Fertility Decisions and Endogenous Residential Sorting*

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Abstract

This paper develops a theoretical framework to consider fertility decisions within an endogenous sorting model of neighborhood effects. The models in the literature typically assume that each family is endowed with children whose expected schooling outcomes are determined by parental preferences on neighborhood quality. However, empirical studies report that the fertility dimension is also endogenous. We extend the model originally developed by Nesheim (2001) to account for endogenous fertility in a framework featuring endogenous social effects. Altruistic parents jointly choose how many children to produce and which neighborhood to live. We show that the fertility decision is an integral part of hedonic housing prices. We discuss its implications for segregation and argue that the model can accommodate alternative definitions of neighborhood quality.

JEL codes: J13, R21, R23.

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1 Introduction

In this paper, we investigate how parental decisions on fertility and residential location interact and how this interaction affects the formulation of hedonic housing prices. As a first step in this direction, we study a version of the equilibrium sorting model originally developed by Nesheim (2001) and extended by Ioannides (2010). In this class of models, parents choose which neighborhood to live recognizing that the human capital levels of their neighbors will be correlated with their own or their children's human capital levels. Neighborhood effects have been successfully endogenized in this literature, but fertility is typically held fixed by the assumption that each family consists of parents and a given number of children. However, there is evidence that fertility is also endogenous in this relationship. Specifically, fertility is documented to be negatively related to neighborhood quality.

To incorporate the fertility choice, we combine Nesheim's model with a modified version of the Barro-Becker model [see Becker and Barro (1988)].[1] In this setup, altruistic parents maximize utility by jointly determining how many children to produce and which neighborhood to live. We present conditions under which parents with higher human capital levels sort themselves into higher-quality neighborhoods and choose to have lower fertility on average. We go beyond this preferred result and describe the role of the sorting equilibrium in strengthening, weakening, or even reversing this natural negative relationship between neighborhood quality and fertility. The effect of neighborhood characteristics on the cost of child rearing is the key to understanding the main forces at work. The equilibrium housing price is a function of the parameters governing the fertility and neighborhood choices of parents as well as the endogenous neighborhood quality measured in terms of the average education among the adult residents living in the neighborhood.

Our research is motivated by a comprehensive list of stylized facts that have emerged from a large number of studies. The theoretical framework we develop offers a coherent interpretation of these facts. We focus on five related—but distinct—pieces of empirical evidence.

[1] In Section 2.2, we explain how we modify the Barro-Becker framework to postulate a fertility choice model for endogenously formed neighborhoods. The Appendix presents the versions of our model with original Barro-Becker assumptions and elaborates why we need to deviate from some of these assumptions.

Stylized fact 1: Fertility is lower in higher-quality neighborhoods.

Hank (2001) reports that, in western Germany, the lowest-fertility areas are the university towns (after correcting for student over-representation). In support of this argument, Kopp (2000) and Hank (2002) find that most of the regional variation in fertility result from differences in the spatial distribution of individual characteristics, mainly the average education. Iyer and Weeks (2010) document using data from Kenya that the effect of social interactions on fertility is mediated by group-level average education. They argue that the regions, which have had early access to colonial education and have the largest average education among women, have the least preference for larger families.

Stylized fact 2: Fertility is lower for parents earning higher wages and, therefore, for better-educated parents.

Empirical studies document a clear negative cross-sectional relationship between income and fertility, which is quite stable over time. An integrated economic analysis of this empirical regularity was first provided by Becker (1960) (although there were earlier studies mentioning the issue). See Jones and Tertilt (2008) for a comprehensive survey.

Stylized fact 3: Average educational attainment per child is lower in larger families.

This is referred to as the "quantity-quality tradeoff" and is often offered as an explanation to the stylized fact 2. Among the most recent studies, Goux and Maurin (2005), Li, Zhang, and Zhu (2008), and Lien, Wu, and Lin (2008) find that educational attainment is negatively correlated with family size.[2]

Stylized fact 4: Schooling outcomes are better in higher-quality neighborhoods.

Some of the key stylized facts about positive neighborhood quality effects on schooling are reported by Case and Katz (1991), Brooks-Gunn et al. (1993), Aaronson (1998), and Raaum, Salvanes, and Sorensen (2006).[3]

[2] For early evidence in support of this hypothesis, see Leibowitz (1974), Gove, Hughes, and Galle (1979), Rosenzweig and Wolpin (1980), Knodel and Wongsith (1991), and Hanushek (1992). It is important to note that whether the effect of quantity on quality is causal or not is controversial. See Angrist, Lavy, and Schlosser (2005) for an argument against a causal relationship.

[3] A related strand of literature, including the papers Black (1999), Bayer, Ferreira, and McMillan (2007), and Clapp, Nanda, and Ross (2008), argue that the effect of school quality on housing prices are biased upward by unobserved neighborhood characteristics, since schooling outcomes are better in higher-quality neighborhoods.

Stylized fact 5: Higher-quality parents choose to live in higher-quality neighborhoods. This means that there is a positive correlation between average human capital in a neighborhood and the human capital of the parents who choose to live in that particular neighborhood. See, for example, Herrnstein and Murray (1994), Kremer (1997), Nesheim (2001), Sampson, Morenoff, and Gannon-Rowley (2002), and Ioannides (2010).

The theoretical models of endogenous sorting explain only the stylized facts 4 and 5. Bringing these five pieces together yields the following set of results. Parents care about their children's schooling outcomes. They prefer living in higher-quality neighborhoods because of increased likelihood of better schooling outcomes for children. People living in higher-quality neighborhoods—these are typically better-educated individuals—tend to produce better-educated but a smaller number of children.[4] Endogenous sorting models of housing prices bring satisfactory explanation on positive sorting between high-quality parents and high-quality neighborhoods, but they are silent when the fertility dimension kicks in. Our goal in this paper is to develop a theoretical model that not only brings all these empirical pieces together but also is coherent and consistent with the standard models in the endogenous sorting literature.

Endogenous sorting models of housing prices explain how parents sort themselves into neighborhoods based on their children's expected schooling achievements [Nesheim (2001), Graham (2009), Ioannides (2010)]. These models recognize that when individuals move into a higher-quality neighborhood by paying a price, they buy a bundle of commodities including social influences; that is, social influences are priced along with other neighborhood characteristics in a hedonic setting. Heterogeneous parents—differing in human capital along the lines of Loury (1981)—care about their own (non-housing) consumption and the expected schooling of their children. Schooling of children is determined by four variables: (1) schooling of own parents, (2) own ability of children, (3) neighborhood quality measured in terms of the average education among the adult residents living in that neighborhood, and (4) a random shock. Complementarity between family and neighborhood characteristics in the production

[4]Notice that this is a version of the quantity-quality tradeoff along the lines of the Beckerian tradition [Becker (1960, 1993), Becker and Lewis (1973)].

of child training induces stratification across neighborhoods. The trade-off between non-housing consumption and children's expected schooling attainment is the fundamental idea in these models. Parents make a residential location choice to equate the marginal cost of neighborhood quality and marginal return from it. At the end, higher-quality parents sort into higher-quality neighborhoods. This paper incorporates parental fertility decisions into such a framework.

Our analysis suggests that endogenous sorting may operate through multiple channels. In particular, we show that fertility and neighborhood quality are jointly determined within each family, and that fertility is an important determinant of willingness to pay for neighborhood quality. We derive closed form solutions for fertility differentials across residential areas and we discuss the implications for hedonic housing prices. The "cost of quality" is an important element in our analysis. Whether child rearing is more expensive in higher quality neighborhoods or not determines the sign and the strength of the equilibrium relationship between fertility and neighborhood quality.

Incorporating the fertility dimension opens up an interesting path in the theoretical analysis of residential sorting. In Nesheim's version, sorting operates through parental human capital. We propose an alternative sorting explanation. Higher-quality parents rationally invest more in their children's human capital. Hence, all children raised in the neighborhood are exposed to other children that are also invested in heavily. This implies that there is a feedback between fertility decisions and neighborhood quality measured in terms of the average ability of adults or children raised in the neighborhood. To reflect this feedback, we extend Nesheim's model to allow for residential sorting to operate through children's human capital.

The plan of the paper is as follows. Section 2 presents the basic setup of the model and describes the solution in detail. Section 3 discusses the main predictions of the model and links housing prices to parental fertility decisions. Section 4 provides an alternative formulation of neighborhood quality. Section 5 concludes.

2 Model

In this section, we first describe the basics of the model and, then, we provide conditions for the optimal solution to be the one in which parents with higher human capital sort themselves into higher-quality neighborhoods to maximize their children's expected schooling outcomes in exchange for lower fertility. We discuss the economic forces at work that could lead to a change in our basic results.

2.1 Self-consistent bunching

At the root of the model is a self-consistent bunching argument which features positive sorting between parental human capital and neighborhood quality. Let s denote neighborhood quality. Under standard assumptions, which will be made clear below, we show that better-educated parents bunch together to form a high-quality neighborhood, then comes the next best neighborhood which consists of parents with somewhat lower schooling on average, and so on. This is a self-selection argument. Higher-quality parents self-select into higher quality neighborhoods in order to maximize the expected schooling outcomes of their children. The self-consistency assumption requires that the equilibrium neighborhood quality, s, should be equal to the average human capital level in the neighborhood, where the average is calculated conditional on the decision rules and with respect to human capital (i.e., schooling) levels of the adult residents. We will come back to this fundamental idea after we describe our model and its solution.

2.2 Basic setup

We consider a static economy populated by a continuum of heterogeneous households consisting of parents and children. Each household is described by a vector of attributes $\boldsymbol{\omega} = (\omega_p, \omega_c) \in \mathbb{R}^2$, where e^{ω_p} is parental schooling and e^{ω_c} is children's ability in school. Children in the same family are assumed to be of identical ability. Households consume a unit of housing each. Parents are altruistic in the sense that they have preferences over their children's expected schooling outcomes in addition to their own non-housing consumption. Parental utility function is

$$V\big(x, s, n|\boldsymbol{\omega}\big) = x + \alpha n^{-\epsilon}\Big(n\mathbb{E}_{\theta}\big[q|s, \boldsymbol{\omega}\big]\Big), \tag{2.1}$$

where $x > 0$ denotes parents' non-housing consumption, s is neighborhood quality, and n is the number of children. $\mathbb{E}_{\theta}\big[q|s, \boldsymbol{\omega}\big]$ is the expected schooling outcome of each child given neighborhood quality s and household characteristics $\boldsymbol{\omega}$. The mathematical expectation is taken over the random shock $\theta = e^u$, which exogenously affects children's schooling outcomes. The parameter $0 < \alpha < 1$ represents the degree of pure altruism and $0 < \epsilon < 1$ captures the feature that altruism toward each child has a constant (negative) elasticity with respect to the number of children.

Residential locations are indexed by neighborhood quality, s. Each location s contains an inelastic supply of residential units by competitive landlords who rent to the parents at the price $\pi(s)$ per unit. Parents allocate their income Y over non-housing consumption, unit housing rent at location s, and child-rearing costs. Therefore, the parental resource constraint is

$$Y = x + \pi(s) + G(n, s), \tag{2.2}$$

where $G(n, s)$ is the goods cost of raising n children in location s. We focus on the special case $G(n, s) = ng(s)$. In this setup, the sign of g' is the key and whether it is negative or positive has different implications. When $g' < 0$, goods cost per child decreases with neighborhood quality. One argument in support of this assumption is that social influences in high-quality environments facilitates child raising and, therefore, costs are reduced. When $g' > 0$, on the other hand, the cost of child rearing per child is increasing in neighborhood quality. A natural explanation is that higher rents in high-quality neighborhoods may cause local retail and service activities to have higher prices. Therefore, the cost of child rearing is greater in better neighborhoods. To reflect these ideas in a parsimonious setting, we assume $g(s) = s^{\beta}$. Under this formulation, $\beta < 0$ implies $g' < 0$ and $\beta > 0$ implies $g' > 0$. We will discuss the further implications of both cases. Note that the cost of child rearing is a convex function of neighborhood quality if $|\beta| > 1$, and it is concave if $|\beta| < 1$.

Schooling achievement, q, of a child with ability e^{ω_c}, whose parents have e^{ω_p} years of school training and live in location s is given by

$$q = \left[s^{\eta} e^{\phi\omega_p}\right] e^{\omega_c} e^{u}. \tag{2.3}$$

The term $s^{\eta} e^{\phi\omega_p}$ is the effect of "environment" in each child's education as a Cobb-Douglas production function with factor shares $\eta > 0$ and $\phi > 0$. Implicit in this formulation is that the environment, child's own ability e^{ω_c}, and the luck component e^{u} are equally important for success in school. We interpret e^{ω_c} as heterogeneous parental expectations on each child's ability, where parental decisions on fertility and neighborhood quality are made prior to the realization of children's ability. This interpretation naturally allows for a positive correlation between ω_p and ω_c. We ignore child spacing and birth-order effects. We assume $u \sim \mathcal{N}(0, \sigma_{uu})$, which implies that $\theta = e^{u}$ is log-normally distributed. We also assume that $u \perp\!\!\!\perp \boldsymbol{\omega}$, where the symbol $\perp\!\!\!\perp$ denotes statistical independence. Under these assumptions, the children's expected schooling level is formulated as

$$\mathbb{E}_{\theta}\left[q|s, \boldsymbol{\omega}\right] = \left[s^{\eta} e^{\phi\omega_p}\right] e^{\omega_c} \mathbb{E}_{\theta}\left[e^{u}\right] = \left[s^{\eta} e^{\phi\omega_p}\right] e^{\omega_c} \Psi_0, \tag{2.4}$$

where $\mathbb{E}_{\theta}\left[e^{u}\right] = \Psi_0 = e^{\frac{1}{2}\sigma_{uu}}$ by log-normality.

Notice that this is a modified version of the Barro-Becker model, where the modification is performed to reconcile the Barro-Becker setup with a standard neighborhood formation framework. In the Appendix, we discuss in detail the similarities and differences between our model and the original Barro-Becker model. We explain in what ways we deviate from the basic Barro-Becker assumptions. The Appendix also develops two different models which are closer to the Barro-Becker setup than our model. Using the results from these alternative models, the Appendix explains why we prefer to stick with the version we present below.

2.3 Parents' problem

After plugging the resource constraint into the parental utility function, parents' problem can be formulated as follows:

$$\max_{s,n}\Big\{Y - \pi(s) - ns^{\beta} + \alpha n^{1-\epsilon}\left[s^{\eta}e^{\phi\omega_p}\right]e^{\omega_c}\Psi_0\Big\}. \tag{2.5}$$

The first-order conditions with respect to s and n are

$$\pi'(s) + \beta n s^{\beta-1} = \eta\alpha n^{1-\epsilon}\left[s^{\eta-1}e^{\phi\omega_p}\right]e^{\omega_c}\Psi_0 \tag{2.6}$$

and

$$s^{\beta} = (1-\epsilon)\alpha n^{-\epsilon}\left[s^{\eta}e^{\phi\omega_p}\right]e^{\omega_c}\Psi_0, \tag{2.7}$$

respectively. The second-order conditions are given by

$$\pi''(s) + \beta(\beta-1)ns^{\beta-2} > \eta\alpha(\eta-1)n^{1-\epsilon}\left[s^{\eta-2}e^{\phi\omega_p}\right]e^{\omega_c}\Psi_0, \tag{2.8}$$

$$0 > -\epsilon(1-\epsilon)\alpha n^{-\epsilon-1}\left[s^{\eta}e^{\phi\omega_p}\right]e^{\omega_c}\Psi_0, \tag{2.9}$$

where the latter inequality is satisfied by default. The first-order conditions say that, in the equilibrium, (i) the marginal cost of moving into a better residential area should be equal to the marginal return from it and (ii) the cost of producing an additional child should be equal to the return from that additional child in terms of educational attainment.

2.4 Endogenous neighborhood effects and housing prices

In this section, we present the derivation of endogenous neighborhood effects arising from our model and we put more structure on household heterogeneity. We start our analysis by plugging the first-order condition for n [Equation (2.7)] into the first-order condition for s [Equation (2.6)], which, after trivial algebra, gives,

$$\pi'(s) = \kappa\left[e^{\phi\omega_p+\omega_c}\right]^{1/\epsilon} s^{\frac{1}{\epsilon}(\eta-\beta)+\beta-1}, \tag{2.10}$$

where

$$\kappa = \left[\Psi_0 \alpha (1-\epsilon)\right]^{1/\epsilon} \left(\frac{\eta}{1-\epsilon} - \beta \right). \tag{2.11}$$

Equation (2.10) measures individual-level willingness to pay for neighborhood quality. We assume, for the moment, that the second-order conditions for a global maximum are satisfied and we proceed to the derivation of the aggregate willingness to pay measure. We will examine whether they are satisfied or not after this derivation. Taking natural logarithms of both sides of this equation yields

$$\epsilon \left(\ln\left[\pi'(s)\right] - \left[\frac{1}{\epsilon}(\eta - \beta) + \beta - 1 \right] \ln\left[s\right] - \ln\left[\kappa\right] \right) = \phi\omega_p + \omega_c. \tag{2.12}$$

Let $T(s)$ denote the index of neighborhood quality, which is described by the left-hand side of (2.12). This index describes the willingness to pay for neighborhood quality in family $\boldsymbol{\omega}$. Let $\boldsymbol{b} = (\phi, 1)$. Then the decision rule (2.12) can be rewritten as

$$T(s) = \boldsymbol{b}\boldsymbol{\omega}'. \tag{2.13}$$

For every family, the productivity of neighborhood quality is monotonic in $\boldsymbol{b}\boldsymbol{\omega}'$. Households sort into neighborhoods based on this productivity and every household in a given neighborhood s must have the same weighted sum $\boldsymbol{b}\boldsymbol{\omega}'$. If we assume that $\boldsymbol{\omega} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ is a bivariate normal distribution describing the population distributions of parental education and children's ability, where

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_p \\ \mu_c \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{pp} & \sigma_{pc} \\ \sigma_{cp} & \sigma_{cc} \end{bmatrix}, \tag{2.14}$$

then the conditional distribution of parental schooling for families who rationally choose to live in neighborhood s and raise n children can be formulated as

$$\omega_p \big| \left(T(s) = \boldsymbol{b}\boldsymbol{\omega}' \right) \;\sim\; \mathcal{N}\left(\tilde{\mu}_p, \tilde{\sigma}_{pp}\right). \tag{2.15}$$

This is a truncated normal distribution, where the truncation is performed based on the selection rule (2.13). Standard calculations based on the properties of the conditional normal distribution yield

$$\tilde{\mu}_p = \mu_p + \big(T(s) - \boldsymbol{b\mu}\big)\frac{\phi\sigma_{pp} + \sigma_{pc}}{\phi^2\sigma_{pp} + 2\phi\sigma_{pc} + \sigma_{cc}} \tag{2.16}$$

and

$$\tilde{\sigma}_{pp} = \sigma_{pp} - \sigma_p\frac{\left(\phi\sigma_{pp} + \sigma_{pc}\right)^2}{\phi^2\sigma_{pp} + 2\phi\sigma_{pc} + \sigma_{cc}}, \tag{2.17}$$

where σ_p is the standard deviation of parental education, i.e., $\sigma_p = \left|\sqrt{\sigma_{pp}}\right|$.

Following the discussion in Section 2.1, we endogenize neighborhood quality by using the following self-consistency idea:

$$s = \mathbb{E}\left[e^{\omega_p}|T(s) = \boldsymbol{b\omega'}\right] = e^{\tilde{\mu}_p + \frac{1}{2}\tilde{\sigma}_{pp}}, \tag{2.18}$$

where $\tilde{\mu}_p$ and $\tilde{\sigma}_{pp}$ are given by the Equations (2.16) and (2.17), respectively. The intuition is simple. Self-selection of high-quality parents into high-quality neighborhoods is captured by the correlation between parental quality and neighborhood quality. The average schooling of adults living in a particular residential area s must represent the quality of that neighborhood. This way of thinking is essential in the social interactions literature based on self-consistent equilibria [see, for example, Brock and Durlauf (2001a,b)].

After plugging in the expressions for $\tilde{\mu}_p$ and $\tilde{\sigma}_{pp}$, the endogeneity condition (2.18) can be rewritten as

$$s = \Psi_1\left[\pi'(s)\right]^{\epsilon\Psi_2} s^{-\epsilon\Psi_2\left(\frac{1}{\epsilon}(\eta-\beta)+\beta-1\right)}, \tag{2.19}$$

where

$$\Psi_1 = e^{\mu_p - \Psi_2(\epsilon\ln[\kappa] + \boldsymbol{b\mu}) + \frac{1}{2}\tilde{\sigma}_{pp}} \tag{2.20}$$

and

$$\Psi_2 = \frac{\phi\sigma_{pp} + \sigma_{pc}}{\phi^2\sigma_{pp} + 2\phi\sigma_{pc} + \sigma_{cc}}. \tag{2.21}$$

Clearly, $\Psi_1 > 0$ and $\Psi_2 > 0$ are functions of both the economic parameters and the parameters of the distributions of exogenous household characteristics.

The interpretation of Ψ_2 is interesting and it will have important implications for our discussion in Section 4. Analogous to the calculations in the self-selection literature [see, for example, Heckman and Honore (1990)], Ψ_2 has a regression coefficient interpretation. To see this more clearly, suppose we regress ω_p on $\phi\omega_p + \omega_c$ as

$$\omega_p = \Psi_2(\phi\omega_p + \omega_c) + \nu, \tag{2.22}$$

where Ψ_2 is the regression coefficient and ν is an error term statistically independent of $\phi\omega_p + \omega_c$. This is a regression of parental education on willingness to pay for neighborhood quality. It is clear that $\Psi_2 = \text{Cov}(\omega_p, \phi\omega_p + \omega_c)/\text{Var}(\phi\omega_p + \omega_c)$, which yields the expression in Equation (2.21). Ψ_2 is positive, which means that parents with more education pay more for neighborhood quality on average. More precisely, Ψ_2 maps the unconditional distribution of ω_p to a conditional distribution of ω_p, where the conditional terms come from parental decisions on how many children to produce and where to raise them. In other words, it defines the rules of self-selection into neighborhoods. Notice that it appears in the formulas for both $\tilde{\mu}_p$ and $\tilde{\sigma}_{pp}$.

Two important results related to $\tilde{\mu}_p$ and $\tilde{\sigma}_{pp}$ should perhaps be reemphasized. Since $\Psi_2 > 0$, the truncated mean $\tilde{\mu}_p$ of parental education is greater than the population mean μ_p of parental education for high-quality neighborhoods. Reverse is true for low-quality neighborhoods; that is, the truncated mean of parental education is smaller than the population mean for low-quality neighborhoods. The punchline is that $\Psi_2 > 0$ is associated with positive sorting. The conditional variance of $\tilde{\sigma}_{pp}$ parental schooling is smaller than the population variance of parental schooling. Self-selection of higher-educated parents into higher-quality neighborhoods reduces the variation in parental human capital. Again, Ψ_2 drives how small this variance is relative to the population variance.

Housing prices can easily be derived from the equilibrium condition (2.19). The "differential rents equation" or the "bid rent curve" is given by

$$\pi'(s) = (\Psi_1)^{-1/(\epsilon\Psi_2)} s^{\frac{1}{\epsilon\Psi_2}+\frac{1}{\epsilon}(\eta-\beta)+\beta-1}, \tag{2.23}$$

which is the first derivative of the hedonic function of housing prices. Since $\Psi_1 > 0$, we conclude that $\pi'(s) > 0$; that is, housing prices is an increasing function of the neighborhood quality. Integrating the differential equation (2.23) with respect to s and assuming $\pi(0) = 0$ give us the pricing equation

$$\pi(s) = (\Psi_1)^{-1/(\epsilon\Psi_2)} \left(\frac{1}{\epsilon\Psi_2} + \frac{1}{\epsilon}(\eta-\beta) + \beta\right)^{-1} s^{\frac{1}{\epsilon\Psi_2}+\frac{1}{\epsilon}(\eta-\beta)+\beta}. \tag{2.24}$$

This aggregate equation defines a unique family of equilibrium pricing rules. The second-order conditions are globally satisfied for every consumer if $\Psi_2 > 0$ and $\frac{1}{\epsilon}(\eta-\beta)+\beta > 0$. $\Psi_2 > 0$ is satisfied by default and it ensures, by the regression interpretation, that better-educated parents are willing to pay more for neighborhood quality on average. $\frac{1}{\epsilon}(\eta-\beta)+\beta > 0$ ensures that the slope of the household-level willingness to pay measure (2.10) is positive. For this condition to hold, we need to impose the following restriction.

Assumption 1. *$\eta/(1-\epsilon) > \beta$.*

The function $\pi(s)$ is naturally connected to the notion of bid rent curve which is well known in the urban economics literature. Clearly, Assumption 1 joined with the result $\Psi_2 > 0$ makes $\pi(s)$ strictly positive and strictly increasing in s. This is needed for the sorting equilibrium to arise. In a sorting equilibrium, parents with higher schooling levels will win the bid for land in higher-quality neighborhoods. If the curvature coefficient $\frac{1}{\epsilon\Psi_2} + \frac{1}{\epsilon}(\eta-\beta)+\beta$ were negative, low human capital parents would outbid high human capital parents for high quality neighborhoods. The resulting equilibrium would involve mixing rather than stratification and the hedonic pricing function (2.24) would not be observed.

Positive sorting into higher-quality neighborhoods—in terms of the average education among the resident adults—is the main feature of the model developed by Nesheim (2001). The extension that we study in this paper has been able to replicate this central idea under certain assumptions. Our main question is to identify the conditions under which this model can accomodate the empirical fact that fertility declines as neighborhood quality increases. We are also interested in the question: under what conditions the endogenous neighborhood effects could actually work against this empirical regularity. Section 3 seeks answers to these questions and states precise conditions to describe the economic forces at work. Before we proceed to Section 3, we provide an interpretation of our hedonic function of housing prices along with a thorough comparison to Nesheim's original model.

2.5 Interpretation

In this subsection, we provide a basic interpretation of our hedonic pricing equation. We compare this equation to the analogous equation derived by Nesheim. We discuss the implications of considering fertility decisions as an integral part of hedonic housing prices. It is instructive to start with restating the utility function and the budget constraint of Nesheim's model in our notation. Parents choose s to maximize utility function

$$V\big(x, s|\boldsymbol{\omega}\big) = x + \left[s^{\eta} e^{\phi\omega_p}\right] e^{\omega_c}\Psi_0, \tag{2.25}$$

subject to the budget constraint $Y = x + \pi(s)$. Notice that fertility is fixed in this setup. Nesheim's version produces the pricing formula

$$\pi(s) = (\bar{\Psi}_1)^{-1/(\Psi_2)} \left(\frac{1}{\Psi_2} + \eta\right)^{-1} s^{\frac{1}{\Psi_2}+\eta}, \tag{2.26}$$

which is quite similar to (2.24) except that our version includes parameters ϵ and β in the slope coefficient, and $\bar{\Psi}_1$ is different than our Ψ_1 while Ψ_2's are the same. In Nesheim's version, the elasticity of housing prices with respect to neighborhood quality is $\rho_N = \frac{1}{\Psi_2} + \eta$. In our version [see Equation (2.24)], this elasticity is $\rho = \frac{1}{\epsilon\Psi_2} + \frac{1}{\epsilon}(\eta - \beta) + \beta$. Therefore, the relationship between ρ and ρ_N is simply

$$\rho = \frac{1}{\epsilon} \left[\rho_N - (1 - \epsilon)\beta \right]. \tag{2.27}$$

This equation says that ρ will be larger than ρ_N for all negative values of β and for some positive values of β only if $\beta < \rho_N/(1-\epsilon)$. In other words, when the cost of child rearing declines (or only slightly rises) with neighborhood quality, the elasticity of housing prices with respect to neighborhood quality in our model will tend to be greater than that in Nesheim's model. This has direct implications on curvature. In Nesheim's version, the pricing function is convex if $\rho_N > 1$ and concave if less than 1. Our $\pi(s)$, on the other hand, is convex if $\rho > 1$ and concave if ρ is less than 1.

The following proposition provides a complete description of curvature in our model.

Proposition 1. *The hedonic function of housing prices $\pi(s)$ tends to be more convex (or less concave) as β, ϵ, Ψ_2 goes down and η goes up.*

What happens to the curvature when η, β, and ϵ change is obvious. If η (the return parameter) goes up or β (the cost parameter) goes down, the attractiveness of higher-quality neighborhoods increases, and, therefore, $\pi(s)$ tends to be more convex (or less concave). A smaller ϵ means that parents' degree of altruism toward each additional child declines more slowly as n goes up. In other words, children have more weight in parental utility when ϵ is low. So, smaller ϵ raises the attractiveness of higher-quality neighborhoods, which makes the pricing equation more convex.

The case for Ψ_2 is subtler. By the regression interpretation, Ψ_2 measures the strength of the relationship between willingness to pay for neighborhood quality and parental education. From expressions (2.16) and (2.17), a decline in Ψ_2 is associated with a compression in average neighborhood quality across neighborhoods (i.e., neighborhoods become similar in terms of mean parental education). In other words, smaller Ψ_2 means that positive sorting is weaker. Weakened association between parental education and the demand for neighborhood quality busts the overall demand for higher quality neighborhoods since people in the left tail will also raise their bids. Therefore, the pricing function becomes more convex (or less concave).

There is an interesting link between curvature of (2.24) and segregation. When $\pi(s)$ is convex, the slope is small for lower values of s and it is large for higher values of s. In other words, incremental improvements are easier for small s and harder for large s. Therefore, segregation is harder. When $\pi(s)$ is concave, on the other hand, segregation is easier by the same logic. We will come back to this discussion in the next section.

3 Fertility and Neighborhood Quality

In this section, we formally establish the key properties of the equilibrium relationship between fertility and neighborhood quality. From (2.7), the equilibrium fertility level for a family with characteristics $\boldsymbol{\omega} = (\omega_p, \omega_c)$, who lives in neighborhood s, is determined according to

$$n = [\alpha \Psi_0 (1-\epsilon)]^{1/\epsilon} \, e^{\frac{1}{\epsilon}(\phi\omega_p + \omega_c)} s^{\frac{1}{\epsilon}(\eta - \beta)}. \tag{3.1}$$

This equation describes the equilibrium relationship between n and s for each family $\boldsymbol{\omega}$. Let $\bar{n}$ denote the average fertility across families living in the neighborhood s, i.e., $\bar{n} = \mathbb{E}[n|s]$. Alternatively, we can write $\bar{n} = \mathbb{E}[n|T(s) = \boldsymbol{b\omega'}]$. Then, by using (2.12) and (2.23),

$$\bar{n} = \frac{1}{\frac{\eta}{1-\epsilon} - \beta} (\Psi_1)^{-1/(\epsilon\Psi_2)} s^{\frac{1}{\epsilon\Psi_2} + \frac{1}{\epsilon}(\eta - \beta)}. \tag{3.2}$$

Obviously, for $\bar{n} > 0$, the condition $\eta/(1-\epsilon) > \beta$ must be satisfied. But we already impose this restriction by Assumption 1, which is required for the second-order conditions to hold. Notice that we still have not imposed any restrictions on the sign of β. Let ρ_f denote the elasticity of fertility with respect to neighborhood quality, which is key to our analysis in this section. From (3.2), this elasticity is

$$\rho_f = \frac{1}{\epsilon} \left[\frac{1}{\Psi_2} + \eta - \beta \right]. \tag{3.3}$$

The following proposition formally establishes the sign of the relationship between $\bar{n}$ and s.

Proposition 2. *The elasticity of fertility with respect to neighborhood quality is negative if $\beta > \eta + \frac{1}{\Psi_2}$ and it is positive otherwise.*

This result is intuitive. β is the cost curvature and η is the expected schooling curvature of neighborhood quality. $\eta + \frac{1}{\Psi_2}$ is positive, which means that to get the preferred result that $\rho_f < 0$: (1) we need to have $\beta > 0$ and (2) β needs to be somewhat large. The interpretation is the following. To have $\rho_f < 0$, the cost of child rearing needs to be an increasing function of neighborhood quality and it must increase rapidly with neighborhood quality. Using our results from Section 2.5, this means that $\rho_f < 0$ when small increments in housing prices can easily lead to segregation. We conclude that the quantity-quality tradeoff in the neighborhood dimension holds when it is more costly to raise children in higher-quality neighborhoods.

It is interesting to discuss the economic forces that may lead to $\rho_f > 0$. Fertility is an increasing function of neighborhood quality when $\beta < \eta + \frac{1}{\Psi_2}$. The following conditions must jointly hold to get this result: (1) β can be positive or negative and (2) if $\beta > 0$, it must be small. We interpret this as follows. In order to obtain the result that fertility goes up with neighborhood quality (i.e., $\rho_f > 0$), we need to have a negative or only a slightly positive value for the cost curvature β. In terms of the housing prices, this means that larger incremental price movements are needed to generate residential segregation.[5]

We conclude that the structure of the cost of child rearing is an integral component of the relationship between fertility, neighborhood quality, and housing prices. If it is the case that greater child-rearing resources are required in higher-quality neighborhoods, the sign of the relationship between fertility and neighborhood quality becomes negative. If high-quality neighborhoods facilitates child rearing by encouraging peer interactions and spillovers (which lower costs), then our setup predicts that fertility may rise with neighborhood quality. This latter result seems to be irrelevant when consistency with the empirical evidence is the concern (stylized fact 1). Nevertheless, it is an interesting result and may hold under specific

[5]The parameters η, ϵ, and Ψ_2 are also important for our results. η represents the benefits of neighborhood quality and its impact can be interpreted similar to β but in the opposite direction. An increase in ϵ means that parents put a higher weight on the utility they get from their children's success. Thus, with a higher ϵ, the association between fertility and neighborhood quality will be weaker (i.e., $|\rho_f|$ will be smaller). As we describe in Section 2.5, an increase in Ψ_2 leads to an overall decline in the demand for higher-quality neighborhoods. Therefore, relationship between fertility and neighborhood quality tend to be negative as parents will be more willing to tradeoff quality and quantity.

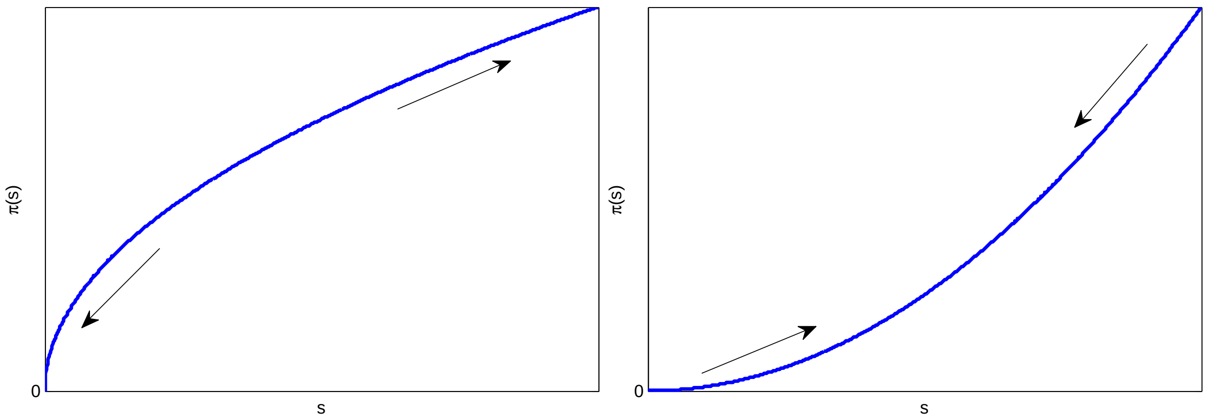

Figure 3.1: Implications of curvature for the link between fertility and neighborhood quality.

circumstances.

There is an interesting link between the curvature of the hedonic function of housing prices and the fertility-neighborhood quality relationship. This link is a joint implication of Propositions 1 and 2. Notice that ρ and ρ_f are closely related. This relationship can be expressed by equation $\rho = \rho_f + \beta$. Any factor that leads to an increase in ρ_f also increases ρ.[6] This relationship connects the neighborhood selection problem to the quantity-quality theory of children. Specifically, it allows us to examine whether the quantity-quality tradeoff holds at the neighborhood level or not.

We compare two different cases: (1) negative ρ_f and (2) positive ρ_f (in particular, $\rho_f > 1-\beta$). Under the first scenario (left panel in Figure 1), the hedonic pricing function tends to be more concave (or less convex); that is, at very low neighborhood quality levels (near zero), an incremental improvement in neighborhood quality requires a much larger increase in the willingness to pay measure than it does at high quality levels. This means that in regions where quantity-quality tradeoff is strong, segregation is a serious problem. Segregated neighborhoods produce more children who are likely to be segregated in the future, whereas higher quality neighborhoods produce less children who are better educated. This leads to a vicious circular pattern. In contrast, under the second scenario (right panel in Figure 1), the pricing function tends to be more convex (or less concave). In this case, when neighborhood quality is close to zero,

[6]This statement is true for β as well. Notice that $\partial\rho/\partial\beta = \partial\rho_f/\partial\beta + 1$, and $\partial\rho_f/\partial\beta < 0$. But clearly, $\partial\rho_f/\partial\beta = -1/\epsilon < -1$, which implies that $\partial\rho/\partial\beta < 0$.

incremental improvements are easier than they are at higher quality levels. The implications for segregation is reversed now; that is, in regions where high-quality neighborhoods facilitate child rearing by reducing costs (i.e., suburban school districts), segregation is much harder and population is much more homogeneous in terms of fertility and education.

The class of models we discuss in this paper is econometrically identifiable. See Nesheim (2001), Ekeland, Heckman, and Nesheim (2004), and Heckman, Matzkin, and Nesheim (2010) for further discussion on identification, estimation, and more.

4 An Alternative Formulation of Neighborhood Effects

Up to this point, we have closely followed Nesheim's model to consider fertility decisions within the analysis of housing prices. One important aspect of the neighborhood selection problem is missing in the current setup. We summarize this missing point as follows. Incorporating fertility decisions brings another dimension of human capital investment into consideration: peer effects are important in schooling success and, therefore, parents tend to move toward the locations where high-quality children are produced to increase their children's exposure to high-quality peer effects. This means that explicitly modeling the feedbacks between peer effects and neighborhood quality mediated through fertility is of interest. In the light of this interpretation, this section reformulates Nesheim's model by considering that neighborhood quality is described by the average ability of children living in the neighborhood rather than the average human capital of adults. We discuss the advantages and disadvantages of this alternative approach.

To differentiate notation, we let $\tilde{s}$ denote neighborhood quality. All the other properties of the model we studied in Sections 2 and 3 hold, except the way we endogenize neighborhood quality with self-consistent bunching. Now we define

$$\tilde{s} = \mathbb{E}\left[e^{\omega_c} | T(\tilde{s}) = \phi\omega_p + \omega_c\right] = e^{\tilde{\mu}_c + \frac{1}{2}\tilde{\sigma}_{cc}}, \tag{4.1}$$

whereas in the previous version, we had

$$s = \mathbb{E}\left[e^{\omega_p} | T(s) = \phi\omega_p + \omega_c\right] = e^{\tilde{\mu}_p + \frac{1}{2}\tilde{\sigma}_{pp}}. \tag{4.2}$$

In words, $\tilde{s}$ describes neighborhood quality in terms of the average ability of the children produced in the neighborhood, while s describes neighborhood quality in terms of the average education of adults living in the neighborhood. The feedback between fertility and neighborhood quality is present in both version and it comes from the conditioning term $T(\cdot) = \boldsymbol{b\omega}'$. This expression has been derived from the parents' problem and it is a combination of the first order conditions with respect to neighborhood quality and fertility. For s, the fertility decisions of parents affect neighborhood quality through the interaction between parents' education and the child-rearing conditions in the neighborhood. For $\tilde{s}$, on the other hand, the fertility decisions affect neighborhood quality through the interaction between children's ability and the child-rearing conditions in the neighborhood.

Next we sketch out the solution for this extended model. The parameters $\tilde{\mu}_c$ and $\tilde{\sigma}_{cc}$, which are the mean and the variance for the conditional distribution of ω_c, are calculated similarly. The formulas for $\tilde{\mu}_c$ and $\tilde{\sigma}_{cc}$ will be only slightly different than those for $\tilde{\mu}_p$ and $\tilde{\sigma}_{pp}$, but their interpretations will differ significantly. We get

$$\tilde{\mu}_c = \mu_c + \left(T(\tilde{s}) - \boldsymbol{b\mu}\right) \underbrace{\frac{\phi\sigma_{pc} + \sigma_{cc}}{\phi^2\sigma_{pp} + 2\phi\sigma_{pc} + \sigma_{cc}}}_{\tilde{\Psi}_2}, \tag{4.3}$$

and

$$\tilde{\sigma}_{cc} = \sigma_{cc} - \sigma_c \frac{\left(\phi\sigma_{pc} + \sigma_{cc}\right)^2}{\phi^2\sigma_{pp} + 2\phi\sigma_{pc} + \sigma_{cc}}. \tag{4.4}$$

This means that we will have $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$, which will be different from Ψ_1 and Ψ_2 in that $\tilde{\mu}_p$ and $\tilde{\sigma}_{pp}$ will be replaced by $\tilde{\mu}_c$ and $\tilde{\sigma}_{cc}$. So the hedonic function of housing prices is formulated as

$$\tilde{\pi}(\tilde{s}) = (\tilde{\Psi}_1)^{-1/(\epsilon\tilde{\Psi}_2)} \left(\frac{1}{\epsilon\tilde{\Psi}_2} + \frac{1}{\epsilon}(\eta - \beta) + \beta \right)^{-1} \tilde{s}^{\frac{1}{\epsilon\tilde{\Psi}_2} + \frac{1}{\epsilon}(\eta-\beta)+\beta}. \tag{4.5}$$

As a result, all the qualitative features of our results, including Propositions 1 and 2, will remain unaltered. But the magnitude of ρ_f (the elasticity of fertility with respect to neighborhood quality) will change depending on whether we use s or $\tilde{s}$ in the analysis. To understand this point, it is useful to understand how $\tilde{\Psi}_2$ differs from Ψ_2, since the effect of the analyst's choice between s or $\tilde{s}$ on these elasticities operates only through Ψ_2 (or $\tilde{\Psi}_2$). Remember that this parameter has a regression coefficient interpretation and it maps the unconditional distribution (of ω_p or ω_c) to the corresponding conditional distribution. More precisely, it defines the rules of self-selection into neighborhoods and the larger it is, the stronger the degree of selectivity. Whether $\tilde{\Psi}_2$ is larger than Ψ_2 or not comes down to a comparison between

$$\phi\sigma_{pc} + \sigma_{cc} \quad \text{and} \quad \phi\sigma_{pp} + \sigma_{pc}. \tag{4.6}$$

If the one on the LHS is larger, then $\tilde{\Psi}_2 > \Psi_2$. From Propositions 1 and 2, the greater Ψ_2, the smaller ρ_f, and the more concave (or less convex) the hedonic function of housing prices (i.e., segregation is easier). It is not possible to derive exact conditions for this comparison. Nevertheless, it will be useful to work out some special cases. For example, let $\sigma_{pc} = 0$, then $\tilde{\Psi}_2 > \Psi_2$ only if $\sigma_{cc} > \phi\sigma_{pp}$. Suppose now that $\sigma_{pp} = \sigma_{cc}$. Then, $\tilde{\Psi}_2 > \Psi_2$ only if $\sigma_{pc} > \sigma_{pp}$ and $\phi > 1$.

It is important to briefly discuss the pros and cons of these two alternatives. The advantage of introducing $\tilde{s}$ over s is that it has a potential to more effectively handle the problems highlighting *peer effects*. Moreover, it fits better to the quantity-quality theory of children; that is, $\tilde{s}$ is a more direct measure of child quality. The advantage of s is that it relates parental education and, therefore, parental income to neighborhood selection in a more appropriate way. Using one versus another also affects the interpretation of our key parameter β. For s, positive β means that cost of child rearing goes up with average human capital of adults in the neighborhood, whereas, for $\tilde{s}$, it means that the child-rearing cost increases with average ability of children in the neighborhood. Positive β may look sensible for the version with s, but it is not quite relevant for the one with $\tilde{s}$. As a result, both the research questions to answer and the interpretations of modeling elements differ across two versions, although the mechanics are similar.

5 Concluding Remarks

In this paper, we develop a theoretical framework in which altrustic heterogeneous parents sort into neighborhoods based on the average human capital of the adult residents living in those neighborhoods. Our main contribution is that we relax the fixed fertility assumption which is typical in the endogenous sorting literature. We build on Nesheim's equilibrium sorting model to incorporate the stylized fact that fertility is lower among high-educated parents and, therefore, in high-quality neighborhoods. We also provide a detailed analysis of the economic forces that could strengthen or weaken this result. The model provides a rich theoretical basis to study the relationship between family's fertility and location choices. We state sufficient conditions for an equilibrium to exist and analyze some of the patterns that result in an equilibrium of this model.

A Appendix

To understand the similarities and differences between our model and the Barro-Becker model, it is instructive to briefly introduce the basic features of the Barro-Becker model first. In the standard Barro-Becker framework, parents receive utility from own consumption, the number of children, and children's consumption. For concreteness, it is possible to describe parental optimization problem in the Barro-Becker model in two steps. In the first step, parents allocate goods between own consumption and child rearing. In the second step, given child-rearing resources—determined in the first step—parents face another tradeoff: whether to produce a lot of children child each with modest earning potentials or invest heavily in the earning potential (and, therefore, future consumption) of a smaller number of children.

There are two crucial assumptions in the Barro-Becker model: (1) the goods cost of producing an additional child is an increasing function of parental education and (2) children's education does not explicitly depend on parental education. The first assumption is a direct implication of the well-known time allocation problem between working and child rearing activities. The second one, which is subtler, is an artifact of the assumption that children's earnings potentials depend on bequests (which do not directly depend on parents' education) and children's raw ability (which is uncorrelated with parental education). We modify both of these assumptions since our goal is to develop a fertility model at the neighborhood level. Models in the endogenous neighborhood effects literature have their own specific structures and our goal is to embed endogenous fertility into these models.

We start our comparison by stating three major similarities between our model and the Barro-Becker model. First, the formulation of parental altruism is identical across the two models. Specifically, both models have a fixed pure altruism parameter (α) along with a variable altruism component ($n^{-\epsilon}$) as a decreasing function of the number of children. Second, in both models, parents get utility from three sources: own consumption, the number of children, and children's consumption.[7] Finally, the goods cost of producing an additional child is

[7] In our model, children's future earnings potential (education) is used as a proxy for their future consumption.

an increasing function of parental education, which is implicitly related to parental time allocation. This is the assumption (1) we state in the previous paragraph.

There are three main differences between our model and the Barro-Becker model. First, we eliminate bequests which are basically direct investments in children's earnings potential and, instead, we introduce an alternative investment component: neighborhood quality (s). Barro-Becker model formulates bequest as a perfectly substitutable (i.e., additive) cost and outcome component, whereas neighborhood quality is a complementary (i.e., multiplicative) input both as a cost component and an outcome component. The intuition is the following. Our goal is to perform an analysis of self-selection into neighborhoods while recognizing that fertility dimension is also endogenous. "Self-selection into a neighborhood" roughly means that parents with similar education levels choose to live close to each other. This implies a positive sorting between parental quality and neighborhood quality, which naturally arises via a complementarity assumption. This is the key to reconcile Nesheim's model and the Barro-Becker model. Second, unlike the assumption (2) in the Barro-Becker model (see above), our model allows for a positive correlation between parental education and children's outcomes. This is the direct consequence of the complementarity assumption, which is needed to generate positive sorting, as we discuss above. In Appendix A, we shut down the correlation between parental human capital and children's outcome to demonstrate that assumption (2) in the Barro-Becker model kills the potential for positively assortative matching between parental quality and neighborhood quality, which lies at the heart of neighborhood selection models. Finally, our model is static whereas the original Barro-Becker model is dynamic. They use a dynastic utility structure in which the dynastic head cares about all future generations subject to a decreasing weight. A static formulation is more convenient for our purposes.

A.1 One

Therefore, the parental resource constraint is

$$Y = x + \pi(s) + G(n, s, \omega_p), \tag{A.1}$$

where $G(n, s, \omega_p)$ is the goods cost of raising n children for parents with education level ω_p in location s. We focus on the special case in which $G(n, s, \omega_p) = ng(s)e^{\delta\omega_p}$, where $\delta > 0$.

A.2 Two

The Barro-Becker model assumes that children's outcomes are independent of parental quality, whereas in our model we need parental quality to be an integral part of children's outcomes. This is needed to generate positive sorting between parents and neighborhoods. In this appendix, we show that imposing independence between parents' education and children's outcomes may lead to negative sorting, which is against the stylized facts.

Under the independence assumption, parents' problem becomes

$$\max_{s,n}\Big\{Y - \pi(s) - ns^{\beta}e^{\delta\omega_p} + \alpha n^{1-\epsilon}s^{\eta}e^{\omega_c}\Psi_0\Big\}. \tag{A.2}$$

Notice that we set $\phi = 0$ in this version. We further set $\sigma_{pc} = 0$; that is, the covariance between parents' education and children's ability is zero. These two assumptions shut down all the dependence.

The derivations will be the same but the implications of this model will be substantially different. Most importantly, we will have $\zeta = -\delta(1-\epsilon)$ which is necessarily negative. To see how this alters the predictions of the model, it suffices to write the sorting condition (analogue of 2.16) and the self-selection condition (analogue of 2.17) as follows:

$$\tilde{\mu}_p = \mu_p + \big(T(s) - \boldsymbol{b\mu}\big)\frac{\zeta\sigma_{pp}}{\zeta^2\sigma_{pp} + \sigma_{cc}}, \tag{A.3}$$

and

$$\tilde{\sigma}_{pp} = \sigma_{pp} - \sigma_p\frac{\zeta^2\sigma_{pp}^2}{\zeta^2\sigma_{pp} + \sigma_{cc}}. \tag{A.4}$$

Notice that the self-selection condition $\tilde{\sigma}_{pp} < \sigma_{pp}$ still holds since $\frac{\zeta^2\sigma_{pp}^2}{\zeta^2\sigma_{pp}+\sigma_{cc}} > 0$. But the sorting condition implies that the direction of sorting may be reversed. To see this more clearly, suppose that we fix $T(s) = t > 0$ such that in our original model

$$t - \zeta^* \mu_p - \mu_c > 0, \tag{A.5}$$

where ζ^* is given by (2.11); that is, it is positive. Under the original model, clearly $\tilde{\mu}_p > \mu_p$ for $T(s) = t$. Consider now the version we discuss in this appendix. Observe that $\frac{\zeta \sigma_{pp}}{\zeta^2 \sigma_{pp} + \sigma_{cc}} < 0$ since $\zeta < 0$. Obviously, for $T(s) = t$,

$$t - \zeta^* \mu_p - \mu_c > 0 \quad \Rightarrow \quad t - \zeta \mu_p - \mu_c > 0. \tag{A.6}$$

As a result, $\tilde{\mu}_p < \mu_p$. For this reason, we deviate from the independence assumption in the Barro-Becker model and we adopt Nesheim's formulation (i.e., pozitive ϕ and σ_{pc}).

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