Sunk Costs of R&D, Trade and Productivity: the moulds industry case*

Carlos Daniel Santos†

University of Alicante and Centre for Economic Performance

September 27, 2010

Abstract

Evidence suggests that trade improves industry productivity via (i) selection of the best firms and (ii) within firm productivity growth. While the selection effect has been well explained by the literature, the productivity growth is not accounted for in standard models. Trade liberalization creates economies of scale in the R&D process that can explain the observed growth. I estimate a model of industry dynamics with endogenous productivity, capital accumulation and aggregate uncertainty. I use data from the Portuguese moulds industry which experienced an exogenous trade shock in 1993 (establishment of the Common Market) and a significant increase in foreign demand afterwards. Firms exploited the increase in demand from other European countries to increase R&D spending. The final results confirm the *trade-induced innovation* effect.

Keywords: Industry Dynamics, Innovation, Markov Equilibrium, Moulds Industry, R&D, Structural Estimation, Sunk Costs, Trade

JEL Classification: C61, D92, L11, L22, O31

1 Introduction

I investigate the effects of trade on productivity in the presence of sunk costs for Research and Development. The mechanism works as follows. When trade barriers are reduced, firms have access to external markets. This increase in market size gives firms enough scale to devote

*__Acknowledgments:__ I would like to thank John Van Reenen and Philipp Schmidt-Dengler for all their guidance and support. I would also like to thank Jérôme Adda, Stephen Bond, Ulrich Doraszelski, Cristian Huse, Sorin Krammer, Vadym Lepetyuk, Mark Roberts, Bernard Salanié, Adam Sanjurjo and Gabriel Weintraub and participants at several meetings and conferences for very useful comments and suggestions. This manuscript is part of my Ph.D. dissertation at The London School of Economics and was previously named "Recovering the Sunk Costs of R&D: the moulds industry case". Research supported by the Portuguese Science and Technology Foundation grant SFRH/BD/12092/2003.

†University of Alicante, Campus San Vicente, 03080 Alicante, Spain; csantos@ua.es

more resources towards R&D and pay the sunk costs. R&D generates future innovations and productivity growth - further increasing the gains from trade.[1] I estimate the sunk costs using data from the Portuguese moulds industry after the country joined the EU in 1986 and the Common Market came in place in 1993. I then illustrate the *trade-induced innovation* effect by exogenously varying market size and calculating the new stationary equilibrium (using the estimated structural parameters). If market size is reduced to the level of 1994, the model predicts a reduction in R&D performance, average productivity, capital stock and number of firms - in line with the evidence. To my knowledge this paper is the first to propose and estimate a dynamic game with many players. The estimation method used can be applied to other studies with microdata on industry dynamics, trade or macroeconomics.

Recent advances in the trade literature were quite successful in explaining the important role of firm level productivity and technology in international trade (Eaton and Kortum (2002), Melitz (2003), Bernard et al. (2003)). However, these models consider productivity to be exogenous and are not able to reconcile with some of the empirical evidence. In particular, the models cannot account for three of the stylized facts: the within firm productivity growth after trade liberalization (Pavnick (2002) and Bernard et al. (2006)), the important role of firm size as a determinant of export behavior, even after conditioning on productivity, and the increase in average sales per firm with market size (Eaton, Kortum and Kramarz (2008)).[2] To rationalize the evidence I introduce endogenous choice of technology and size (physical capital) by modeling the adjustment costs for investment and sunk costs for R&D. By so doing, I extend the analysis of the recent trade literature along the technology dimension while abstracting from the role of geography. Economic integration reduces uncertainty. Adjustment costs for capital and aggregate uncertainty are used to generate market size effects. Physical capital and aggregate uncertainty create an option value for investment that generates the observed non-neutrality to market size. Further, I show that this non-neutrality leads to an increase in average size and thus a consequent increase in R&D.[3] Trade then provides the same opportunities to an open economy as would an increase in country size to a closed economy.

There is also a relatively large literature documenting firms' joint decisions of export and innovation. For example, Aw, Roberts and Xu (2009) develop a single agent framework in which firms decide whether or not to enter the export market and whether or not to invest in R&D. They find an interaction between R&D and export status due to the tendency of high productivity plants to self select into exporting and R&D. While I do not model export

[1] Pavcnik (2002) estimates the trade gains for the Chilean industry and De Loecker (2007) for the Belgian textile industry.

[2] The industrial organization literature has for a long time recognized that market structure depends on market size. This dependence can be due to the competition effects induced in larger markets (Bresnahan and Reiss (1991)). Melitz and Ottaviano (2008) introduce endogenous markups in a trade model to rationalize market size effects.

[3] Sutton (1991, 1998) explains how in the presence of sunk costs, market structure does not become fragmented with increases in market size; on average, firms become larger as market size increases.

decisions separately from internal sales, I would expect similar size effects on export decisions and innovation. Finally, there is some evidence for "learning-by-exporting" with productivity improvements following export market participation (see the survey in Lopez (2005)). It is not clear how we could disentangle the effects of R&D from pure learning. In a certain way, learning can be regarded as a form of R&D if it involves dedicated effort. In this case, learning and R&D are similar and both lead to higher productivity.

In setting up the model, I bring together the literature of endogenous innovation and investment. It is by now commonly accepted that in most industries innovating firms are more productive, larger and less likely to exit. Klette and Kortum (2004) propose an elegant general equilibrium model of innovation which can explain these stylized facts about R&D and industry dynamics. I build on their model and extend it to allow for physical capital investment. Such extension provides a better fit for the empirical analysis but comes at the cost of loosing analytical solutions.[4]

The investment literature has considered decisions in which investment is lumpy. Authors have attempted to explain this investment lumpiness with non-convex adjustment costs such as irreversibilities and fixed costs (see Cooper and Haltiwanger (2006)). Besides providing a good fit to the microevidence, such costs can rationalize the large persistence in size, as well as the investment reaction to risk and volatility.

The model is specified as to capture some of the empirical facts. Size is a considerable determinant of R&D decisions, and innovations resulting from R&D expenditures later translate into productivity increases.[5] The decision to export depends on both firm size and productivity. For example, capital stock and productivity explain 15% and 8% of the variation in exporting behavior. Finally, a high persistence in capital and productivity motivates the use of a dynamic model.

My model is similar in spirit to the industry equilibrium models of Hopenhayn (1992) and Ericson and Pakes (1995). I introduce incomplete information and assume that individual players do not observe the state or actions of their competitors. Players' strategies and beliefs are Markovian and depend on payoff relevant states. This assumption allows us to break the "curse of dimensionality" and solve the model with a large number of players. Estimation is done in three steps using the method developed by Bajari, Benkard and Levin (2008), henceforth BBL. In the first step productivity is recovered using production function estimation methods. In the second step, payoff, policy and transition functions are estimated. Finally, in

[4] I use a variation of their model. Similarly to Melitz (2003) I use productivity as the measure of knowledge whereas Klette and Kortum (2004) use the number of products. Using the number of products together with assuming a monopoly market for each product, has some benefits in their case since it allows the return function to be linear in the knowledge stock. Linearity simplifies the dynamic problem with a value function that is also linear in knowledge stock and can be characterized analytically. A second difference from their work is the introduction of discrete innovation decisions.

[5] Productivity here is broadly defined as either total factor productivity or price-cost margins.

the third step I search for parameters to rationalize the estimated policy functions as optimal. The available data determines the parametrization and the structural elements that can be identified. Although the model is quite general, here I adopt a specific parametrization for empirical purposes. In the last section, I evaluate the impact of changes in (i) market size, (ii) sunk costs of R&D, and (iii) entry costs on investment, productivity and market structure. The results confirm that average firm size and R&D are lower with either a smaller market size or larger sunk costs.

One issue I do not address is the existence of persistent unobserved heterogeneity. The first reason I avoid this problem is that estimated productivity captures part of the unobserved heterogeneity. The second reason is that persistence in marginal costs or benefits for investment or R&D is a difficult topic to address in the estimation of structural models. Because the models used here are nonlinear, it seems that dealing with persistent unobserved heterogeneity can only be done with sufficiently large time series.

2 The moulds industry

The Portuguese moulds industry is an interesting success case. The ability to develop itself even in the absence of a national market and the fact that most of the production has always been exported illustrates the importance of foreign markets. Between 1994 and 2003 the growth in exports was mainly to other European countries in detriment of the US, traditionally the larger market (Table I). After 2001 the industry stabilized and 90% of total production is currently exported (out of which 72% to the automotive industry).

Rank	1970	1980	1985	1990	1995	2000	2003
1	USA	USA	USA	USA	USA	France	Germany
2	UK	UK	UK	France	France	USA	France
3	W. Germ.	Sweden	Russia	Germany	Germany	Germany	Spain
4	Canada	Mexico	Israel	UK	UK	Spain	USA
5	Venez.	W. Germ.	Venez.	Holland	Holland	UK	UK
6	Nd	France	France	Spain	Israel	Sweden	Sweden
7	Nd	Holland	Holland	Sweden	Belg./Lux.	Holland	Holland
8	Nd	Venez.	Sweden	Israel	Sweden	Israel	Romania
9	Nd	Spain	Spain	Belg./Lux.	Brazil	Belg./Lux.	Switzerland

Source: CEFAMOL (2008)

Table I: Ranking of export destinations.

Portuguese mouldmakers benefit from a good international reputation. A report by the US international trade commission (USITC (2002)) emphasizes the fast delivery, technology, quality and competitive price as their main strengths.

> *"Despite Portugal's small size, it has emerged as one of the world's leading exporters of industrial molds. In 2001, despite limited production of dies, Portugal was the*

eighth largest producer of dies and molds in the world and it exports to more than 70 countries. The Portuguese TDM (Tools, Dies and Moulds) industry's success in exporting, and in adoption of the latest computer technologies, has occurred despite the fact that Portugal has a small industrial base on which the TDM industry can depend. Since joining the EU in 1986, Portugal has focused on serving customers in the common market." (USITC (2002)

Entering the *European Economic Community* in 1986 and the establishment of the *Common European Market* in 1993 were two important factors driving the observed growth. With such events, firms were able to reach important European producers and later increase R&D and innovation. Beira et al. (2003) and IAPMEI (2006) document how the close collaboration with car makers constituted a strong push for the development of new processes and products, normally in the form of research projects. These research projects are typically implemented over a given time frame and have well defined targets and objectives. Projects have to be completed, and upon completion the gains become permanent. This type of project-based R&D motivates the sunk cost specification. An alternative current expenditures, or fixed costs, approach would be better suited for industries where instead of projects, R&D is performed continuously (for example the chemical or pharmaceutical industries). Such fixed cost specification would not rationalize the fact that expenditures occur in spans. The data in Tables II and III support these claims. R&D spans last on average 2.5 years and most firms report uninterrupted R&D over the R&D span.[6]

		$R_{i,t+1}$	
		0	1
R_{it}	0	93%	7%
	1	45%	55%

Table II: R&D transition probabilities.

R&D spans								
Number of years with positive reported R&D	1	2	3	4	5	6	7	8
Number of firms	25	12	6	6	2	5	2	1

Table III: Size of R&D spans.

Brief industry history

The history of the industry dates back to the 1930's and 1940's when the development of plastics created a demand for moulds. The Portuguese moulds industry grew in the late

[6] Only 16 firms out of 59 interrupt R&D and of these, eight interrupt it for one year, seven for two years and one for five years. The transition probability into R&D is 7.4% and the transition probability out of R&D is 45%.

1950's as a major supplier of moulds for the glass (where it inherited some of its expertise) but mainly for the toy industry. In the late 1980's the production started shifting from toy manufacturing towards the growing industries of automobiles and packaging. During the 1990's the largest markets shifted from the US to France, Germany and Spain (Table I, IAPMEI (2006)).

The industry underwent several changes with the introduction of new technologies (*e.g.* CAD, CAM, Complex process, In-mould Assembling) and an increasing importance of innovation and R&D. For example, the technology used in computer operated machines radically changed from the 1980's to the 1990's. State of the art machinery allows flexibility at a low cost. It also allows a close collaboration with the client in the design phase. Design teams can work closely with the engineers and produce 3D virtual versions of the mould which is produced in the final phase. Controlling the technology was a major requirement for car-manufacturers and an advantage of Portuguese producers.

2.1 Sample description

The sample is part of the *Central de Balanços* compiled by Portuguese Central Bank for the period between 1994 to 2003. It contains financial information (balance sheet and P&L) together with other characterization variables: number of workers, total exports, R&D, founding year and current status (*e.g.* operating, bankrupt, etc). The five-digit industry code (NACE, revision 1.1) is 29563. Industry aggregate variables for sales, number of firms, employment and value added are collected from the Portuguese National Statistics Office (INE (2007)) and industry prices from IAPMEI (2006). The Appendix provides a description of the sample and variable construction.

There are 1,290 observations for 231 firms, out of which 265 observations with positive R&D corresponding to 59 firms and 49 R&D start-ups (Table A.II). The average firm in the sample sells goods worth 1.5 million Euros and employs 32 workers with an average labor productivity of 20,381 Euros. The industry is populated by many small and medium firms with no market leaders. R&D firms are three times larger, 20% more productive and export more (Table A.I).

Stationarity Over the period, real sales grew at an average 8.9% per year and labor productivity at 5.7% (Tables A.III and A.I) raising some concerns over non-stationarity. Growth in average sales is mostly concentrated in the 1994 to 1998 period with a stabilization after 1998 occurring due to the increase in the number of firms (Figure A.1). The growth and stabilization are consistent with a structural shock occurring somewhere before 1994 with the observed years of 1994 to 1998 being a period of transitional dynamics. This structural shock was the European economic integration that culminated with the establishment of the *Common Market* in 1993. Thus, I will abstract from any non-stationarity concerns.

3 The framework

I will start by outlining the model to be estimated and describe the elements of the general case together with a characterization of the equilibrium in the Technical Appendix. The framework captures most of the industry features described above, namely, the high investment rates and the increase in Research and Development. Firms can increase size by investing in physical capital or increase productivity by engaging in R&D activities. R&D and innovation takes the form of projects, implemented over a certain period. Since the projects cannot be undone, the cost of improving productivity is sunk and cannot be recovered. The firms can also enter the market by paying an entry fee and exit the market and collect a scrap value. To close the model we have to define an equilibrium for the dynamic game played by the firms. Since the industry is very fragmented (largest market share is below 10% in any given year), I will abstract from strategic interactions and summarize industry competition by an aggregate state. Firms form rational expectations about how competition will look like in the future. In equilibrium, the expectations (beliefs) should match actual behavior. Aggregate uncertainty leads to a stochastic equilibrium.[7]

The three features that distinguish this from previous work are the introduction of size (capital), endogenous productivity and aggregate uncertainty. Changes in aggregate uncertainty generate investment sensitivity. Having access to larger markets reduces uncertainty and this will trigger the size effects induced by endogenous capital choices. Finally, changes in size will influence R&D choices and consequently productivity.

Notice that the unimportant internal market allows me to ignore import competition and abstract from separately modeling export decisions and sales. Since size was a necessary condition for the introduction of innovations to be profitable, the increase in demand generated by the trade shocks can explain the large investment rates observed.

3.1 States and actions

Each firm is defined along four dimensions. The size represented by the capital stock ($K \in \mathcal{K}$), its productivity level ($\omega \in \Omega$), whether it is an R&D firm ($R \in \{0,1\}$) and whether it is an incumbent or a potential entrant ($\chi \in \{0,1\}$). We can now define the state of firm i in period t, s_{it}

$$s_{it} = (K_{it}, \omega_{it}, R_{it}, \chi_{it})$$

There is an aggregate state S_t that summarizes the state of the industry and is equal to the average industry deflated sales ($\tilde{Y}_t$ is average industry sales and $\tilde{P}_t$ is the average industry price)

[7] The models of Hopenhayn (1992) or Melitz (2003) remove any aggregate uncertainty to obtain a deterministic equilibrium.

$$S_t = \frac{\tilde{Y}_t}{\tilde{P}_t}$$

To fit the model statistically I specify unobserved heterogeneity. Payoff shocks are privately observed by the firm, unobserved by the econometrician and include shocks to investment costs ε_{it}^{I} , to the sunk cost ε_{it}^{RD}, to the scrap value ε_{it}^{scrap}, and to profits ε_{it}^{π}. The distribution of these shocks is nonparametrically unidentified (see section 4.4). The shocks also have to satisfy conditional independence (Rust (1994)). Thus, I will assume the vector of payoff shocks $\boldsymbol{\varepsilon}_{it} = (\varepsilon^{I}, \varepsilon^{RD}, \varepsilon^{entry}, \varepsilon^{scrap}, \varepsilon_{it}^{\pi})$ are draw from a distribution, $F(\varepsilon)$.

Firms' available choices are the investment level ($I \in \mathbb{R}_+$) which will add to the current capital stock and generate growth, and R&D ($R \in \{0,1\}$) which will lead to productivity improvements. Firms can also decide to enter or exit ($\chi \in \{0,1\}$) from the industry. Summarize these choices in the vector of actions a_{it}

$$a_{it} = (I_{i,t+1}, R_{i,t+1}, \chi_{i,t+1})$$

For convenience I will sometimes use the short notations for the vector of states and actions (s_{it}, a_{it}) and in other occasions I use the actual states and actions.

3.2 State transition

Individual states evolve over time with a controlled first order Markov transition. Most naturally, the evolution of capital is deterministic while productivity evolves stochastically.

Productivity R&D firms have better prospects for their productivity than non-R&D firms (in a probabilistic sense). Unless firm level prices are observed, we cannot separate "true" productivity from price margins. In the absence of observed firm level prices, we have to use a broad definition for productivity.

The *internal* source of uncertainty distinguishes R&D investment from other decisions like capital investment, labor hiring, entry and exit which have deterministic outcomes and where the sources of uncertainty are *external* to the company (*e.g.* due to the environment, competition, demand, etc.). Productivity follows a controlled first order Markov process.

$$\begin{aligned} \omega_{i,t+1} &= E(\omega_{i,t+1}|\omega_{it}, R_{it}) + \nu_{i,t+1}^{\omega} \qquad (1) \\ &= P^{\cdot,\omega}(\omega_{it}, R_{it}) + \nu_{i,t+1}^{\omega} \end{aligned}$$

where v_{it}^{ω} is independently and identically distributed across firms and time and $P^{\cdot,\omega}(.)$ is a parametric function used to approximate the conditional expectation function. The transition for individual productivity is estimated separately for R&D and non-R&D firms and several functional forms for $P^{\cdot,\omega}(.)$ will be reported, including polynomial function of order n.

Capital stock The capital stock depreciates at rate δ and investment adds to the stock with transition

$$K_{i,t+1} = (1-\delta)K_{it} + I_{i,t+1}$$

R&D R&D is an absorbing state and the sunk cost is paid only once. A non-R&D firm ($R_{it} = 0$) can decide to do R&D ($R_{is} = 1$) and this will remain for all $s \geq t+1$. R&D expenditures can be seen as a technological upgrading with a permanent effect. I can relax this assumption and allow an R&D transition where firms can switch in and out of R&D. In the sample, firms rarely switch in and out of R&D (see Tables II and III) and to relax the absorbing state assumption I would need a longer time series. Furthermore, we are interested in understanding the impact of trade liberalization on R&D behavior and the permanent shift is sufficient to capture these effects.

Entry/exit A potential entrant can enter and an incumbent remain in the market, $\chi_{i,t+1} = 1$. Alternatively, an incumbent can exit and a potential entrant can stay out, $\chi_{i,t+1} = 0$. Exiting or staying out are also absorbing states.

3.3 Payoffs

Firms compete with the other firms to sell their products. The profits obtained are the solution to the static pricing game. As mentioned, the industry is very fragmented so, let competition be summarized by the aggregate state, S_t. The profit function, π, is bounded and additively separable in the payoff shocks

$$\pi(s_{it}, S_t, a_{it}, \boldsymbol{\varepsilon}_{it}) = \pi(s_{it}, S_t, a_{it}) + \boldsymbol{\varepsilon}_{it}(a_{it}) \tag{2}$$

There are several adjustment costs to consider.

Investment cost Lumpy investment at the plant level is well documented. It can be rationalized by the existence of strong non-convex adjustment costs and irreversibilities. However, the disinvestment reported in the sample (at book value) contains too much noise and there are few (39) observations with zero investment. Thus, there is insuficient variation in the data to allow identification of a very flexible adjustment cost function and I will adopt quadratic adjustment costs with total irreversibility

$$C^K(I_{it}, K_{it-1}) = \left[\mu_1 I_{it} + \mu_2 \frac{I_{it}^2}{K_{it-1}}\right] + \varepsilon_{i,t-1}^I I_{it} \quad \text{if} \quad I_{it} \geq 0 \tag{3}$$

where μ_2 indexes the degree of convexity. Total irreversibility is sufficient to create an option value for investment which will generate the market size effects. This simple parametrization still captures the important investment features: option value of investment and periods of inaction. I also report estimates for the model where the quadratic term is set to zero and the results show that the estimates of the remaining parameters are not affected. A more general form for the investment function is possible at the cost of increasing the number of parameters to estimate.

R&D costs R&D decisions take the form of projects implemented over a given period - technological upgrading, catchup innovation or introduction of new technologies. If the whole project is not implemented the returns are likely to be small while they become permanent upon completion. Sunk costs are a good first order approximation to the underlying R&D cost structure. We can write them as $\lambda+\ \varepsilon_{it}^{RD}$ where λ is the average sunk cost and ε_{it}^{RD} is a private cost shock. Not modeling current expenditures (variable or fixed costs) allows me to avoid considering an extra policy function. As mentioned above, the evidence supports a sunk cost specification.[8]

Entry cost Potential entrants are short lived and cannot delay entry. Upon entry they must pay a (privately observed) sunk entry fee of $E+\varepsilon_{it}^{entry}$ to get a productivity/capital draw from the distribution $p^{E}(\omega_{t+1},K_{t+1}|\chi_t=0)$. Since entry effects are captured in the equilibrium transition for the aggregate state, it is not necessary to model entry for the estimation. When the equilibrium is calculated (as in the analysis of counterfactuals), the entry process will be explicitly modeled.

Exit value Every period the firm has the option of exiting the industry and receive a scrap value of $e+\varepsilon_{it}^{scrap}$. Changes to how exit is modeled have negligible effects to the value function and estimates of investment and sunk costs. Furthermore, the estimates for the exit value are likely to be imprecise since there is little variation in observed exit.

Payoff function Let $\theta=(\mu_1,\mu_2,\lambda,e)$ be the vector of cost parameters. Using the specified cost structure the per period return function of an incumbent is

$$\begin{aligned} & \pi(\omega_{it},K_{it},R_{it},S_t,R_{i,t+1},\chi_{i,t+1},I_{i,t+1},\boldsymbol{\varepsilon}_{it};\alpha,\theta)= \\ = \quad & \tilde{\pi}(\omega_{it},K_{it},S_t,\alpha,\varepsilon^{\pi})+\varepsilon_{it}^{\pi}-\mu_1 I_{i,t+1}-\mu_2\frac{I_{i,t+1}^2}{K_{it}}-\varepsilon_{it}^{I}I_{i,t+1} \\ & -(\lambda+\varepsilon_{it}^{RD})(R_{i,t+1}-R_{it})R_{i,t+1}+(1-\chi_{i,t+1})(e+\varepsilon_{it}^{scrap}) \end{aligned} \tag{4}$$

[8] In principle, sunk, fixed and variable costs are identifiable. The sunk costs would be identified off the first time it was decided to do R&D, fixed costs would be identified off the switches in and out of R&D and finally the variable costs would be identified off the variations in observed R&D expenditures. In practice, this can only be achieved if there is sufficient variation in the R&D data and a long time series.

Where $\tilde{\pi}(.)$ is the gross profit function (gross of adjustment costs) and the payoff shocks enter additively (Rust (1994)). The two aggregate variables are market size ($\tilde{Y}$) which evolves exogenously and average industry price ($\tilde{P}$) which is determined endogenously.[9]

There are two options to specify the gross profit function, $\tilde{\pi}(.)$. First we could parametrize demand and the production function and write the reduced form sales as the solution to the static pricing game. Examples that are consistent with the aggregate state specification are monopolistic competition or symmetric Cournot competition. Alternatively, if gross profits are observed, the profit function can be left unspecified and estimated directly. The second route is followed here. Reported cash flows are used to estimate the profit function. Since cash flows can take negative values, I allow for fixed production costs

$$\tilde{\pi}(\omega_{it}, K_{it}, R_{it}, S_t, \alpha) = \alpha_0 e^{\alpha_1 \omega_{it}} K_{it}^{\alpha_2} S_t^{\alpha_3} + \alpha_4 + \alpha_5 K_{it} + \alpha_6 R_{it} \tag{5}$$

where the vector of parameters, $\alpha = (\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)$ are estimated by nonlinear least squares.

3.4 Value function

We can finally write down the dynamic problem faced by an individual firm. Players choose actions to maximize the discounted sum of profits

$$\begin{aligned} V(s_{it}, S_t) &= \max_{\{a_{is}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \pi(s_{is}, S_s, a_{is}, \varepsilon_{is}) \\ &= \max_{a_{it}} \pi(s_{it}, S_t, a_{it}, \varepsilon_{it}) + \beta E_t^{a_{it}} V(s_{i,t+1}, S_{t+1}) \end{aligned}$$

where the expected continuation value is integrated over the possible values of the state variables conditional on the current state and actions

$$E_t^{a_{it}} V(s_{i,t+1}, S_{t+1}) = \int_{s_{i,t+1}} \int_{S_{t+1}} V_{i,t+1} p(a_{it}, s_{it}, s_{i,t+1}) q(s_{it}, S_t, S_{t+1}) ds_{i,t+1} dS_{t+1}$$

the transition $p(a_{it}, s_{it}, s_{i,t+1})$ is the probability of reaching state $s_{i,t+1}$ from state s_{it} when action a_{it} is chosen, and the function $q(s_{it}, S_t, S_{t+1})$ is the probability of reaching state S_{t+1} from state (s_{it}, S_t). The individual transition ($p(.)$) is a primitive of the model assumed known to all players, whereas the industry state transition ($q(.)$) represents the beliefs about the evolution of the industry state.

[9] Individual prices are determined in the static pricing game. Individual pricing strategies map individual states into the price space ($P_i^* = P(\omega_i, K_i, \tilde{P}, \tilde{Y})$). The aggregate state then puts all individual prices together $S_t = \frac{\tilde{Y}_t}{\tilde{P}_t(\omega, K, R, \tilde{Y})}$.

Equilibrium In equilibrium, beliefs $q(s_{it}, S_t, S_{t+1})$ are consistent with optimal behavior (see the Technical Appendix) . The (stochastic) *equilibrium* transition for the aggregate state is $q^*(s_{it}, S_t, S_{t+1})$.

4 The estimation procedure

The estimation is processed in three steps. In the first step, I estimate unobserved productivity using several alternatives. In the second step I estimate the parameters, α, of profit function ($\tilde{\pi}(\omega_{it}, K_{it}, R_{it}, S_t, \alpha)$), the firm-level and industry-level state transitions, ($p(\omega_{i,t+1}|\omega_{it}, R_{it}, \chi_{it})$ and $q(S_t, S_{t+1})$) and the equilibrium policy functions for investment ($I(\omega_{it}, K_{it}, R_{it}, S_t)$), R&D ($R(\omega_{it}, K_{it}, R_{it}, S_t)$) and exit ($\chi(\omega_{it}, K_{it}, R_{it}, S_t)$).[10] Finally, in the third step the dynamic parameters (μ_1, μ_2, λ, e) are recovered using the equilibrium conditions. Since the model has no analytical solution, I use computational methods.

4.1 Step 1: Productivity

Total factor productivity is an unobserved state variable that can be estimated as the residual from a production function.[11] To estimate the production function I use the method proposed by De Loecker (2007) that can be used in situations where revenues are observed instead of quantities and there is imperfect competition (see also Klette and Griliches (1995)). The method was developed for single agent models and I propose a slight extension that can be applied to equilibrium models. The difference in equilibrium models is that the input demand function depends on the industry state, more precisely, the aggregate industry state (see Appendix for a discussion A.2).[12]

4.2 Step 2: Policies and transitions

4.2.1 Policies

The policy functions are estimated with the observations for the state variables and actions. The optimal solution to the dynamic problem faced by an individual firm is the following investment function

$$\frac{I_{i,t+1}}{K_{it}} = \frac{1}{2\mu_2}\left(\beta\frac{\partial E(V(s_{i,t+1}, S_{t+1}|s_{it}, S_t))}{\partial I_{i,t+1}} - \mu_1\right) - \frac{1}{2\mu_2}\varepsilon_{it}^I$$

which I estimate separately for R&D and non-R&D firms using a flexible polynomial

[10] Assuming players own effect on the aggregate state is negligible we can write $q(s_{it}, S_t, S_{t+1}) = q(S_t, S_{t+1})$. This is the case when players are infinitesimal.

[11] Ackerberg et al. (2007) provide an excellent survey of the literature for estimating production functions.

[12] There is also the selection problem resulting from exit of less productive firms from the sample. Since the number of exits observed in the data is very small, the correction for selection is negligible.

$$\frac{I_{i,t+1}}{K_{it}} = P^{n,i}(\omega_{it}, K_{it}, S_t, R_{it}) - \frac{1}{2\mu_2}\varepsilon_{it}^I \tag{6}$$

A lower choice of n to approximate the polynomial function $P^{n,\cdot}(.)$ is preferred. Higher order polynomials can create large distortions and generate a poor approximation (e.g. Runge's phenomenon). This is more likely to occur in intervals of the data with less observations that are normally at the tails of the distribution. Since the estimation procedure averages over the states, the large distortions in the policy functions' estimates at the tails can create a large bias in the final (averaged) results. As a further check on the goodness of the approximations, I evaluate how well the estimated policies match the data by comparing the simulated predictions with the actual behavior.

The probability of doing R&D is

$$\begin{aligned}\Pr(R_{i,t+1} &= 1|R_{it} = 0, s_{it}, S_t) = \\ &\Phi\left(-\lambda + \beta \begin{bmatrix} E\{V(s_{i,t+1}, S_{t+1})|R_{i,t+1} = 1\} \\ -E\{V(s_{i,t+1}, S_{t+1})|R_{i,t+1} = 0\} \end{bmatrix}\right)\end{aligned}$$

which I approximate by

$$\Pr(R_{i,t+1} = 1|R_{it} = 0) = \Phi\left(P^{n,rd}(\omega_{it}, K_{it}, S_t, R_{it} = 0)\right) \tag{7}$$

The same argument in favor of lower order polynomials is in place here. Finally, the exit function is treated similarly

$$\Pr(\chi_{i,t+1} = 0|\chi_{it} = 1) = \Phi\left(P^{n,\chi}(\omega_{it}, K_{it}, S_t, R_{it})\right) \tag{8}$$

Equation (6) is estimated by OLS and equations (7) and (8) by maximum likelihood.

4.2.2 The transition function

There are two stochastic transition functions to consider: productivity and aggregate state. The productivity transition is a primitive of the model, specified in equation (1) and can be estimated by OLS. The transition for the aggregate state is an equilibrium object resulting from optimal behavior but can be estimated directly from the data. I parametrize it as

$$\ln(S_{t+1}) = \mu_{SS} + \rho_S \ln(S_t) + \nu_{t+1}^S \tag{9}$$

where ν_{t+1}^S is a zero mean normal random variable with variance $\sigma_{SS}^2 = \sigma_S^2(1 - \rho_S^2)$. The variance of the aggregate state represents the aggregate uncertainty affecting investment. The intercept is $\mu_{SS} = (1 - \rho_S)\mu_S$, and $(\mu_S, \sigma_S, \rho_S)$ are the unconditional mean, variance and autocorrelation for the $\ln(S)$ process.

4.3 Step 3: Minimum distance estimator

Using the estimated profits, policies and transition functions, I now recover the cost parameters as follows:

1. Choose n_s different starting values $\{s_{it}^{(j)}, S_t^{(j)}\}_{j=1}^{n_S}$ for $t = 0$. For example the starting values can be the observations in the sample;

2. For each value $(s_{it}^{(j)}, S_t^{(j)})$, draw a vector of payoff shocks, $\boldsymbol{\varepsilon}_{it} = (\varepsilon_{it}^{I}, \varepsilon_{it}^{RD}, \varepsilon_{it}^{scrap}, \varepsilon_{it}^{\pi})$.

3. Calculate actions $(a_{it}^{(j)})$ at each state $(s_{it}^{(j)}, S_t^{(j)})$ using the estimated policy functions and the payoff shocks drawn in 2.;

4. Draw shocks to productivity $(\nu_{i,t+1}^{\omega})$ and to the aggregate state (ν_{t+1}^{S}) and update states $(s_{i,t+1}^{(j)}, S_{t+1}^{(j)})$ using the shocks drawn, the estimated transition functions and the actions calculated in 3.;

5. Repeat steps 2. to 4. for $\bar{T}$ periods, and construct a sequence of actions and states: $\{a_{it}(s_{i0}^{(j)}, S_0^{(j)}), s_{it}(s_{i0}^{(j)}, S_0^{(j)}), S_t(S_{i0}^{(j)})\}_{t=1}^{\bar{T}}$. Use this sequence to calculate the discounted stream of profits for a given parameter vector θ and for each $j = 1, .., n_s$:

$$V^{(j,m)}(s_{i0}^{(j)}, S_0^{(j)}) = \sum_{t=0}^{\bar{T}} \beta^t \pi(a_{it}, s_{it}, S_t, \boldsymbol{\varepsilon}_{it}; \hat{\alpha}, \hat{P}^n, \theta)$$

6. Repeat steps 2. to 5. for $m = 1, ..., n_J$ times and calculate an average estimate for the expected value at each (j) state:

$$\widehat{EV}(s_{i0}^{(j)}, S_0^{(j)}; \tilde{\alpha}, \theta) = \frac{1}{n_J} \sum_{m=1}^{n_J} V^{(j,m)}(s_{i0}^{(j)}, S_0^{(j)})$$

The equilibrium conditions imply that at equilibrium beliefs, $q^*(.)$, strategy $a(.)$ is an

equilibrium if for all $a' \neq a$ the following condition holds:

$$V(s_{i0}, S_0; a, q^*(S_{t+1}|S_t); \theta) \geq V(s_{i0}, S_0; a', q^*(S_{t+1}|S_t); \theta)$$

Given the linearity of the value function in the dynamic parameters we can write

$$V(s_{i0}, S_0; a, q^*(S_{t+1}|S_t); \theta) = W(s_{i0}, S_0; a, q^*(S_{t+1}|S_t)) * \theta$$

where $\theta = [1, \mu_1, \mu_2, \lambda, e]$, $W(s_{i0}, S_0; a, q^*(S_{t+1}|S_t)) = E_{a|s_{i0},S_0} \sum_{s=0}^{\infty} \beta^s w_{is}$ and $w_{is} = \left[\widetilde{\pi}(s_{is}, S_s; \alpha), I_{is}, I_{is}^2, \mathbf{1}(R_{is+1} = 1, R_{is} = 0), \mathbf{1}(\chi_{is+1} = 0, \chi_{is} = 1)\right]$;[13]

7. Construct alternative investment, R&D and exit policies (a'_{it}). One possible way is to draw a random variable and add it to the estimated policies ($a' = \hat{a} + r.v.$). Using the non-optimal policies, calculate alternative expected values following steps 2. to 6.: $W(s_0, S_0; a', q^*(.))$. Do this for n_a different alternative policies;

8. Calculate the difference between the optimal and non-optimal value functions for each policy/state pair ($X_k, k = 1, ...n_I$), where $X_k = (a'_{it}, s_{i0}, S_0)$ and there are $n_I = n_a * n_s$ such pairs:

$$\hat{g}(X_k; \theta, \hat{\alpha}, \hat{P}^n) = \left[\hat{W}(s_{i0}, S_0; a, q(S_t, S_{t+1})) - \hat{W}(s_{i0}, S_0; a', q(S_t, S_{t+1}))\right] * \theta$$

Since estimated policies should be optimal, the expected value of using strategy a should not be smaller than using alternative a'. A violation of equilibrium conditions occurs when $\hat{g}(X_k, \theta, \hat{\alpha}, \hat{P}^n) < 0$. The empirical minimum difference estimator minimizes[14] the squared violations of these equilibrium conditions

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \hat{J}(\theta; \tilde{\alpha}) = \arg\min_{\theta \in \Theta} \frac{1}{n_I} \sum_{k=1}^{n_I} \left(\min\left\{\hat{g}(X_k; \theta, \hat{\alpha}, \hat{P}^n), 0\right\}\right)^2$$

I set the time of each path is at $\bar{T} = 75$, the discount factor at $\beta = 0.96$, the number of starting configurations is the total number of observations ($n_s = 1,017$), the number of simulations for each configuration $n_J = 200$ and the number of alternative policies $n_a = 125$, giving a total number of "differences", $n_I = 127,125$.

Choice of alternative policies The vector of cost parameters, θ, must rationalize the estimated strategy profile, $\hat{a}$. In general, θ can be point or set identified, depending on the model, available sample and alternative policies. Bajari et al. (2007) also propose an estimation method when the parameters are set identified.

The inequality in the objective function, $J(\theta)$, arises from comparing $\hat{a}$ with alternative policies, a'. The way alternative policies are chosen will influence estimation and identification.

[13] When the paid sunk cost is $\lambda + \varepsilon_{is}^{RD}$, the conditional expected value is $\widetilde{\lambda} = \lambda + E[\varepsilon_{is}^{RD}|R_{is+1} = 1, R_{is} = 0]$. The unconditional expected value is larger than the conditional and using linearity we can only recover the later since we do not know (and we cannot identify) the standard deviation of ε^{RD}. We can easily incorporate the conditional expected sunk cost in the policy simulations and the interpretation for the conclusions should also account for this, i.e., we identify the average cost paid by a firm that decided to do R&D. The same holds for the exit value.

[14] When the objective functions is not smooth (*e.g.* problems with discontinuous, non-differentiable, or stochastic objective functions) using derivative based methods might produce inaccurate solutions. Using derivative free methods (for example, the Nelder-Mead algorithm) to minimize the empirical minimum distance ($\hat{J}$) helps to circumvent these problems. Non-smooth functions occur with finite n_I, because of the min operator in the empirical objective function, $\hat{J}$. This can create discontinuities even when $g()$ is continuous in θ.

If we choose alternative policies very *far* from $\hat{a}$, the identified set increases while if we choose alternative policies very *close* to $\hat{a}$, the identified set shrinks. Since we can produce as many alternative policies as required, we can improve the estimation by choosing the alternative policies and artificially generating non-optimal observations.

This raises the question of how should we choose the alternative policies? One option is to add a slight perturbation to the estimated policy function. If the perturbation is positive, all alternative actions will be larger and the parameters are only identified in a one sided set. For example, if alternative R&D start-up decisions are more frequent (i.e. positive perturbations added to the optimal policy function), the alternative policies will generate high levels of R&D behavior. Firms will do more R&D than actually observed in the sample which can only be rationalized if the sunk costs are high (bounded below). However, the sunk costs are not bounded above unless we also add negative perturbations. I add zero-mean normally distributed errors to the investment, R&D and exit policies with standard deviations of 1, 0.5 and 0.5. This translates in a 95% probability for the alternative policies for investment to be in the interval $\pm 200\%$ from the estimated one. I also evaluate the sensitivity of the estimates to other choices of a'.

4.4 Identification

Identifying restrictions Assuming agent's optimal dynamic behavior, imposes no testable predictions. Without further restrictions, a given reduced form (observed) model can be rationalized by more than one parametric form for the structural model. The identification problem in dynamic models is well known (Rust (1994), Magnac and Thesmar (2002), Pesendorfer and Schmidt Dengler (2008) and Bajari et al. (2008)).

The unknown structural objects are the period returns, distribution for the shocks, state transition function and discount factor: $\pi(a_{it}, \mathbf{s}_t, \varepsilon_{it}), F(\varepsilon_{it}), p(s_{i,t+1}|s_{it}, a_{it}), \beta$. These objects cannot be separatelly identified. Pesendorfer and Schmidt Dengler (2008) extend the work of Magnac and Thesmar (2002) on single agent models to dynamic games and provide solutions to the identification problem. The solutions are to normalize the period returns for some outside alternative and to use exclusion restrictions (i.e. state variables that can be excluded from the period returns). Even so, without further restrictions the discount factor and distribution of cost shocks are nonparametrically unidentified. There are also other alternatives. One is the use of parametric restrictions on the return function . Another alternative is to estimate part of the return function directly when some measure of returns is observed.

Out of the four structural objects to be estimated, I use observed cash flows to estimate part of the return function ($\widetilde{\pi}(.;\alpha)$) in the second step. Assuming agents have rational expectations, I recover their beliefs by estimating the evolution for the states in the second step. The discount factor and the distribution of shocks ($\beta, F(\varepsilon)$) are set exogenously. The only object left to estimate is the cost function. This function takes the value of zero when there is inaction (no investment and no R&D start-up), $C(s_{it}, a_{it} = 0) = 0$ and is unafected by productivity and the aggregate state. Thus, the cost function satisfies both the normalization and exclusion restrictions.

Sample features The cost function is identified from simple sample features. The cost parameters are estimated to rationalize the observed choices given the observed returns and state transitions. For example, the estimated sunk costs compare the profits earned by the firms that decided to do R&D at a given state with the profits of the firms that decided not to do R&D. Had these costs been higher, we would have observed less R&D and had these costs been lower, we would have observed more R&D.

Beliefs Firms operate in a dynamic and uncertain environment and have to form beliefs about the future. In general, these beliefs are not known or even estimable by the econometrician. For this reason, I maintain the hypothesis of full rationality. However, this can be relaxed in some situations. For example, if firms use a forecasting method that is known to the econometrician, we can try to estimate the beliefs from the data. Full rationality is just one particular forecasting method that we can estimate from the data. Thus, any solution we attempt must assume beforehand what beliefs firms are adopting when doing their choices.[15]

5 Results

As explained above the estimation is done in three steps. An evaluation of the sensitivity to errors and bias in the profits, policies and transitions is also conducted. As a robustness check I also try alternative polynomials in the second step, different static profit functions and different productivity measures. Alternative specifications for the dynamic cost parameters are also reported. Overall, there is evidence of relatively large sunk costs of R&D.

5.1 First step

5.1.1 Productivity

Total factor productivity is calculated as the residual from a production function. The method is discussed in Appendix A.2. Three specifications for the transition function $E(\omega_{it}|\omega_{it-1}, R_{it-1})$ are reported: linear and cubic polynomial and sigmoidal function. All three yeld similar results and share the same relevant feature - higher productivity for R&D firms.

Firms will be willing to pay the sunk cost if they expect a gain in the future, in this case, higher productivity. Productivity for R&D firms is on average 26% higher than for no-R&D firms. (Figure 1)

[15] An alternative recently explored in Aradillas-Lopez and Tamer (2009) is the use of rationalizability to derive bounds on the parameters by using weaker concepts than full rationality. Fershtman and Pakes (2009) also provide some extensions to the rationality concept.

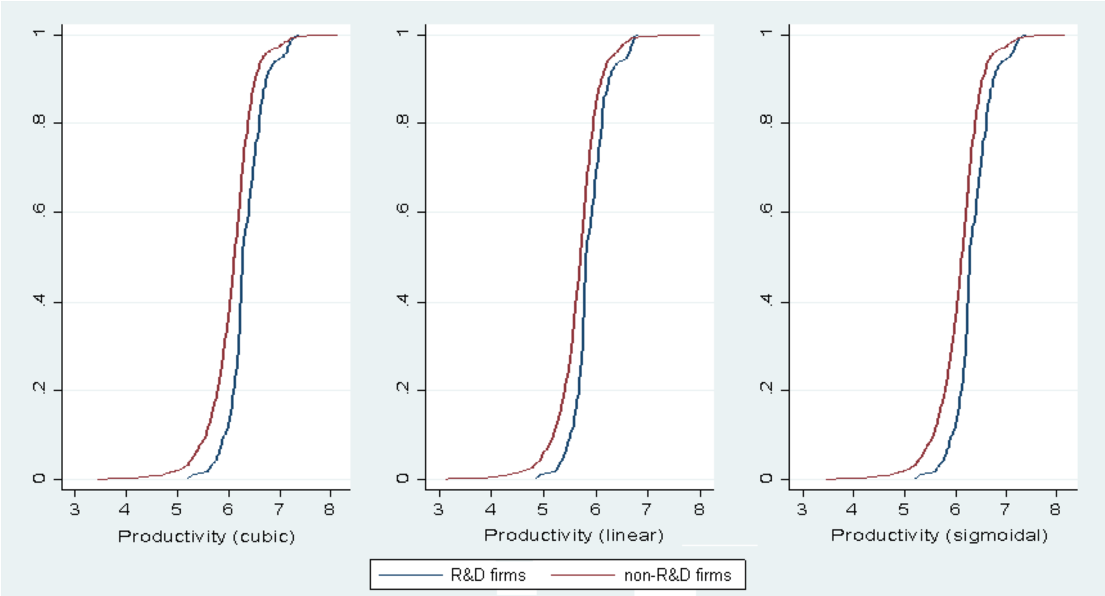

Figure 1: Productivity distribution (cubic, linear and sigmoidal approximation)

5.2 Second step

5.2.1 Static profits

	(i)		(ii)		(iii)		(iv)		(v)	
Dependent Variable: Cash flows $\tilde{\pi}_{it}$										
	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*
$\hat{\alpha}_0$	-5.03	*1.21*	-4.02	*1.08*	-6.44	*0.85*	-6.48	*0.08*	-6.79	*0.88*
$\hat{\alpha}_1$	0.95	*0.16*	0.80	*0.15*	1.22	*0.04*	1.23	*0.04*	1.25	*0.04*
$\hat{\alpha}_2$	0.74	*0.04*	0.76	*0.04*	0.64	*0.02*	0.65	*0.02*	0.66	*0.02*
$\hat{\alpha}_3$	0.14	*0.05*	0.13	*0.05*	0.19	*0.06*	0.19	*0.06*	0.18	*0.06*
$\hat{\alpha}_4^a$	-8.65	*13.47*	-7.19	*13.82*	-2.45	*0.14*			-5.94	*13.29*
$\hat{\alpha}_5$	-0.12	*0.08*	-1.98	*1.03*						
$\hat{\alpha}_6^a$	109.5	*23.3*							115.8	*23.1*
R^2	87%		87%		87%		87%		87%	

Notes: [a]Coefficients and standard errors divided by 1,000. Results for the profit function (equation (5)) using different specifications in columns (i) to (v).

Table IV: Reduced form profit function estimates.

The reduced form profit function (gross of adjustment costs) in equation (5) is estimated using the reported cash flows. Firms report average cash flows of 366 thousand Euros, ranging from -740 thousand to more than 14 million Euros.

The results of Table IV suggest that a 10% increase in physical capital translates in a 8% increase in profits, consistent with diminishing returns on capital (7% net increase in profits if we include the capital fixed cost component). A 10% increase in productivity leads to a 9% increase in profits (13% net increase in profits if we include the capital fixed cost component) while a 10% increase in average market size leads to a 2% increase in average profits. Finally, the fixed cost component is increasing for firms with larger capital stocks and a 100,000 Euros increase in capital stock leads to a 15,000 Euro increase in fixed costs. On average, R&D firms earn 105 thousand Euros more over the productivity gains.

Overall the results illustrate the importance of both physical capital and productivity as determinants of profitability. The fit is very good and over 85% of the total variation in profits is explained by the state variables (capital and productivity). Since both variables are quite persistent (see Table V), they characterize the persistence of profits and the dynamical system. The results are also robust across all specifications as reported in the remaining columns of Table IV.

5.2.2 Transition function

Aggregate state The aggregate state transition (9) is fully specified by three parameters: the mean, variance and autocorrelation of the aggregate state. The estimated sample values are $\hat{\mu}_S = 13.18$, $\hat{\sigma}_S = 0.28$ and $\hat{\rho}_S = 0.79$.

Productivity Using the productivity values I estimate equation (1) for the R&D and non R&D firms. Two parametric specifications are reported in Table V and the results with four specifications are plotted in Figure A.3. Two main features emerge: strong persistence in productivity across all specifications, and mean reversion. These two features are totally absent in models with static productivity and suggest that productivity adjusts slowly, yet it converges. Models with static productivity will thus overestimate the value of firms with high current productivity and underestimate the value of firms with low current productivity.

	(i)		(ii)		(iii)		(iv)	
Dependent Variable: Productivity ($\hat{\omega}_{it}$)								
	Non-R&D		R&D		Non-R&D		R&D	
	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*
$\hat{\omega}_{i,t-1}$	-3.55	*1.06*	-11.54	*5.91*	0.92	*0.01*	0.95	*0.02*
$\hat{\omega}^2_{i,t-1}$	0.72	*0.18*	1.92	*0.92*	0.48	*0.09*	0.32	*0.13*
$\hat{\omega}^3_{i,t-1}$	-0.04	*0.01*	-0.10	*0.05*				
const.	9.66	*2.05*	27.31	*12.58*				
$\hat{R}^2$	85%		93%		84%		93%	
Obs.	790		254		790		254	
Firms	197		59		197		59	
$\hat{s.e.}(\nu^\omega)$	0.19		0.11		0.20		0.12	

Notes: Results for the productivity transition using a 3rd order polynomial are reported in columns (i) and (ii) and for the linear case in columns (iii) and (iv).

Table V: Productivity transition estimates.

Evaluating the different approximations we conclude that the 6th order polynomial performs poorly at the tails of the distribution. For the R&D firms, the results for the linear and cubic polynomials are similar, while the sigmoid function is non-stationary in the region with low productivity. For the non-R&D firms, the 6th order polynomial is again non-stationary as well as the sigmoid function. The linear case does not give a good fit in the low productivity region while the cubic polynomial fits the data well both in terms of average fit ($\hat{R}^2 = 85\%$) and at the tails of the distribution.

5.2.3 Investment, R&D and Exit policies

Policy function estimates are reported in Table VI while Figures A.3 to A.4 in the Appendix report the fit and observations in a two-dimensional plane for productivity and capital. The R&D function in equation (7) is estimated with a probit model, and the investment function in equation (6) is estimated by ordinary least squares. Due to the limited number of observations, a linear probit is used for the exit policies. Again, the preference for low order polynomials is related with the poor fit of the high order polynomials in regions with few observations (particularly at the tails) and averages are very sensitive to incorrect predicted actions at the extremes. In some situations we would like the fitted functions to preserve basic properties, like monotonicity.

	(i)		(ii)		(iii)		(iv)		(v)	
Dep. Var.:	R&D				Investment				Exit	
Model:	Probit				OLS				Probit	
					Non R&D firms		R&D firms			
	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*
s_{t-1}	-0.07	*0.22*			-0.20	*0.16*	-0.09	*0.33*	0.07	*0.46*
$k_{i,t-1}$	2.09	*1.07*	2.08	*0.91*	-0.24	*0.55*	-1.28	*1.82*	0.00	*0.11*
$k_{i,t-1}^2$	-0.12	*0.05*	-0.07	*0.03*	0.00	*0.02*	0.24	*0.08*		
$\hat{\omega}_{i,t-1}$	-2.23	*2.53*	0.08	*0.22*	1.32	*1.65*	14.50	*6.28*	-0.51	*0.30*
$\hat{\omega}_{i,t-1}^2$	-0.01	*0.25*			-0.20	*0.13*	-0.35	*0.68*		
$\hat{\omega}_{i,t-1} * k_{i,t-1}$	0.18	*0.19*			0.14	*0.06*	-0.71	*0.36*		
Const.	-8.77	*9.05*	-16.76	*6.21*	4.25	*6.19*	-31.69	*17.54*	-0.39	*6.18*
$\hat{R}^2$	24.2%		23.9%		40%		55%		7.4%	
Obs.	838		838		801		204		1044	
Firms	212		212		208		51		223	

Notes: Results for the R&D start-up probit regression are reported in columns (i) and (ii) and for the investment OLS regressions in columns (iii) and (iv). Finally, results for the exit probit regression are reported in column (v).

Table VI: Policy function estimates: R&D, investment and exit.

Exit As expected, larger and more productive firms are less likely to exit. However, the small number of observations for exit make the results statistically insignificant. Alternatively we could fit an exogenous exit probability with aggregate moments. Doing so, would not account for selection due to exit and overestimate the value of firms with low productivity, and vice-versa. Even though this effect is on average small, the estimated parameters are preferred to an exogenous exit probability.

R&D probit There are strong and significant size effects. Small firms have a probability of doing R&D close to zero, which increases strongly with capital and levels out for large capital stocks (Figure A.2). The strong size effect supports evidence for the potential gains from trade. However, conditional on size there is no evidence that more productive firms are more likely to do R&D. The non significant productivity effect suggests the selection of more productive firms into R&D is not meaningful.

Investment For non-R&D firms, investment is increasing in both size and productivity. For R&D firms, the size effect is milder while the productivity effect is stronger. This suggests size-driven investment for non-R&D firms, up until the optimal level. Once the optimal level is reached and R&D is started, the size effect becomes less pronounced and productivity is the main driver of investment.

Summarizing, investment of non-R&D firms is strongly influenced by size and productivity. The R&D decision is affected by size but not by productivity. Finally, large and more productive firms are less likely to exit, although none of the coefficients in the exit probit is significant.

Implications The results suggest the following model. Relatively more productive firms start by investing in physical capital. Once they reach a certain size, R&D becomes a profitable investment. After R&D is done, productivity increases and since firms are now larger, investment *rates* decrease. The same mechanism is suggested by the descriptive statistics (Table A.I). Growth rates are larger for non-R&D firms (for sales, value added and labor productivity) and investment rates are lower for R&D firms. Furthermore, R&D firms are larger and more productive. Even though labor productivity *growth* is higher for non-R&D firms, labor productivity *level* is higher for the R&D firms. Taken together, these features characterize firm level dynamics and motivate the use of a dynamic model with productivity and capital investment.

5.3 Third step

The minimum distance estimator outlined above is now implemented to recover the dynamic parameters reported in the first row (model 1) of Table VII: the linear and quadratic investment cost, R&D sunk cost and exit value. Confidence intervals were constructed using the bootstrap.[16]

The values are estimated with the expected signs. Sunk costs are estimated at about 3.4 million Euros, almost two times the average sales and more than one year worth of sales for an

[16] Step 3 only produces simulation error. This error is reduced as the number of simulation draws increase ($n_J \to \infty$). Since bootstrapping requires intense computations, the number of simulations is kept within a manageable value and set $n_J = 125$. The bootstrapped confidence intervals will thus contain some noise from the simulations.

Parameters	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	$\hat{e}$	$\hat{F}$
Model 1 - baseline					
Coefs	1.08	3.02	3,369,600	1,606,800	-
5th percentile	*-5.29*	*0.05*	*-1,423,855*	*-5,163,630*	-
95th percentile	*1.66*	*48.73*	*16,657,150*	*9,788,270*	-
Model 2 - linear investment costs					
Coefs	1.61	-	3,896,300	993,400	-
5th percentile	*0.95*	-	*-3,105,840*	*-4,857,300*	-
95th percentile	*3.35*	-	*23,086,895*	*9,222,970*	-
Model 3 - operating fixed costs					
Coefs	1.61	-	3,740,000	110,450,200	-4,874,290
5th percentile	*0.95*	-	*-3,542,330*	*-692,556,855*	*-30,613,683*
95th percentile	*3.38*	-	*28,332,270*	*490,108,195*	*20,868,684*

Notes: Estimates for the cost parameters and bootstraped confidence intervals.

Table VII: Cost parameters estimates.

average R&D firm. Investment costs are increasing and convex but they are only statistically significant when we set the quadratic component to zero. The exit value is estimated at around 1.8 million Euros but due to the small number of exits this estimate is very imprecise. Alternative specifications where the quadratic investment cost term, μ_2, is dropped or a fixed operating cost, F, added are also reported in the second and third rows (models 2 and 3). Overall the precision of the linear cost improves when we drop the quadratic adjustment cost. The estimate of 1.6 Euros for each Euro of investment suggests indirect investment costs are about 60%.

When I introduce a fixed cost in model 3, the exit value becomes unrealistically large. This result nicely illustrates the importance of the identification conditions. In this case, the normalization condition is violated making it difficult to separately identify $\hat{F}$ and $\hat{e}$.[17] Estimating the fixed operating cost in the first step using observed profits is a better approach since the profits for the outside alternative (i.e. not exiting) are then normalized.

How reasonable are the estimated parameters? Doing a "back of the envelope" calculation we can compare average profits earned by an R&D firm in the period before it started doing R&D with the average profits earned afterwards. The difference is $230,000$ Euros. The value of this difference discounted over a 40 year horizon is 4.6 million Euros, slightly above the estimated sunk costs from the structural model. The slight upward bias might be due to a selection effect that is accounted and corrected in the structural model.

5.4 Robustness

As explained above, bias in the policy function estimates can create a bias in the third step. To evaluate how robust the results are, I estimate the model under alternative specifications for the discount factor, profit function and policies.

[17] The estimated values of $\hat{e} = 110,450,200$ of model 3 and $\widetilde{e} = 1,606,800$ of model 1 with $\beta = 0.96$ rationalize a "net" value (i.e. $(1-\beta)(\hat{e}-\widetilde{e}) = 4,353,736$) close to the fixed cost estimate, $\hat{F} = 4,874,290$. If identification is not possible, there will be an infinite combination of pairs $(\hat{F}, \hat{e})$ that can rationalize the observed decisions. For example, in a non-stochastic setting, a firm can decide to exit today and collect the exit value $\hat{e}$ or stay one more period and collect $\pi + \hat{F} + \beta\hat{e}$. For an indifferent firm $\pi + \hat{F} + \beta\hat{e} = \hat{e}$. In the absence of fixed costs (F), the indifference condition is $\pi + \beta\widetilde{e} = \widetilde{e}$. Replacing and solving the two conditions we get $\hat{e} - \widetilde{e} = \frac{1}{1-\beta}\hat{F}$.

Discount factor Lower discount factors reduce the continuation value and, as expected, decrease the estimated investment and sunk costs. For example, with a discount factor of 0.9 the sunk costs are estimated at 1.2 million Euros and the investment costs at 0.83 for the linear and 1.41 for the quadratic component.

β	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	$\hat{e}$
0.90	0.83	1.41	1,299,200	-574,500
0.91	0.87	1.58	1,477,800	-504,100
0.92	0.91	1.77	1,702,700	-368,300
0.93	0.95	2.00	1,989,500	-131,900
0.94	1.00	2.27	2,355,300	264,100
0.95	1.04	2.61	2,808,200	835,700
0.96	1.08	3.02	3,368,800	1,607,700
0.97	1.12	3.51	4,028,200	2,731,000
0.98	1.15	4.16	4,355,900	4,733,300
0.99	1.13	5.10	3,042,900	8,481,800

Table VIII: Cost parameters estimates for different discount factors.

Profit function Estimated dynamic cost parameters are also quite robust when the alternative reduced form profit functions from Table IV are used. Overall, the sunk costs are estimated between 1.1 and 3.7 million Euros while the investment costs are only marginally affected.

	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	$\hat{e}$
Baseline	1.08	3.02	3,369,600	1,606,800
$\alpha_6 = 0$	1.03	3.27	1,111,600	1,156,500
$\alpha_5 = \alpha_6 = 0$	1.16	2.77	1,423,500	1,922,300
$\alpha_4 = \alpha_5 = \alpha_6 = 0$	1.16	2.77	1,403,600	1,946,700
$\alpha_5 = 0$	1.17	2.70	3,672,700	1,979,000

Table IX: Cost parameters estimates for different specifications of the profit function.

Policy functions The policy functions play a central role in the estimation. Due to the nonlinearities, even small bias in the estimated policies can get magnified into a large bias in the structural parameters. To evaluate how sensitive the results are, I try simpler (linear) policies. Using a linear investment function, sunk cost are estimated at about 3 million Euros and investment costs at -1.98 for the linear and 29.09 for the quadratic component. Alternatively, using a linear probit for R&D, sunk cost are estimated at 4.5 million Euros and adjustment costs at 1.18 for the linear and 2.01 for the quadratic component. If we allow both the investment and R&D policies to be linear, the sunk cost is estimated at 2.9 millions.

Overall, the results are relatively robust to alternative specifications. A final validation exercise is to evaluate how well the estimated model matches the actual data.

	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	$\hat{e}$
Linear R&D probit	1.18	2.01	4,475,100	-992,000
Linear investment	-1.98	29.09	2,987,600	1,174,100
Linear R&D probit and investment	-1.64	26.98	2,920,100	1,489,900

Table X: Cost parameters estimates for different policy functions.

Validation How well does the model fit the evidence? As opposed to other estimation methods, the minimum distance estimator does not match the aggregate moments.[18] Thus, I can use the comparison between the actual data and the simulated aggregate moments as an exercise of cross validation to the model. Before progress is made I must set some parameters: the distribution for the entry cost, the productivity distribution of entrants, market size, wage rate and the standard deviation for the sunk costs. I also set exogenously the average exit value because its estimate is imprecise and too large to rationalize the evidence, generating too much exit.

The parameters are set as follows. Mean and variance for the entry costs are set together with the mean and variance for the exit value to fit the average number of firms and entry/exit rates. The values are 1.3 million Euros for the average entry cost and 750 thousand Euros for the exit value with standard deviations of 300 and 100 thousand. The mean and variance for the productivity of entrants is matched with the actual value from the sample: 5.8 and 0.35. Market size is 450 million Euros, similar to the market size in 2004 and the wage rate is 16,000 Euros, also in line with reported average industry wages. The standard deviation for sunk costs is chosen at 337,500 Euros, to match the 30% of R&D firms.

		Data		Simulated
Moments	All	Pre-'97	Post-'97	
Average number of firms	680	536	753	719
Average log-sales	13.18	12.94	13.30	13.11
Percentage of R&D firms	0.21	0.13	0.29	0.30
Average productivity	6.13	6.07	6.18	6.22
St. dev. of productivity	0.45	0.39	0.50	0.47
Average log-capital stock	12.77	12.43	13.00	12.93
St. dev. of log-capital	1.57	1.49	1.58	0.64
Capital-prod. correlation	0.46	0.42	0.47	0.42
*Average investment rate**	0.22	0.25	0.20	0.13
*St. dev. of invest. rate**	0.23	0.24	0.22	0.22
Entry rate (%)	3.70	5.12	2.30	1.7
Exit rate (%)	-	-	-	1.3

Notes: Sample moments pre and post 1997 and moments for the simulated sample using the estimated parameters.
*Conditional on investment rate smaller than 1.

Table XI: Sample and simulated moments.

Using the full set of parameters (estimated and exogenously set), I solve the model and sim-

[18] BBL also propose a method of moments estimator to reduce the bias in the estimates by averaging out first stage estimation error.

ulate the industry for 120 periods. To do the simulations I am now required to solve the model and calculate the new equilibrium industry evolution, $q(S_t, S_{t+1})$. The computational burden of solving a full dynamic game would be prohibitive while I can solve the proposed aggregate state model in a reasonable time.[19] After dropping the first 60 periods I calculate the moments for the stationary market structure, reported in Table XI: the mean and standard deviation for productivity, capital, and investment rate, and the correlation between productivity and capital. Also reported are the number of firms, aggregate state and entry rate but these values are not free since, as explained, they where "calibrated" by setting the distributions of entry and exit values.

The model can explain well the first and second moments for capital and productivity but generates too little variation in capital. The little variation in capital is probably because all firms are ex-ante identical and there is no source of persistent unobserved heterogeneity in the model. The model does not provide such a good fit for the investment rates. This is because we observe a high investment transition period. If we consider only the later years (after 2000), investment rates and variation decrease to 0.13 and 0.18, more in line with the predicted values. Finally, the model explains well the correlation between capital and productivity. Using these values as a benchmark, we can now evaluate the effects of some policy changes.

6 Counterfactual Experiments

This section reports the results for three policy experiments to assess the impact on industry R&D, productivity and investment. The experiments consist of changes in (i) market size, (ii) sunk costs of R&D, and (iii) entry costs. Because the model is stylized, the particular numbers generated by the counterfactual simulations should be seen as merely suggestive as the effects of any policy change might depend on other factors not captured by the model.[20]

Reduction in market size The goal is to access the models' predictions in case we return to the pre-trade liberalization era, by evaluating the effects of a reduction in market size. I can use changes in market size as a parallel for changes in trade barriers because the internal market is small and the industry has always been a heavy exporter. In particular, since 90% of total production is exported, I can abstract from import competition effects. One implicit assumption is that foreign competitors do not react to changes at the national level. Since Portugal is a small country, worldwide industry structure is unlikely to be affected by changes

[19] A MATLAB algorithm to solve the model is available from the author. Solving the model takes about 120 minutes on a 2.6 Ghz Phenom Quad-core computer with 3GB RAM.

[20] Also, the standard deviation of the payoff shocks is not identified and this object influences the magnitude (but not the sign) of the effects. This is particularly relevant for the sunk costs of R&D where we identify the average cost paid by a firm who decided to do R&D and not the average unconditional cost. A decrease in the conditional mean is stronger than an equivalent decrease in the unconditional mean.

at the national level. Thus, for the Portuguese producers the evolution of the world market can be taken as exogenous.

The results in Table XII suggest that a reduction in market size from 450 million Euros to the level in 1994 of 150 million Euros will cause a reduction in average R&D, capital and productivity. The figures predicted by the model are actually similar to the values observed in the sample for the early periods. Also, entry and exit rates increase and the correlation between capital and productivity decreases. More importantly, however, the number of firms does not decrease linearly with market size. Thus, firms get smaller as market size shrinks. This is the important size effect documented in other studies (*e.g.* Eaton, Kortum and Kramarz (2008)) that is not captured by the models of Melitz (2003) or Eaton and Kortum (2003).

Market size $(Y) : 10^6 EUR$	540	495	*450 base*	405	345	285	225	150	105
Average log-sales	13.15	13.13	*13.11*	13.08	13.06	13.02	12.97	12.92	12.85
Average number of firms	824	771	*719*	647	557	467	374	250	160
Percentage of R&D firms	0.32	0.31	*0.30*	0.26	0.25	0.22	0.19	0.14	0.09
Average productivity	6.23	6.23	*6.22*	6.22	6.21	6.21	6.19	6.17	6.14
St. dev. of productivity	0.47	0.47	*0.47*	0.47	0.47	0.47	0.47	0.47	0.47
Average log-capital stock	12.97	12.95	*12.93*	12.88	12.85	12.79	12.73	12.62	12.38
St. dev. of log-capital	0.63	0.64	*0.64*	0.63	0.65	0.65	0.66	0.73	0.79
Capital-prod. correlation	0.42	0.42	*0.42*	0.42	0.42	0.41	0.40	0.39	0.39
Average investment rate	0.13	0.13	*0.13*	0.13	0.12	0.12	0.11	0.09	0.06
St. dev. of invest. rate	0.23	0.23	*0.22*	0.21	0.22	0.22	0.21	0.23	0.26
Entry rate (%)	1.0	1.3	*1.7*	2.2	3.1	4.3	6.0	10.3	17.4
Exit rate (%)	0.7	1.0	*1.3*	1.8	2.7	3.8	5.5	9.8	17.1

Table XII: Predicted moments from changes in market size.

The mechanism causing the size effect illustrates the need of (i) adjustment costs for capital and (ii) aggregate uncertainty. The first justifies the need of a dynamic model and the second, the need of an equilibrium model with aggregate uncertainty. Models without aggregate uncertainty (and constant markups) predict market size neutrality.[21] As the simulated results show, this no longer holds in the general case. Aggregate uncertainty together with adjustment costs for capital create an option value for investment and generate two effects from changes in market size. First, as market size increases, uncertainty is reduced pushing investment up. The equilibrium average capital stock becomes larger which, in the presence of sunk costs, leads to an increase in R&D. Second, as market size increases more firms can fit in the market. In equilibrium, the increase in market size leads to an increase in the number of firms in the market, but because firms are on average larger, the increase is less than proportional. This generates the concave relation between market size and number of firms which is present in Table XII and has been well documented in the industrial organization literature. In the limit, if we allow firms to endogenously set the level of R&D, and productivity is increasing with R&D, we can

[21] An exception here is the work of Melitz and Ottaviano (2008) that allows for varying markups.

have a Sutton (1998) style effect where industry concentration is bounded below as market size increases.

Reduction in the sunk costs of R&D A reduction in sunk costs can also have effects on productivity and trade. An example of a policy to reduce the sunk costs is a direct subsidy to R&D start-up. Other, more general, policies could be more effective in reducing sunk costs. For example, a public R&D lab could explore economies of scale and avoid R&D duplication costs. The results in Table XIII show that a reduction in sunk costs is expected to lead to an increase in R&D, productivity, capital and investment. More importantly average sales are increasing which suggests that reducing the sunk costs of R&D can also promote firm exports. These effects are similar to an increase in market size and either of them is able to rationalize the observed industry change.

Sunk costs $(\lambda) : 10^6 EUR$	2.89	3.06	3.23	3.32	*3.40 base*	3.57	3.74	4.08	4.42
Average log-sales	13.23	13.19	13.13	13.14	*13.10*	13.06	13.07	13.06	13.05
Average number of firms	1000	991	893	792	*714*	621	576	563	552
Percentage of R&D firms	0.98	0.88	0.60	0.46	*0.29*	0.11	0.03	0.01	0.00
Average productivity	6.36	6.34	6.28	6.25	*6.22*	6.19	6.17	6.17	6.16
St. dev. of productivity	0.42	0.43	0.45	0.46	*0.46*	0.47	0.47	0.47	0.47
Average log-capital stock	13.44	13.36	13.14	13.06	*12.92*	12.80	12.76	12.74	12.73
St. dev. of log-capital	0.66	0.65	0.68	0.68	*0.63*	0.56	0.54	0.52	0.51
Capital-prod. correlation	0.39	0.40	0.44	0.44	*0.41*	0.37	0.35	0.34	0.33
Average investment rate	0.14	0.14	0.14	0.13	*0.13*	0.12	0.12	0.12	0.11
St. dev. of invest. rate	0.31	0.29	0.26	0.24	*0.22*	0.19	0.19	0.18	0.17
Entry rate (%)	0.0	0.1	0.6	1.2	*1.7*	2.5	2.9	3.0	3.2
Exit rate (%)	0.0	0.0	0.4	0.9	*1.3*	2.0	2.5	2.6	2.7

Table XIII: Predicted moments from changes in sunk costs of R&D.

Reducing the sunk costs only affects the value of non-R&D firms and leaves the value of R&D firms unchanged (excluding equilibrium effects). Changes in sunk costs affect the value function through three different mechanisms. First, smaller sunk costs cause a direct reduction in costs. Second, the probability of doing R&D increases with a decrease in sunk costs having an indirect effect and increasing expected benefits. Finally, there are the equilibrium effects that operates in the opposite way in the value function. As sunk costs decrease, the value of entering is increased and more firms enter the market leading to an increase in the number of firms. The decrease in sunk costs leads to an increase in R&D and productivity. As a consequence, firms invest more and become larger. Summarizing, a decrease in the sunk costs lead to more firms in equilibrium that are on average larger and more productive.

Increase in entry costs Lastly, there is often political support for the creation of large firms under the argument that this might spur innovation. One way to create large firms is by increasing entry costs. Large entry costs will decrease entry rates and protect incumbent firms from competition, allowing them to grow. Larger firms will then be willing to pay the sunk cost and start innovating.

As reported in Table XIV, an increase in entry costs boosts innovation by generating an increase in average size, investment and R&D. However, it also leads to a reduction in the number of firms and entry/exit rates and bad firms are now less likely to exit and be replaced by better firms. Overall, this selection effect might lead to a decrease in average industry productivity, specially if the boost in innovation is not sufficiently strong (or all firms are already innovating). As expected, the increase in entry costs protects firms and leads to an increase in average size. Alternatively, increasing entry costs during a trade liberalization period could mitigate the negative competition effects from the increase in entry costs, while still keeping the positive effects from trade liberalization.

Entry costs $(e) : 10^6 EUR$	0.98	1.04	1.17	*1.3* *base*	1.43	1.56	1.63	1.69
Average log-sales	12.94	12.98	13.02	*13.10*	13.19	13.26	13.32	13.36
Average number of firms	876	815	778	*713*	661	646	629	599
Percentage of R&D firms	0.16	0.21	0.25	*0.29*	0.37	0.46	0.51	0.57
Average productivity	6.19	6.21	6.22	*6.22*	6.24	6.25	6.26	6.27
St. dev. of productivity	0.47	0.47	0.47	*0.46*	0.46	0.46	0.45	0.44
Average log-capital stock	12.71	12.82	12.87	*12.92*	13.00	13.10	13.18	13.23
St. dev. of log-capital	0.68	0.64	0.62	*0.64*	0.65	0.67	0.70	0.70
Capital-prod. correlation	0.39	0.40	0.41	*0.41*	0.42	0.44	0.44	0.41
Average investment rate	0.10	0.12	0.12	*0.13*	0.13	0.14	0.14	0.14
St. dev. of invest. rate	0.22	0.22	0.21	*0.22*	0.23	0.26	0.28	0.27
Entry rate (%)	6.7	4.0	2.6	*1.7*	1.0	0.5	0.3	0.3
Exit rate (%)	6.5	3.8	2.3	*1.3*	0.5	0.1	0.1	0.0

Table XIV: Predicted moments from changes in entry costs.

7 Conclusion

The evidence suggests that technology is endogenous and size is an important explanation for the technology decision. I have proposed and estimated an equilibrium model with endogenous choices of technology and size. One disadvantage compared with simpler models is the lack of analytical solutions. I estimate the model for a sample of firms from the Portuguese moulds industry and find evidence of relatively large sunk costs of R&D. These costs are particularly relevant in this industry since firms are very small. Getting access to large markets allows the firms to exploit the economies of scale in R&D and introduce more innovations. Overall, sunk costs explain why R&D is done by larger firms and why successful firms invest in physical capital before doing R&D. The two results put together can explain the observed performance of some industries after trade liberalization episodes, like the Portuguese moulds industry.

There are some challenging questions left unanswered. First, how relevant are the sunk costs on a macro scale. The existence of sunk costs can be relevant on a macroeconomic level if they are important within each industry. Second, the model proposed here is partial equilibrium. A trade liberalization event will certainly trigger other general equilibrium effects on wages and returns on capital. How these effects affect the innovation decisions is a question to be considered. Third and finally, the model does not incorporate persistent unobserved heterogeneity either in returns or costs. Incorporating persistent heterogeneity in dynamic models is probably the most important question to be addressed in the future.

A Appendix

A.1 Sample construction and descriptive statistics

The sample comes from three sources: Aggregate variables (sales, value added, employment) come from the Portuguese National Statistics Office (INE); industry price deflators are collected from IAPMEI (2006); and the firm level sample was extracted from the Bank of Portugal database on firms across the economy (*Central de Balancos*, five digit NACE code industry 29563 - moulds industry).

Some notes on the *Central de Balancos*: The sample has been collected by the Central Bank since 1986. However, due to changes in accounting rules, it is only comparable from 1990. The data between 1990 and 1994 is not considered reliable. From 2000 the sampling method (simple random sampling) was changed to stratified sampling causing a drop in the number of observed (mostly smaller) firms in 1999 and 2000.

Representativeness: The sample is representative of the whole industry, in particular for the early periods. It covers 90% of total sales and industry employment in 1994 and the coverage decreases to a minimum of 50% of sales (40% of employment) in 2003. This reduction is mainly due to changes in the sampling procedure as explained above. There is an obvious gap in productivity trends between the sample and the industry (also total sales and employment). Labor productivity increased by roughly 60% in the sample and only 40% in the industry. None of the aggregate variables are calculated using the sample but come directly from the collected industry wide variables.

Variable construction:

Capital stock was calculated using the perpetual inventory method with a 8% depreciation rate

$$K_{i,t+1} = (1 - depreciation) * K_{it} + I_{i,t+1}$$

Value added is equal to sales subtracted from materials and external services expenditures

$$VA_{it} = Y_{it} - M_{it} - ESE_{it}$$

R&D dummy variable takes a value equal to one whenever positive R&D was reported in the past or present and zero otherwise.

Cash flow is constructed as the sum of accounting profits plus depreciation and amortization.

Both aggregate and individual sales and value added were deflated with the industry price deflator.

In 11 observations the number of workers reported was zero and were dropped.

There were 9 holes identified in the sample, i.e. firms that interrupt reporting for 1 or more consecutive years. Either the earlier or later periods are dropped, minimizing the total number of observations lost.

	Mean	Std. Dev.	Min	Max
All firms: 1274 observations				
Sales (EUR)	1,574,073	2,869,201	3,292	34,700,000
Exports (EUR)	891,333	2,483,554	0	31,800,000
Capital stock (EUR)	1,058,104	2,130,734	135	23,800,000
Employment	32	39	1	258
Labor productivity (EUR)	20,381	9,044	359	74,632
Investment rate	0.20	0.25	0.00	5.32
Sales growth	8.9%	34.5%	-195.8%	469.0%
Value added growth	9.4%	40.5%	-289.3%	477.0%
Labor productivity growth	5.7%	37.1%	-289.3%	284.3%
Non R&D firms: 1009 observations				
Sales (EUR)	1,198,854	2,321,233	3,292	26,800,000
Exports (EUR)	640,879	1,919,257	0	25,200,000
Capital stock (EUR)	835,706	1,854,294	135	20,600,000
Employment	27	35	1	230
Labor productivity (EUR)	19,609	9,178	359	74,632
Investment rate	20.9%	27.5%	0.0%	531.7%
Sales growth	9.9%	37.9%	-195.8%	469.0%
Value added growth	10.4%	45.2%	-289.3%	477.0%
Labor productivity growth	6.2%	41.1%	-289.3%	284.3%
R&D firms: 265 observations				
Sales (EUR)	3,002,735	4,066,477	99,206	34,700,000
Exports (EUR)	1,844,947	3,811,178	0	31,800,000
Capital stock (EUR)	1,904,897	2,802,605	53,161	23,800,000
Employment	52	45	3	258
Labor productivity (EUR)	23,321	7,861	7,148	59,923
Investment rate	16.7%	14.2%	0.0%	77.5%
Sales growth	5.6%	20.1%	-101.8%	123.3%
Value added growth	6.3%	19.6%	-113.3%	102.4%
Labor productivity growth	3.9%	19.9%	-87.2%	116.9%
R&D to sales ratio	0.9%	3.4%	0.0%	46.5%

Source: Central de Balanços, Bank of Portugal

Table A.I: Summary statistics.

There are few observations on entry and exit. Due to the way the data was collected some firms might not be reported in the sample but still be active in the industry complicating the identification of entry and exit. Firms could have been operating in the market before first appearing in the sample and they might still be operating after leaving the sample. The problem is addressed with two variables to identify entry and exit. For entry, firms report their founding year and I consider it to be an entry if the firm first appeared in the sample within a 2 year window from the reported founding year (values reported in Table A.II under the column "entry"). Regarding exit the central bank collects a variable for the "status" of the firm. The quality of the exit variable is poor and some firms might have closed down and still be reported as "active", so only a fraction of actual exits is captured. I was able to identify a total of 48 entries and 7 exits from the panel.

Year	*Number of firms*	*Number of non R&D firms*	*Number of R&D firms*	*R&D start-ups*	*Entry*	*Entry in the dataset*	*Exits*
1994	144	134	10	-	2	3	0
1995	157	137	20	10	12	14	2
1996	165	141	24	4	8	14	0
1997	170	145	25	2	11	20	2
1998	164	135	29	7	9	33	0
1999	136	108	28	3	2	46	1
2000	92	68	24	7	2	8	0
2001	88	56	32	9	1	5	0
2002	88	53	35	4	1	2	0
2003	86	48	38	3	0	0	2
Total	**1290**	**1025**	**265**	**49**	**48**	**145**	**7**

Table A.II: R&D, Entry and Exit: number of observations per year.

A.1.1 Aggregate State

Market definition: The market is defined as total worldwide demand for Portuguese moulds. The total worldwide demand would be a better definition but observations for total world sales are not available. The sample contains observations for total industry sales (national plus exports). One restriction added by this market definition is that Portuguese firms take as exogenous the evolution of foreign competition. Let Y^W be the world supply of moulds, Y^{NP} the world supply of foreign moulds and $Y = Y^W - Y^{NP}$ the Portuguese supply of moulds. The definition restricts foreign competition to be exogenous and Y^{NP} to evolve exogenously.

Aggregate state definition: The aggregate state is defined as average deflated total industry sales: $\frac{Y/N}{\widetilde{P}}$. It can be divided into three components. The first is total industry sales (Y) and can easily be assumed to evolve exogenously. As explained above, it is total demand for Portuguese moulds. The second variable is the industry price ($\widetilde{P}$) and being the solution to the static pricing game it evolves endogenously. The pricing strategies are a mapping from states onto the pricing space. Finally, the total number of incumbents (N) is also endogenous and it depends on market size. Modeling the three variables separately would involve taking into account (and estimating) all cross correlations.

In Figure A.1 I plot the evolution of all variables. The market was growing between 1993 and 2000 and stopped until 2003. The number of firms share a similar pattern. On the other side prices were increasing slightly and decreased in the later years. The industry grew substantially after 1994 due to the strong increase in demand for Portuguese moulds. We also observe an increase in labor productivity and R&D. The cross correlations are as expected with prices being negatively correlated with number of firms and market size, and the number of firms being positively correlated with market size. The evolution of the three variables is summarized by the evolution of the single index variable, $\frac{\widetilde{Y}}{P}$(average deflated sales).

	Number of firms	Production (EUR mio)	Exports (EUR mio)	Exports % of sales	Employment	Value Added (EUR mio)	Price (EUR/ton)
1994	644	171	132	77%	5,133	101	24.43
1995	570	193	151	78%	5,796	114	25.25
1996	452	244	191	78%	7,316	143	25.71
1997	477	293	220	75%	7,821	166	25.73
1998	461	322	232	72%	7,740	167	24.62
1999	549	362	250	69%	8,429	208	25.23
2000	604	412	277	67%	8,879	228	26.49
2001	612	421	328	78%	8,919	240	26.74
2002	722	378	310	82%	9,312	235	24.97
2003	738	403	303	75%	8,766	227	22.86
2004	1109	455	340	.	9,846	259	20.33
2005	1230	468	298	.	10,108	256	18.69

Source: National statistics office, INE (2007)

Table A.III: Aggregate variables

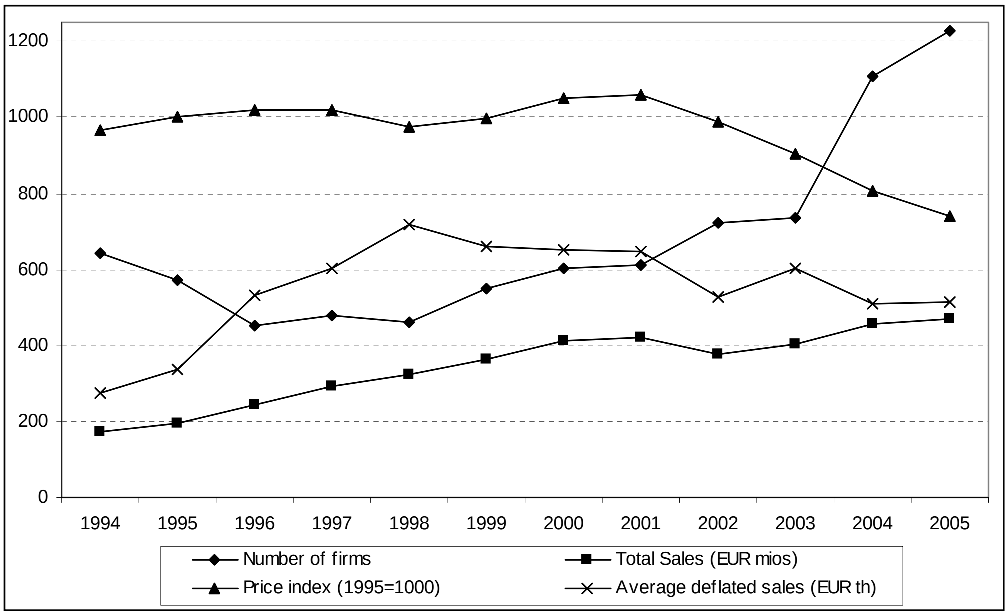

Figure A.1: Aggregate variables: Number of firms, price index, average and total sales

A.2 Production function estimation

Here, I explain how total factor productivity is estimated. Productivity is not directly observed and there are several available methods to estimate it as the residual from a production function. De Loecker (2007) proposes an estimator for both the production function parameters and the demand elasticity under imperfect competition when one uses deflated sales instead of quantities (see also Klette and Griliches (1995). The estimator, however, cannot be directly applied to an equilibrium model.[22] Input demand functions depend on the industry state, more precisely, the aggregate industry state. For example investment functions depend on equilibrium beliefs about industry evolution. Specifying the investment policy as originally proposed by Olley and Pakes (1996), $i(\omega)$, is a misspecification since the equilibrium policy for investment in a Markov Perfect Equilibrium depends on the state of all players in the industry $i^*(\omega_1, .., \omega_N)$. Therefore, elasticity of demand cannot be recovered in the first step as proposed by De Loecker (2007) since the input demand is also a function of aggregate sales.[23] Using a Cobb-Douglas production function with labor and capital ($Q = e^{\omega_i} L^{\alpha_l} K^{\alpha_k}$), adding a demand function ($Q = \frac{\tilde{Y}}{\tilde{P}} \left(\frac{P}{\tilde{P}} \right)^{-\eta}$) with market size ($\tilde{Y}$) and price index ($\tilde{P}$) and taking the logs of firm level deflated sales ($\frac{P_{it} Q_{it}}{\tilde{P}}$)

$$y_{it} - \tilde{p}_t = q_{it} + p_{it} - \tilde{p}_t = \frac{1}{\eta}(\tilde{y}_t - \tilde{p}_t) + \frac{\eta - 1}{\eta} (\omega_{it} + \alpha_k k_{it} + \alpha_l l_{it}) \tag{A.1}$$

Using materials to control for the unobservable as in Levinsohn and Petrin (2003), the input demand is a function of the state at time t (individual and industry states)

$$m_{it} = m(\omega_{it}, k_{it}, R_{it}, S_t) \tag{A.2}$$

Assuming invertibility[24]

$$\omega_{it} = \omega(k_{it}, R_{it}, S_t, m_{it}) \tag{A.3}$$

Since productivity is also a function of market conditions ($S_t = \frac{\tilde{Y}_t}{\tilde{P}_t}$) in equation (A.2), demand elasticity (η) is not identified in the first stage, because it enters nonlinearly in the control function (A.3). This is the main difference from a single agent framework as in

[22] This is the true for most methods with similar approaches to Olley and Pakes (1996). Such methods look at single agent problems and disregard the possibility that policy functions (investment or materials) are equilibrium solutions to dynamic games. Therefore, the policy functions will be misspecified and the productivity control function will not include some important industry level variables leading to bias in the parameter estimates due to unobserved heterogeneity. How large this bias is in practice is an empirical question.

[23] There is also the selection problem due to exit as explained in Olley and Pakes (1996). In this paper I will abstract from this problem since the number of exits observed in the data is very small and the correction does not affect my estimates. However, if there were more observations on exits, this problem could be more carefully addressed.

[24] With imperfect competition an increase in productivity might not lead to an increase in output and therefore in materials usage causing concerns over invertibility. For the demand system specified, an increase in productivity is equivalent to a decrease in costs and it translates in a decrease in prices. Both total output and materials usage increase.

De Loecker (2007) where input demand depends solely on individual state variables ($m_{it} = m(\omega_{it}, k_{it}, R_{it})$). Using the controlled first order Markov process assumption for productivity $\omega_{it} = E[\omega_{it}|\omega_{it-1}, R_{it-1}] + \nu^{\omega}_{it}$.

A.2.1 Stage I

From eq. (A.1) using deflated sales as variables $y^p_{it} = y_{it} - \tilde{p}_t$ and $\tilde{y}^p_t = \tilde{y}_t - \tilde{p}_t = \ln(S_t)$ the production function is

$$y^p_{it} = \frac{\eta - 1}{\eta}\alpha_l l_{it} + \phi(k_{it}, R_{it}, \tilde{y}^p_t, m_{it}) + y^{\varepsilon}_{it} \tag{A.4}$$

where y^{ε}_{it} is measurement error in y^p_{it}, and $\widehat{\frac{\eta-1}{\eta}\alpha_l}$ and $\hat{\phi}$ can be estimated nonparametrically or using an nth-order polynomial approximation from

$$\phi(k_{it}, R_{it}, \tilde{y}^p_t, m_{it}) = \frac{1}{\eta}\tilde{y}^p_t + \frac{\eta - 1}{\eta}\alpha_k k_{it} + \frac{\eta - 1}{\eta}\omega(k_{it}, R_{it}, \tilde{y}^p_t, m_{it})$$

A.2.2 Stage II

Let the estimated value be $\hat{\phi}_{it} = \hat{y}^p_{it} - \widehat{\frac{\eta-1}{\eta}\alpha}_l l_{it}$. An estimate of $\frac{\eta-1}{\eta}\omega_{it}$ for a given $\widetilde{\frac{\eta-1}{\eta}\alpha_k}$ and $\widetilde{\frac{1}{\eta}}$ is

$$\widehat{\frac{\eta - 1}{\eta}}\omega_{it} = \hat{\phi}_{it} - \widetilde{\frac{1}{\eta}}\tilde{y}^p_t - \widetilde{\frac{\eta - 1}{\eta}}\alpha_k k_{it}$$

Approximate $E[\omega_{it}|\omega_{it-1}, R_{it-1}]$ with an nth-order polynomial $P^{n,\omega}(\omega_{it-1}, R_{it-1})$. Several approximations are used in the empirical section and in practice a cubic polynomial provides a good fit, in particular at the tails of the observations[25]

$$\begin{aligned} \hat{y}^p_{it} - \widehat{\frac{\eta - 1}{\eta}}\alpha_l l_{it} \quad &= \quad \frac{1}{\eta}\tilde{y}^p_t + \frac{\eta - 1}{\eta}\alpha_k k_{it} + \\ &+P^{n,\omega}(\omega_{it-1}, R_{it-1}) + \nu^{\omega}_{it} \end{aligned} \tag{A.5}$$

Finally, $\frac{1}{\eta}$ and α_k are estimated by nonlinear least squares using eq. (A.5).

A.2.3 Results

Estimates are reported in Table A.IV.[26] The estimated labor and capital coefficients are 0.51 and 0.4, while the estimated demand elasticity implies a price-cost margin of 8%. The values are at a

[25] Instead of using a cubic polynomial, also reported are the results with a sigmoidal function which preserves monotonicity: $E(\omega_{it}|\omega_{i,t-1}) = \frac{\gamma_0}{(1+\gamma_1 \exp(-\omega_{i,t-1}))}$

[26] The results are reported for value added production functions. The approach is identical to the one using sales under the assumption that materials are a constant share of total sales. Since by definition $Y_{it} = VA_{it} + M_{it}$, if $M_{it} = \alpha_m Y_{it}$ then $Y_{it} = VA_{it} + \alpha_m Y_{it}$ so that $Y_{it} = \frac{1}{(1-\alpha_m)} VA_{it}$.

	(i)		(ii)		(iii)		(iv)		(v)		(vi)	
	1st stage		2nd stage		2nd stage		2nd stage		Fixed Effects			
Equation:	(A.4)		(A.5)		(A.5)		(A.5)		(A.1)		(A.1)	
Model:	OLS		NLLS		NLLS		NLLS		FE		FE	
			Cubic Approx.		*Linear Approx.*		*Sigmoidal Approx.*				*Control function*	
	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*
l_{it}	0.47	*0.04*							0.69	*0.04*	0.52	*0.04*
k_{it}			0.37	*0.02*	0.41	*0.03*	0.38	*0.02*	0.37	*0.03*		
$\tilde{y}_t^p$			0.08	*0.04*	0.07	*0.05*	0.07	*0.04*				
const.							3.57	*0.64*				
$\hat{R}^2$	98%		92%		92%		92%		92%		93%	
Obs.	1,271		1,044		1,044		1,044		1,271		1271	
Firms	227		223		223		223		227		227	
Labor coef.			0.51		0.51		0.74		0.69			
Capital coef.			0.40		0.44		0.40		0.37		-	
Price Cost Margin			8%		7%		7%		-		-	

Notes: Results for the first stage of the production function estimates are reported in column (i). Columns (ii)-(iv) report the second stage estimates using a cubic, linear and sigmoidal approximation, respectively. Column (v) reports the results for the fixed effects specification with year dummies. The specification in column (vi) is identical to column (i) with added fixed effects.

Table A.IV: Production function estimates.

reasonable level and within the range of parameters found in the literature for other industries. In columns (iii) and (iv) results are also reported for a linear and a sigmoid parametrization for productivity transition. The advantage is that both preserve monotonicity. Overall the differences are negligible and there is no evidence that functional forms are restrictive.

Results using alternative methods are also reported. In particular using a simple fixed effects specification with time dummies (column (v)) does not perform well due to serial correlation in productivity. Adding the control function $E[\omega_{it}|\omega_{it-1}, R_{it-1}]$ in column (vi) reduces the magnitude of the labor coefficient suggesting the fixed effects and time dummies are not properly capturing the correlation between labor and productivity

A.3 Figures

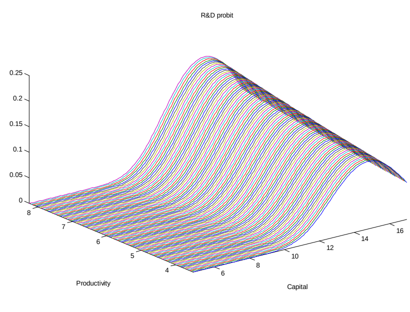

Figure A.2: R&D probit, fit in productivity and capital space

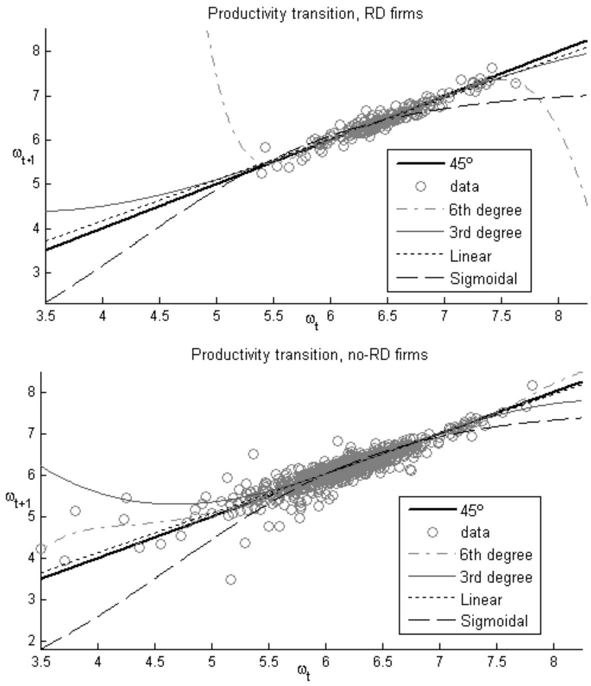

Figure A.3: Productivity transition (observations and fit)

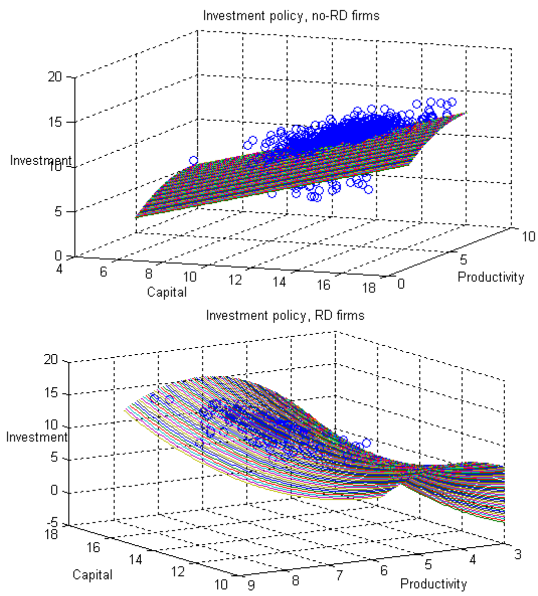

Figure A.4: Investment functions, fit in productivity and capital space

References

[1] Ackerberg, D., L. Benkard, S. Berry, and A. Pakes (2007): "Econometric Tools for Analyzing Market Outcomes", in *Handbook of Econometrics*, vol. 6, ch. 63, ed. by J. Heckman and E. Leamer.

[2] Aradillas-Lopez, A., and E. Tamer (2008): "The Identification Power of Equilibrium in Simple Games", *Journal of Business and Economic Statistics*, 26 (3), 261-283.

[3] Aw, B., M. Roberts, and D. Xu (2009): "R&D Investment, Exporting and Productivity Dynamics", Working Paper 14670, NBER.

[4] Bajari, P., L. Benkard, and J. Levin (2007): "Estimating Dynamic Models of Imperfect Competition", *Econometrica*, 75 (5), 1331-1370.

[5] Beira, E., C. Crespo, N. Gomes, and J. Menezes (2003): "Dos moldes à engenharia do produto, a trajectória de um cluster", in *Momentos de Inovação e Engenharia em Portugal no seculo XX*, ed. by M. Heitor, J. Brito and M. Rollo, Lisboa: Dom Quixote.

[6] Bernard, A., J. Eaton, B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade", *American Economic Review*, 93(4), 1268-1290.

[7] Bernard, A., J. Jensen and P. Schott (2006): "Trade costs, Firms and Productivity", *Journal of Monetary Economics*, 53(5), 917-937.

[8] CEFAMOL (2008): "Current Situation of the Portuguese Mouldmaking Industry", mimeo, http://www.cefamol.pt/.

[9] Cooper, R., and J. Haltiwanger (2006); "On the Nature of the Capital Adjustment Costs", *Review of Economic Studies*, 73(3), 611-633.

[10] De Loecker, J. (2007): "Product Differentiation, Multi-product Firms and Estimating the Impact of Trade Liberalization on Productivity", Working Paper WP13155, NBER.

[11] Doraszelski, U., and M. Satterthwaite (2010): "Computable Markov-Perfect Industry Dynamics", *Rand Journal of Economics*, 41(2), 215-243.

[12] Eaton, J., and S. Kortum (2002): "Technology, Geography and Trade", *Econometrica*, 70(5), 1741-1779.

[13] Eaton, J., S. Kortum, and F. Kramarz (2008): "An Anatomy of International Trade: Evidence from French Firms", Working Paper WP14610, NBER.

[14] Ericson, R., and A. Pakes (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work", *Review of Economic Studies*, 62 (1), 53-82.

[15] Fershtman, C., and A. Pakes (2009): "Finite State Dynamics with Asymmetric Information: A Framework for Applied Work", Working Paper, Harvard University.

[16] Hopenhayn, H. (1992): "Entry, Exit and Firm Dynamics in Long Run Equilibrium", *Econometrica*, 60(5), 1127-1150.

[17] IAPMEI (2006): "A Industria Portuguesa de Moldes no Contexto Internacional - contributos para o conhecimento da actividade", mimeo, www.iapmei.pt.

[18] Jovanovic, B. (1982): "Selection and the Evolution of Industry", *Econometrica*, 50(3).

[19] Klette, T., and Z. Griliches (2000): "Empirical Patterns of Firm Growth and R&D Investment: A Quality Ladder Model Interpretation", *The Economic Journal*, 110, 363-387.

[20] Klette, T., and S. Kortum (2004): "Innovating Firms and Aggregate Innovation", *Journal of Political Economy*, 112 (5), 986-1018.

[21] Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables", *Review of Economic Studies*, 317-342.

[22] Lopez, R. (2005): "Trade and growth: Reconciling the macroeconomic and microeconomic evidence", *Journal of Economic Surveys* 19, 623-648.

[23] Magnac, T., and D. Thesmar (2002): "Identifying Dynamic Discrete Decision Processes", *Econometrica*, 70, 801-816.

[24] Melitz, M. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity", *Econometrica*, 71(6) 1695-1725.

[25] Melitz, M., and G. Ottaviano (2008): "Market Size, Trade and Productivity", *Review of Economic Studies*, 75(1), 295-316

[26] Olley, S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry", *Econometrica*, 64(6), 1263-98.

[27] Pavcnik, N. (2002): "Trade liberalization, exit, and productivity improvement: Evidence from Chilean plants", *Review of Economic Studies*, 69(1), 245-276.

[28] Pesendorfer, M., and P. Schmidt-Dengler (2008): "Asymptotic Least Squares Estimators for Dynamic Games", *Review of Economic Studies*, 75(3), 901-928.

[29] Rust, J. (1994): "Structural Estimation of Markov Decision Processes" in *Handbook of Econometrics*, vol. 4, ch. 51, ed. by C. Engel and D. McFadden, North Holland, 3082–3139.

[30] Ryan, S. (2008): "The Costs of Environmental Regulation in a Concentrated Industry", Working Paper 0510, MIT.

[31] Schmidt-Dengler, P. (2006): "The Timing of New Technology Adoption: The Case of MRI", Working Paper, London School of Economics.

[32] Stiglitz, J. (1987): "Technological Change, Sunk Costs and Competition", *Brooking Papers on Economic Activity*, 3, 883-947.

[33] Sutton, J. (1991): *Sunk Costs and Market Structure*, MIT Press.

[34] Sutton, J. (1998): *Technology and Market Structure*, MIT Press.

[35] United States International Trade Comission (2002): "Tools, dies and industrial molds: competitive conditions in the United States and selected foreign markets", Investigation no. 332-435, USITC Publication 3556.

B Technical Appendix

B.1 The aggregate state dynamic model

This section describes the elements of the general aggregate state dynamic model and characterizes the equilibrium conditions. I draw on Pesendorfer and Schmidt-Dengler (2008) and extend some results for the aggregate state model.

B.1.1 States and actions

Time is discrete, $t = 1, 2, ..., \infty$. There are N players where a player is denoted by $i \in \{1, ..., N\}$.

States Agents are endowed with a state variable $s_{it} \in \mathbf{S}_i = \{1, ...L\}$ and a vector of payoff shocks $\boldsymbol{\varepsilon}_{it} \in \mathbb{R}^K$. Both the state and the shocks are privately observed by the players. The econometrician observes the states, s_{it}, but not the payoff shocks, $\boldsymbol{\varepsilon}_{it}$.

The industry state is $\mathbf{s}_t = (s_{1t}, ..., s_{Nt}) \in \mathbf{S} = S_i^N$. The cardinality of the state space is $m_s = L^N$. The state of competitors $s_{-it} = (s_{1t}, ..s_{i-1,t}, s_{i+1,t}, .., s_{Nt}) \in \mathbf{S}_{-i} = S_i^{N-1}$ with cardinality $m_{s-1} = L^{N-1}$. The vector of payoff shocks is drawn independently from the strict monotone and continuous distribution $F(.|\mathbf{s}_t)$.

Actions Each player chooses an action $a_{it} \in A_i = \{0, .., K\}$. Decisions are simultaneous after players observe their state and payoff shock. A profile $\mathbf{a}_t = (a_{1t}, ..., a_{Nt}) \in \mathbf{A} = A_i^N$ and $\mathbf{a}_{-it} \in \mathbf{A}_{-i} = A_i^{N-1}$. The cardinality of the action space $\mathbf{A}$ is $m_a = (1 + K)^N$ and $\mathbf{A}_{-i}$ is $m_{a-1} = (1 + K)^{N-1}$.

State transition The individual state transition is described by a density function $p : A_i \times S_i \times S_i \to [0, 1]$. A typical element of $p(a, s, s')$ equals the probability that state s' is reached from state s when action a is chosen and $\sum_{s' \in S_i} p(a, s, s') = 1$. The industry transition can be represented by a density function $g : \mathbf{A} \times \mathbf{S} \times \mathbf{S} \to [0, 1]$. A typical element of $g(\mathbf{a}, \mathbf{s}, \mathbf{s}') = \prod_{j=1}^{N} p(a_j, s_j, s'_j)$ equals the probability that state $\mathbf{s}'$ is reached from state $\mathbf{s}$ when action profile $\mathbf{a}$ is chosen and $\sum_{\mathbf{s}' \in \mathbf{S}} g(\mathbf{a}, \mathbf{s}, \mathbf{s}') = 1$.

Per period payoff Firms receive per period returns which depend on the state of the industry, current actions and shocks ($\pi(\mathbf{a}_t, \mathbf{s}_t, \boldsymbol{\varepsilon}_{it})$).

Assumption B.1 *(a) There exists a function ($S : \mathbf{S} \to J$) that maps the vector of firm's individual states ($\mathbf{s}_t$) into an aggregate index ($S(s_{1t}, .., s_{Nt}) \in J$). The cardinality of J is $m_J \leq m_s$.*

(b) Per period payoffs can be written as

$$\pi(\mathbf{a}_t, \mathbf{s}_t, \boldsymbol{\varepsilon}_{it}) = \pi(\mathbf{a}_t, s_{it}, S_t) + \sum_{k=1}^{K} \varepsilon_{it}^k \cdot 1(a_{it} = k)$$

Under this assumption, S_t is the payoff relevant variable commonly observed by all agents. Notice that the payoff relevant shocks ($\boldsymbol{\varepsilon}_{it}$) have no impact on the stage game pricing. One type of demand which meets this assumption is Monopolistic Competition.

Assumption B.2 *(a) (Incomplete Information) Individual states and actions are private information and;*

(b) (Markov beliefs) Players use the current state to form their beliefs: $\sigma(\mathbf{s}|S, s_i)$.

Under Assumption B.2 the only common information is the aggregate state. Moreover, it restricts agents to form beliefs using only the current state. This is the main assumption that distinguishes the aggregate state model from the literature with no privately observed state variables. The restriction on beliefs is important to circumvent the learning problem that would emerge otherwise.

Strategies I consider pure symmetric Markovian strategies, $a_{it}(s_{it}, S_t, \boldsymbol{\varepsilon}_{it})$. These strategies depend only on payoff relevant variables. Using symmetry we can drop the i subscript and imposing stationarity we can drop the t subscript:

$$a_{it}(s_{it}, S_t, \boldsymbol{\varepsilon}_{it}) = a(s_{it}, S_t, \boldsymbol{\varepsilon}_{it})$$

Let $p(a_i|s_i, S; \sigma)$ denote the probability of player i choosing action a_i when he observes state (s_i, S) and has beliefs σ.

Value function We can write the ex-ante value function defined as the discounted sum of future payoffs before player specific shocks are observed and actions taken, as

$$V(s_i, S; \sigma) = \sum_{\mathbf{s}\in\mathbf{S}} \sum_{\mathbf{a}\in\mathbf{A}} p(\mathbf{a}|\mathbf{s}, S; \sigma)\sigma(\mathbf{s}|s_i, S) \left[\pi_i(\mathbf{a}, \mathbf{s}) + \beta \sum_{s'_i, S'} \widetilde{g}(\mathbf{a}, \mathbf{s}, s'_i, S')V(s'_i, S'; \sigma)\right] \quad \text{(B.1)}$$

$$+ \sum_{k=1}^{K} E_\varepsilon[\varepsilon_i^k|a_i = k].p(a_i = k|s_i, S; \sigma)$$

where $\widetilde{g}(\mathbf{a}, \mathbf{s}, s'_i, S') = \left(\sum_{\mathbf{s}'\in\mathbf{S}} g(\mathbf{a}, \mathbf{s}, \mathbf{s}') \cdot \mathbf{1}(S' = S(\mathbf{s}'), s'_i \in \mathbf{s}')\right)$, $p(\mathbf{a}|\mathbf{s}, S; \sigma) = \prod_{j=1}^{N} p(a_j|s_j, S; \sigma) \cdot \mathbf{1}(S = S(\mathbf{s}), s_j \in \mathbf{s})$ and E_ε is the expectation over the payoff shocks. Equation (B.1) is satisfied at each $(s_i, S \in \mathbf{S}_i \times J)$. This can be written in matrix form.

$$\begin{aligned} \mathbf{V}(\boldsymbol{\sigma}) &= (\boldsymbol{\sigma} \cdot \mathbf{P}(\sigma))\boldsymbol{\Pi} + \mathbf{D}(p(\sigma)) + \beta(\boldsymbol{\sigma} \cdot \mathbf{P}(\sigma))\widetilde{G}\mathbf{V}(\boldsymbol{\sigma}) \qquad \text{(B.2)} \\ &= [\mathbf{I}_{LJ} - \beta(\boldsymbol{\sigma} \cdot \mathbf{P}(\sigma))\widetilde{G}]^{-1}[(\boldsymbol{\sigma} \cdot \mathbf{P}(\sigma))\boldsymbol{\Pi} + \mathbf{D}(p(\sigma))] \end{aligned}$$

where $\mathbf{V}(\boldsymbol{\sigma}) = [V(s_i, S; \sigma)]_{s_i, S \in \mathbf{S}_i \times J}$ is the $(L \cdot m_J) \times 1$ dimensional vector of expected discounted sum of future payoffs. $\widetilde{G}$ is the $(m_a \cdot m_s) \times (L \cdot m_j)$ transition matrix with element $\widetilde{g}(\mathbf{a}, \mathbf{s}, s_i', S')$. Π is the $m_a \cdot m_s \times 1$ dimensional vector of profits, $\mathbf{P}(\sigma)$ is the $(L \cdot m_J) \times (m_a \cdot m_s)$ dimensional matrix with element in row (s_i, S) and column $(\mathbf{a}, \mathbf{s})$ equal to $p(\mathbf{a}|\mathbf{s}; \sigma) \cdot \mathbf{1}(s_i \in \mathbf{s}, S = S(\mathbf{s}))$. $\boldsymbol{\sigma}$ is the $(L \cdot m_J) \times (m_a \cdot m_s)$ dimensional matrix with element in row (s_i, S) and column $(\mathbf{a}, \mathbf{s})$ equal to $\sigma(\mathbf{s}|s_i, S) \cdot \mathbf{1}(S = S(\mathbf{s}), (s_i) \in \mathbf{s})$ for any $\mathbf{a}$. $(\boldsymbol{\sigma} \cdot \mathbf{P}(\sigma))$ is the element by element product of the two matrices. I_J is the $L \cdot m_J$ dimensional identity matrix. Finally, $D(p(\sigma))$ is the $(L \cdot m_J) \times 1$ dimensional vector of expected payoff shocks.

B.1.2 Equilibrium

Let $u^S(a_i; \sigma)$ denote the continuation value net of payoff shocks under action a_i and beliefs σ.

$$u^S(a_i; \sigma, \theta) = \pi(a_i, s_i, S)$$

$$+\beta \sum_{\mathbf{s}_{-i} \in \mathbf{S}_{-i}} \sum_{\mathbf{a}_{-i} \in \mathbf{A}_{-i}} p(\mathbf{a}_{-i}|\mathbf{s}, S; \sigma)\sigma(\mathbf{s}|s_i, S) \sum_{\mathbf{s}' \in \mathbf{S}} g(\mathbf{a}, \mathbf{s}, \mathbf{s}') \cdot \mathbf{1}(S' = S(\mathbf{s}'))V(s_i', S'; \sigma) \tag{B.3}$$

where $p(\mathbf{a}_{-i}|s_i, \mathbf{s}_{-i}, S; \sigma) = \prod_{j=1, j \neq i}^{N} p(a_j|s_j, S; \sigma) \cdot \mathbf{1}(S = S(s_i, \mathbf{s}_{-i}), s_j \in \mathbf{s}_{-i})$. The optimality condition requires that action a_i is chosen when

$$u(a_i; \sigma, \theta) + \varepsilon_i^{a_i} \geq u(a_i'; \sigma, \theta) + \varepsilon_i^{a_i'} \quad \text{for all} \quad a_i' \in \mathbf{A}_i \tag{B.4}$$

Writing equation (B.4) as the ex-ante optimal choice probability

$$\begin{aligned} p(a_i|s_i, S; \sigma) &= \Psi(a_i, s_i, S; \sigma) \\ &= \int \left[\prod_{k \in A_i, k \neq a_i} \mathbf{1}(u(a_i; \sigma) - u(k; \sigma) \geq \varepsilon_i^k - \varepsilon_i^{a_i}) \right] dF \end{aligned} \tag{B.5}$$

We can write this system of $(K \cdot \mathbf{S}_i \cdot J)$ equations in vector form

$$\mathbf{p}^\sigma = \boldsymbol{\Psi}(\mathbf{p}^\sigma)$$

A solution exists by Brouwer's fixed point theorem since this equation is a continuous self-map on $[0, 1]^{K \cdot \mathbf{S}_i \cdot J}$ as shown in Schmidt-Dengler and Pesendorfer (2008). The solution is the optimal behavior for given beliefs, $\mathbf{p}^\sigma$.

Stationary distribution Let $\lambda(\mathbf{s};\sigma)$ be the probability of being in state $\mathbf{s}$ and let the transition be

$$\lambda(\mathbf{s}';\sigma)=\sum_{\mathbf{s}}\sum_{\mathbf{a}}p(\mathbf{a}|\mathbf{s};\sigma)g(\mathbf{a},\mathbf{s},\mathbf{s}')\lambda(\mathbf{s};\sigma) \tag{B.6}$$

Define $\mathbf{P}^{\sigma}$ as the $m_s\times(m_s\cdot m_a)$ dimensional matrix consisting of $p(\mathbf{a}|\mathbf{s};\sigma)$ in row $\mathbf{s}$, columns $(\mathbf{a},\mathbf{s})$ and zero in the remaining columns. G is the $(m_a\cdot m_s)\times m_s$ dimensional matrix. Convergence to a unique stationary distribution with $\lambda^*(\mathbf{s};\sigma)>0$ for all $\mathbf{s}\in\mathbf{S}$ occurs if $\mathbf{P}^{\sigma}G$ is a regular matrix. We can write equation (B.6) in vector form as

$$\lambda^{\sigma\prime}=\mathbf{P}^{\sigma}G\lambda^{\sigma}$$

and the stationary distribution is the eigenvector $(\lambda_{\mathbf{s}}^{\sigma})$, solving

$$\left(\lambda_{\mathbf{s}}^{\sigma}\right)^*.(I_{m_s}-\mathbf{P}^{\sigma}G)=0 \tag{B.7}$$

The stationary conditional distribution is

$$\begin{aligned}\lambda^*(\mathbf{s}|S;\sigma) &= \frac{\Pr(\mathbf{s},S|\sigma)}{\Pr(S|\sigma)}=\frac{\Pr(S|\mathbf{s};\sigma)\Pr(\mathbf{s}|\sigma)}{\Pr(S|\sigma)} \\ &= \frac{\mathbf{1}(S=S(\mathbf{s}))\cdot\lambda^*(\mathbf{s};\sigma)}{\sum_{\mathbf{s}\in\mathbf{S}}[\mathbf{1}(S=S(\mathbf{s}))\cdot\lambda^*(\mathbf{s};\sigma)]}\end{aligned} \tag{B.8}$$

$$\lambda^*(\mathbf{s}|s_i,S;\sigma)=\frac{\Pr(\mathbf{s},s_i,S|\sigma)}{\Pr(s_i,S|\sigma)}=\frac{1(S=S(\mathbf{s})))1(s_i\in\mathbf{s})\lambda^*(\mathbf{s};\sigma)}{\sum_{\mathbf{s}\in\mathbf{S}}[\mathbf{1}(S=S(\mathbf{s}),s_i\in\mathbf{s})\cdot\lambda^*(\mathbf{s};\sigma)]}$$

and $\lambda^*(\mathbf{s};\sigma)$ is probability of element $\mathbf{s}\in\mathbf{S}$ occurring under the stationary distribution $(\lambda_{\mathbf{s}}^{\sigma})^*$. We can write this in matrix form

$$\boldsymbol{\lambda}^*_{\mathbf{s}|s_i,S}=\boldsymbol{\Lambda}(\boldsymbol{\sigma})$$

where $\boldsymbol{\lambda}^*_{\mathbf{s}|s_i,S}$ denotes a $(m_s\cdot S_i\cdot J)\times 1$ dimensional vector. In equilibrium, beliefs must be consistent with optimal behavior $\boldsymbol{\lambda}^*_{\mathbf{s}|s_i,S}=\boldsymbol{\sigma}$

$$\boldsymbol{\lambda}^*_{\mathbf{s}|s_i,S}=\boldsymbol{\Lambda}(\boldsymbol{\lambda}^*_{\mathbf{s}|s_i,S}) \tag{B.9}$$

Equation (B.9) is both a necessary and sufficient condition for a Markov perfect equilibrium.

Theorem 1 *An equilibrium exists.*

Equation (B.9) gives a continuous self-map on $[0,1]^{m_s\cdot S_i\cdot J}$. Brouwer's fixed point theorem implies there exists (at least) a fixed point $\boldsymbol{\lambda}^*$ to the function $\boldsymbol{\Lambda}$. This fixed point corresponds to an equilibrium.