

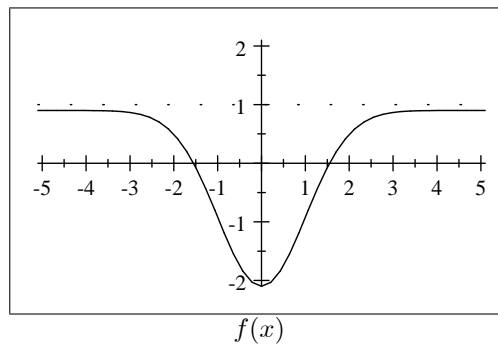
Math 101, Exemption Exam, September 27 2006

Time allowed is 100 minutes.

There are 6 questions worth a total of 100 points.

You must show your work and give explanations, no work equals no credit!

1. (10 points) The graph of  $y = f(x)$  is shown in the figure. Sketch the graph of  $y = -2f(x+1) - 1$ . Label any intercepts or asymptotes that can be determined.



2. (10 points) Find the equation of the tangent line to the curve  $x^3y^2 + y^3 = 2$  at the point  $(1, 1)$ .

3. (30 points) Compute the following. **You must show your work and give explanations, otherwise you get no credit!**

(a)  $\frac{d}{dx} \left( 2xe^x - \frac{1}{\sqrt{x}} \right)$

(b)  $\lim_{x \rightarrow \infty} \frac{e^{-3x}}{e^{-2x} + e^{-x}}$

(c)  $\lim_{x \rightarrow 0} \frac{\sin 2x - 2x}{x^3}$

(d)  $\int \frac{\ln(x)}{x} dx$

(e)  $\int \frac{\cos 2x}{(\sin 2x + 1)^2} dx$

4. **(20 points)** Determine whether the following statements are true or false. If true, explain why. If false, show why by giving a counter example or in another meaningful way. **(Each explanation is worth 4 points)**

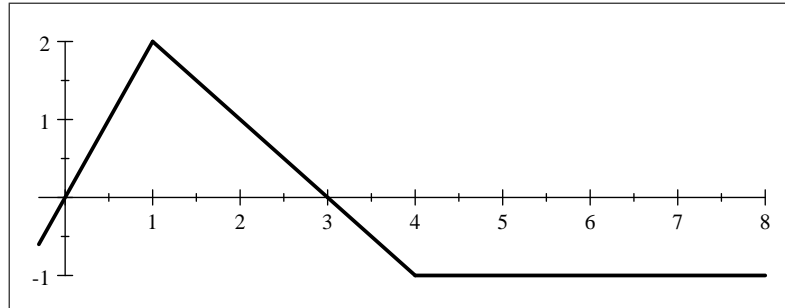
(a) The area under the curve  $y = x^{-3}$  above the  $x$  axis from  $x = 100$  to  $\infty$  is infinite.

(b) The function  $f(x) = \frac{x - \sin x}{x}$  has a vertical asymptote at  $x = 0$ .

(c) If  $y = L(x)$  is the tangent line of  $f(x) = e^x$  at  $x = 0$ , then  $e^x \geq L(x)$  for every  $x$ .

(d) If the sides  $x$  and  $y$  of a rectangle change in time with  $\frac{dx}{dt} = -1$ ,  $\frac{dy}{dt} = 2$ , then the area is decreasing when  $x = 3$  and  $y = 2$ .

5. **(15 points)** The graph of the derivative  $g'(x)$  a function  $g(x)$  is given below.



- (a) Find the value of a number  $b > 0$  so that  $\int_0^b g'(x) dx = 0$ .  
(b) If  $g(3) = 0$ , evaluate the limit

$$\lim_{x \rightarrow 3} \frac{g(x)}{(x-3)^2}.$$

6. **(15 points)** Find a point on the curve  $y = \sqrt{x}$  whose distance to the point  $(\frac{3}{2}, 0)$  is the smallest.