

Appendix C Productivity

The goal is to derive the reduced form for aggregate output

$$Q_j = Z_j N_{pj}$$

where $Q_j = \left[\int_1^\infty q_j(z)^{\frac{\sigma-1}{\sigma}}\mu_j(dz) + \int_{z_x^*}^\infty q_j^*(z)^{\frac{\sigma-1}{\sigma}}\mu^*(dz)\right]^{\frac{\sigma}{\sigma-1}}$ and N_{pj} is labor used for production. Let $n_{dj}(z)$ denote labor for production of units sold domestically and $n_{j,*}(z)$ for exports. With some algebra, we find

$$n_{dj}(z) = (\sigma-1)\frac{\pi_{dj}}{w_j(1+\tau_{lj})}z$$
$$n_{j,*}(z) = (\sigma-1)\frac{\pi_{xj}-\pi_{dj}}{w_j(1+\tau_{lj})}z$$

Labor used in production is

$$N_{pj} = (\sigma-1)\left[\frac{\pi_{dj}}{w_j(1+\tau_{lj})}\int_1^\infty z\mu_j(dz) + \frac{\pi_{xj}-\pi_{dj}}{w_j(1+\tau_{lj})}\int_{z_{xj}}^\infty z\mu_j(dz)\right]$$

From trade balance,

$$\sigma(\pi_{xj}-\pi_{dj})\int_{z_{xj}}^\infty z\mu(dz) = \sigma\frac{\pi_{dj}}{(w_j(1+\tau_{lj}))^{1-\sigma}}X^* \tag{22}$$

where $X^* = (1+\tau_x^*)^{1-\sigma}(w^*(1+\tau_l^*))^{1-\sigma}\int_{z_x^*}^{\infty} z\mu^*(dz)$. The left hand side of equation (22) is exports and the right hand side is imports. X^* is supply of foreign goods, which we take as given following the small open economy assumption. We can rewrite total production labor as

$$N_{pj} = (\sigma - 1)\frac{\pi_{dj}}{(w_j(1+\tau_{lj}))}\left[\int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1}X^*\right]$$

Since $\pi_{dj} = Q_j P_j^{\sigma}(w_j(1+\tau_{lj}))^{1-\sigma}\sigma^{-\sigma}(\sigma-1)^{\sigma-1}$.

$$\begin{aligned}
N_{pj} &= \left(\frac{(\sigma-1)}{\sigma}\right)^{\sigma}\frac{Q_j P_j^{\sigma}}{(w_j(1+\tau_{lj}))^{\sigma}}\left[\int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1}X^*\right] \\
N_{pj} &= \left(\frac{(\sigma-1)}{\sigma}\right)^{\sigma}\frac{Q_j P_j^{\sigma}}{(w_j(1+\tau_{lj}))^{\sigma}}\tilde{Z}_j
\end{aligned}$$

where $\tilde{Z}_j = \int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1}X^*$.

Next consider the price P_j. By definition,

$$\begin{aligned}
P_j^{1-\sigma} &= \int_1^{\infty} p_j(z)^{1-\sigma} + (1+\tau_x^*)^{1-\sigma}\int_{z_x^*}^{\infty} p^*(z)^{1-\sigma} \\
&= \left(\frac{\sigma}{\sigma-1}(w_j(1+\tau_{lj}))\right)^{1-\sigma}\left(\int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1}X^*\right) \\
&= \left(\frac{\sigma}{\sigma-1}(w_j(1+\tau_{lj}))\right)^{1-\sigma}\tilde{Z}_j
\end{aligned}$$

Thus,

$$\begin{aligned}
N_{pj} &= \left(\frac{(\sigma-1)}{\sigma}\right)^{\sigma}\frac{Q_j P_j^{\sigma}}{(w_j(1+\tau_{lj}))^{\sigma}}P^{1-\sigma}\left(\frac{\sigma}{\sigma-1}(w_j(1+\tau_l j))\right)^{\sigma-1} \\
&= \frac{(\sigma-1)}{\sigma}\frac{Q_j P_j}{w_j(1+\tau_{lj})} \\
&= \frac{(\sigma-1)}{\sigma}\frac{Q_j}{w_j(1+\tau_{lj})}\frac{\sigma}{\sigma-1}(w_j(1+\tau_{lj}))\tilde{Z}_j^{\frac{1}{1-\sigma}} \\
&= Q_j\tilde{Z}_j^{\frac{1}{1-\sigma}}
\end{aligned}$$

Rearranging,

$$Q_j = Z_j N_{pj}$$

where

$$Z_j = \left[\int_1^\infty z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1} X^* \right]^{\frac{1}{\sigma-1}}$$

Appendix D Fit of the Approximation

Recall that our solution for the non exporter growth rate involves a differential equation with no closed form solution. Since we need a closed form to derive the distribution of firms, we approximate the non exporter with the following functional form

$$g_{di}(z) = (a_i + b_i z + c_i z^2 + d_i z^3)^{-1}$$

In this section, we discuss the goodness of this fit. Table 13 shows the values we compute for the variables a, b, c, and d for each country. Figures 14 through 18 show how good this approximation is for the growth rates and the non exporter value function.

Table 13: Fitted Values

Country	a	b	c	d
France	39.93	44.58	-54.27	14.69
Germany	23.39	66.24	-62.75	15.83
Italy	-20.83	275.59	-296.97	91.45
Spain	36.33	50.34	-56.99	15.17
U.K.	42.65	31.97	-47.01	13.80

Figure 14: France

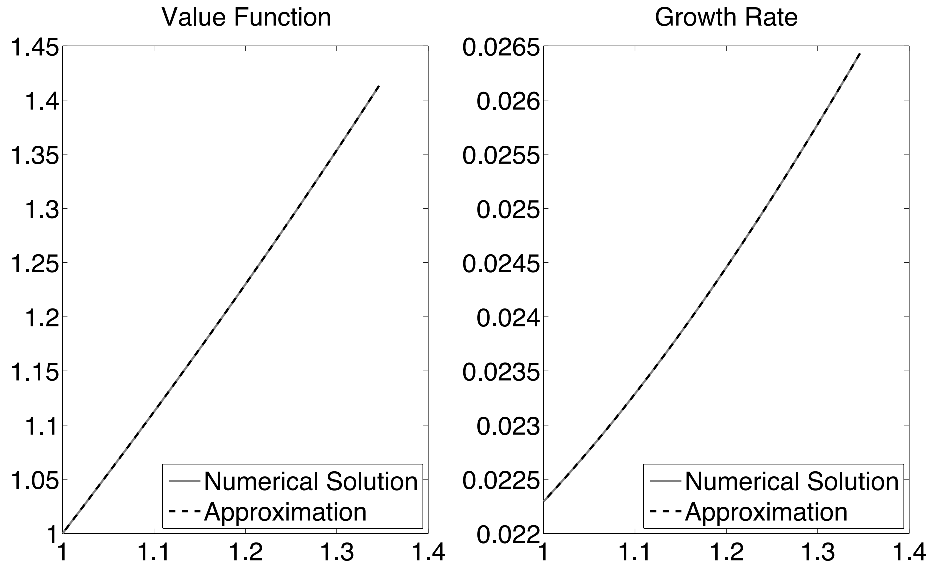


Figure 15: Germany

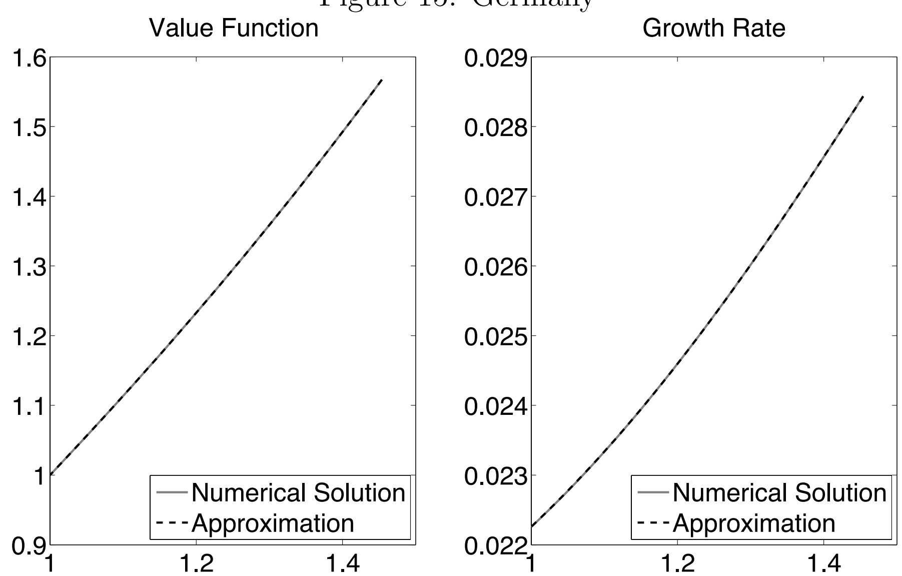


Figure 16: Italy

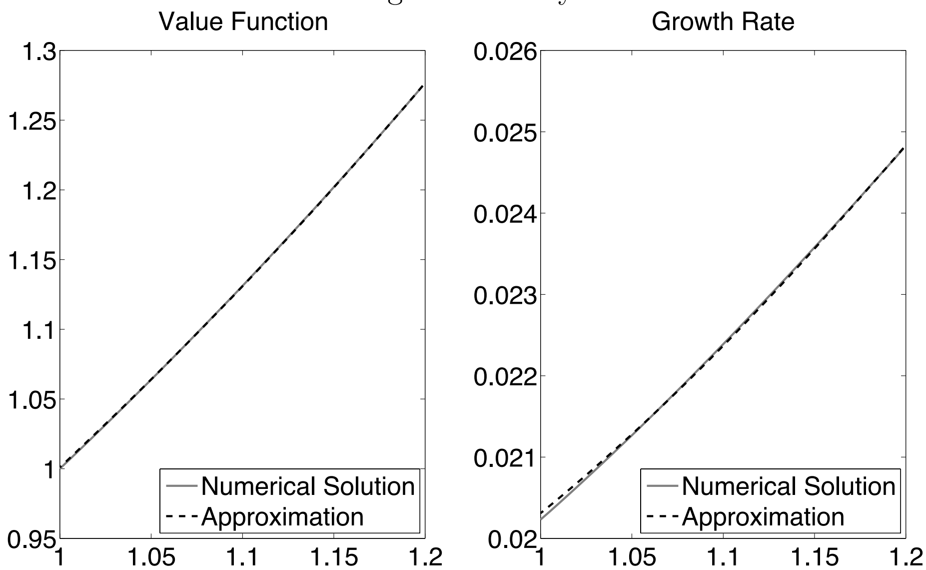


Figure 17: Spain

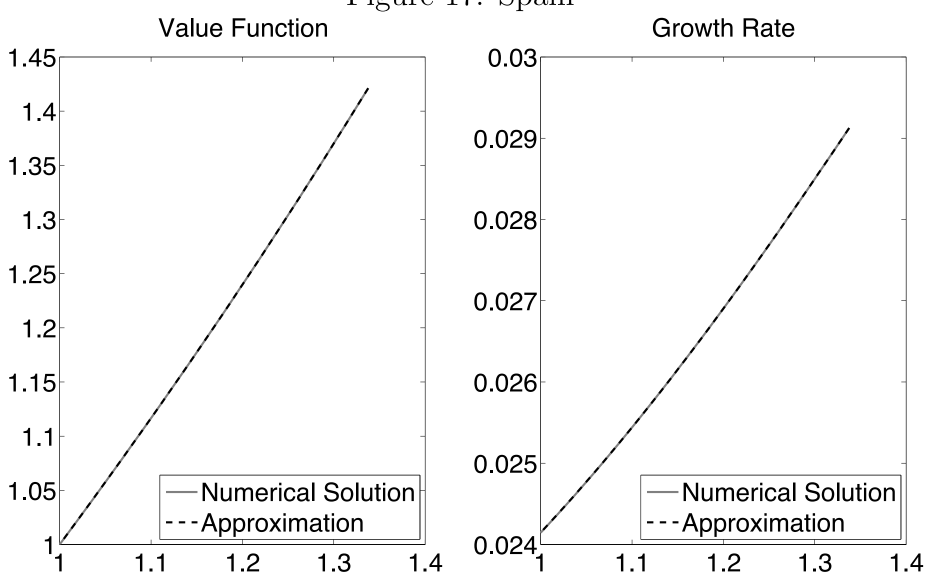
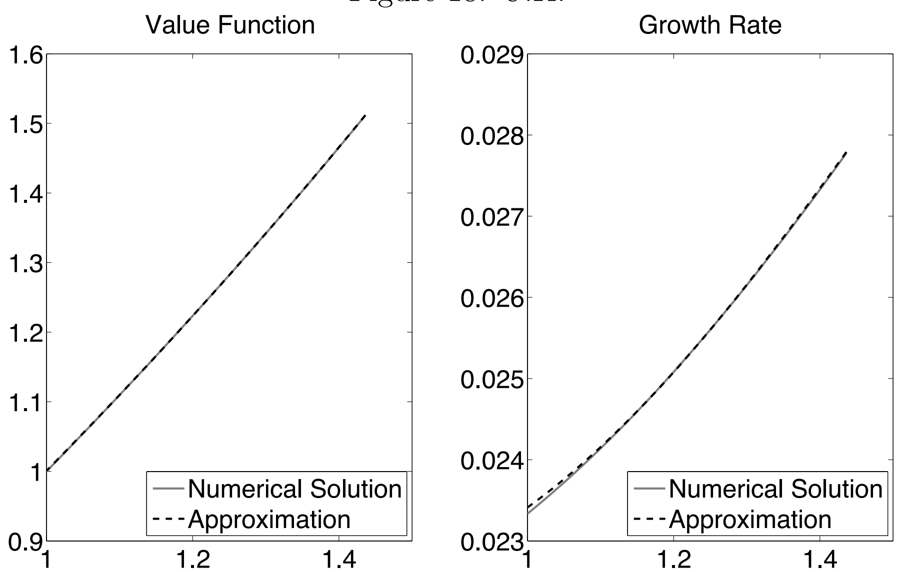


Figure 18: U.K.


Value Function
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
1
1.2
1.4
Numerical Solution
Approximation
Growth Rate
0.029
0.028
0.027
0.026
0.025
0.024
0.023
1
1.2
1.4
Numerical Solution
Approximation