Multi-Unit Auctions with Resale

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Abstract. We study multi-unit auctions in the presence of resale opportunities among bidders who have either single- or multi-unit demand. We analyze equilibrium for Vickrey auctions with separate bidding and with package bidding when the valuations of bidders with single-unit demand are either independently distributed or the same. For both cases, Vickrey auction with package bidding allocates the units efficiently whether the resale markets are present or not. When the valuations of the bidders with single-unit demand are the same, although Vickrey auction with separate bidding allocates the objects efficiently when the resale markets are prohibited, it is no longer the case when resale markets are present. Hence, the resale opportunity leads to resale activity. When the valuations of the bidders with single-unit demand are independent, this auction does not allocate the objects efficiently in any equilibrium with or without resale. (JEL D44)

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1. Introduction

The large scales of privatization of assets such as spectrum licenses, gas and electricity supply attract attention to the multi-unit auctions (see e.g. Ausubel, 2004; Krishna and Perry, 2000). Unlike the auctioning of the non-government owned assets, the efficiency of the allocation rather than the revenue maximization is the main objective of these auctions. For example, the Federal Communications Commission (FCC) as directed by the US Congress primarily considers the efficient allocation of spectrum licenses by the auction (see Ausubel and Cramton, 1999; McMillan, 1994). Absence of post-auction trade (resale activities) between bidders is considered as a success of the auction design when the aim is to allocate objects efficiently at the auction stage (see e.g. Cramton, 1998). This paper investigates how the allowing or preventing of resale markets affects the efficiency in multi-unit-auctions. We show that the possibility of the resale may make an otherwise efficient auction inefficient.[2] However, there are auctions which remain allocating the objects efficiently even if the resale markets are allowed.

In allocating objects efficiently, some versions of Vickrey auctions are often used in practice despite the potential drawback of low revenues. For example, the spectrum licenses were sold by second-price auctions in New Zealand where the highest bids were much higher than the revenue (see Milgrom, 2004). The attractiveness of the Vickrey auction is that it extracts the true value of the bidders via simple strategies that are independent of the underlying distribution of the values (see Ausubel and Milgrom, 2006). In auction theory, equilibrium in this type of strategies are called weakly information invariant equilibrium (see Börgers and McQuade, 2007) or robust equilibrium (see e.g. Hafalir and Krishna, 2008) which is a generalization of ex-post equilibrium to extensive form games. Non-robust equilibria are sensitive to risk attitudes or behavioral biases (see e.g. Filiz Ozbay and Ozbay, 2007). Moreover, it may be too much to expect from the bidders to discover a non-robust equilibrium. For example, in single unit settings, the distribution dependent equilibrium of first-price sealed bid auction is more complicated than truthful value bidding equilibrium of second-price sealed bid

[2] We obtain this result without having speculators in the equilibrium. The speculators are the type of bidders who aim to win the auction in order to make profit in the resale stage rather than the use of object for themselves. In the literature, resale opportunity motivates the speculators to bid in the auction (see e.g. Garratt and Tröger, 2006.)

auction. Although both are efficient in theory, experimentally it is shown that second-price has better efficiency rate. Furthermore, the value bidding robust equilibrium has been always taken as the benchmark for experimental findings in second price auctions (see e.g. Kagel and Levin, 1993, and Garratt and Wooders, 2004). Hence, it is important to study existence of robust efficient equilibria when the main objective of the auctioneer is efficiency.

The aforementioned reasons motivate us to analyze Vickrey auctions. In selling multi unit objects, the auctioneer can use Vickrey auction to sell the objects either in separate auctions or in a single auction allowing package bidding. Hence, we consider two types of Vickrey auction mechanisms:

- *Vickrey Auction with Separate Bidding* where each unit of the good is auctioned separately. The winner of each auction is the highest bidder of that auction, and she pays the second highest bid.
- *Vickrey Auction with Package Bidding* where all the units are sold in a single auction. Each bidder submits bids for each package of units that she is interested and the units are allocated to the bidders who have the highest total bids for two objects. Each winner pays the price that is equivalent to the externality she exerts on other competing bidders.

When multi-unit goods are auctioned, the bidders' demand may differ depending on how large or small they are. For example, in the FCC auctions, some bidders are smaller than the others because of their geographical restrictions, financial constraints, or having different uses of the objects, and therefore they would like to bid only on a small number of the licenses. We model a situation where there are large bidders who would like to buy multi-units and small bidders such that each would like to buy a single unit (see Krishna and Rosenthal, 1996, and Chernomaz and Levin, 2009 for similar settings[3]). There are complementarities between objects such that for large bidders having a package of objects is more valuable than the sum of values of disjoint subpackages. For example, in spectrum auctions, some big telecommunication companies might value multiple licenses to serve large geographical locations more than the sum of the values of each license because marginal cost of serving to larger area can be decreasing. In

[3] Krishna and Rosenthal (1996) develop this model in order to study the FCC auctions of licenses for the radio-frequency spectrum. Chernomaz and Levin (2009) study theoretically and experimentally first price auctions in this setting. Neither of these models allows for post-auction resale.

our model, we consider one large bidder (the global bidder) and N small bidders (the local bidders). There are N units of the good to be sold. The global bidder is interested in all units of the good and each local one is interested in a single unit. Signals regarding the valuation of each bidder are privately observed. We assume that the signals of the global and local bidders are independent. For the signals of the locals, we consider two cases: (i) symmetric locals who have the same signals, (ii) independent locals whose signals are independently drawn. Two small telecommunication firms interested in radio-frequencies in different areas might have independent valuations due to the demand differences in different geographical regions. However, small timber buyers with similar facilities and with similar production costs might have identical valuations for the small units of logs that they are interested in.

We study Vickrey auctions with separate and package bidding and with and without resale possibilities. The resale markets are designed so that the winners of the auction can make take-it-or-leave-it offer to the unsuccessful bidders of the auction as in Hafalir and Krishna (2008).

We show that Vickrey auction with separate bidding allocates the objects efficiently only when the resale market is prohibited and the locals are symmetric. Given that the package bidding may not be feasible in some cases, Vickrey auction with separate bidding becomes an alternative. The problem is that, when the resale markets are allowed, this auction does not have any efficient robust equilibrium. Due to complementarities for the global, there is a room for the locals to bid more than their value in order to resell the object at a higher price. In order to eliminate this speculative bidding, in the equilibrium the global bidder will bid more aggressively. Hence, the existence of the resale markets will lead to resale. This is a striking result because in the literature when the resale possibility leads to resale activity, it happens through speculators. Here, it happens in order to prevent speculators. When the signals of the locals are independent, Vickrey auction with separate bidding does not allocate the objects efficiently under either resale or without resale cases. This result is true even if we consider non-robust Nash Equilibria.

On the other hand, Vickrey auction with package bidding is appealing because it allocates the objects efficiently whether the locals are symmetric or independent. We show that this property of equilibrium carries over when resale markets are allowed. The package bidding will control the speculation of the locals, and hence the resale will not occur when the objects are allocated via Vickrey auction with package bidding.

Although, the revenue is not the main concern of the government auctions, the resale activity is typically considered as a loss of the seller from the gain of the trade. Contrary to this intuition, we show that the resale possibility may increase the seller's revenue from a given mechanism.

Literature Review:

The existing models of auctions with resale in the literature mainly consider single unit problems. Although demand asymmetry together with complementarities makes our model sound analogous to asymmetric single unit auctions, we show that having multi-units creates major differences in terms of results of the two settings. Hafalir and Krishna (2008) find that for the single unit case, the second-price auction with and without resale both have the same efficient equilibrium. Although our global bidder seems similar to their strong bidder in terms of how she values the objects when the locals are symmetric, in Vickrey auction with separate bidding, the global bidder bid differently whether the resale is possible or not. Therefore, the results known for single unit cases do not carry over to multi-unit cases.

The literature on auctions with resale provides five main reasons for resale: (i) New information regarding the values of the objects arrives after the auctions, (ii) new buyers arrive after the auction is over, (iii) asymmetry in auction may lead to inefficient allocation, (iv) presence of speculators in the auction, (v) misperception of resale markets. Our setup is closest to the third type because multi-unit auction setting provides a natural asymmetry in terms of demand of bidders and therefore may lead to inefficient allocation under different formats.

Haile (1999, 2000, 2001, 2003) and Gupta and Lebrun (1999) examine private value auctions where the buyers are uncertain about their values while bidding in the auction and information about the values is received in resale stage. They found that the revenue of the seller increases by the resale opportunity if she has strong bargaining position in the resale market. Lange, List and Price (2004) experimentally study symmetric first price auctions where bidders' valuations are initially noisy and there is a room for resale. Haile (1999) provides a model where new buyers show up after the auction is over.

Maskin and Riley (2000) study effects of asymmetries among bidders' value distributions and provide characterizations of revenue comparisons for first- and second-price auctions without resale. Hafalir and Krishna (2008) analyze first- and second-price auctions with resale under asymmetry. They consider auctions with two bidders and show that truthtelling in second-price is an equilibrium with and without resale. First-price with resale leads to higher revenue. This theory is experimentally tested by Georganas and Kagel (2009). Zheng (2002) shows that revenue maximizing allocation can be implemented via repeated resale. Xu, Levin, and Ye (2009) study sequential auctions to sell two units to two bidders in the presence of synergies and show that the effect of resale on revenue is ambiguous.

Garratt and Tröger (2006) compare English, second-price, Dutch and first-price auctions when there are 0-value bidders (speculators). They show that resale can increase the revenue of the auctions and revenue equivalence does not exist among these formats. Georganas (2003) experimentally observes that in symmetric English auctions with resale there are significant deviations from equilibrium bidding strategies.

2. Model

The basics of the model are first introduced by Krishna and Rosenthal (1996) (see also Rosenthal and Szentes, 2003, and Chernomaz and Levin, 2009). There are N>1 objects for sale and N+1 bidders. One bidder, who is called the global bidder, is interested in all of the objects and the other N bidders, the local bidders, are each interested in one object. Throughout the paper, the global bidder is denoted by g, and local bidder *i* is denoted by l_i. Each bidder is a risk neutral individual. Before the auction, each bidder privately observes a signal. Each bidder's signal is distributed by a distribution function F on $[\underline{s}, \overline{s}]$. The signal of the global is independent of the signals of the locals. In Section 3, the locals are symmetric in the sense that they have the same signal and in Section 4, the signals of the locals are independently drawn.

There are complementarities between objects for the global. If she gets a package of objects, she values it more than the sum of the values of any disjoint subpackages. Formally, the value of receiving one object is the signal of that individual, $v^1(s) = s$; the value of receiving $n \in \{2, ..., N\}$ objects by the global with signal s is denoted by $v^n(s)$ where $v^n(s)$ is strictly increasing and continuous in s and for any signal s, $\frac{v^n(s)}{n}$ is increasing in n.[4] Normalize $v^N(\underline{s}) = N\underline{s}$. The value of the objects for the seller is zero.

The objects are first auctioned to the bidders and then the bidders can trade the objects among each other at a post-auction stage.

Auction Stage:

We consider two types of Vickrey auctions: (i) Vickrey auction with separate bidding where the global cannot submit bids for packages with more than one objects. This is equivalent to the mechanism where objects are sold separately in N simultaneous second-price sealed bid auctions, and (ii) Vickrey auction with package bidding where there is one auction to sell all objects. In this case the bidders are allowed to submit bids for any package of objects that they are interested in.

- *Vickrey Auction with Separate Bidding:* N simultaneous auctions are run, each is called Market *i* denoted by M_i for $i = 1, ..., N$. Local *i* is present only in M_i, for $i \in \{1, ..., N\}$. The global bidder is present in all of the markets. In market M_i, each bidder of that market (local *i* and the global) submits a bid, the highest bidder gets the object *i*, and pays the bid of the unsuccessful bidder (the second highest bid) of that market.
- *Vickrey Auction with Package Bidding*[5]*:* N objects are sold in one auction. Local *i* submits a bid for object *i*, the global submits bids for each package of objects. The

[4] One can easily show that the efficiency results of the paper still hold without the assumption that for any signal s, $\frac{v^n(s)}{n}$ is increasing in n provided that $v^n(\underline{s}) = n\underline{s}$ for all *n*. This is assumed to simplify the notation in the proofs. Under this assumption, the efficient allocation gives all the objects either to the global or to the locals.

[5] This is equivalent to the second-price auction when a single object is auctioned. It is considered as the appropriate generalization of the second-price auction for multi-unit settings (see e.g. Krishna 2002).

objects are allocated to the bidders who have the highest total bids for them. Each winner pays the price that is equivalent to the externality she exerts on other competing bidders. Independent of the auction format, all bids are announced after the auctions.[6]

Resale Stage:

After the auction stage is completed, the winner of an object may offer a price to sell the object to an unsuccessful bidder of the auction stage. All the take-it-or-leave-it offers are made simultaneously and cannot be negotiated.[7] If an offer is accepted, then the trade takes place at the offered price. If it is rejected, the winner of the auction keeps the object.

Equilibrium:

As the reasons discussed in the introduction, throughout the paper, we not only check when auction allocates the objects efficiently but also look for existence of robust equilibrium where the strategies of the buyers (both in the auction stage and in the resale stage when it exists) do not depend on the distribution of signals. This refinement is appealing because it considers only simple strategies rather than the ones that condition on the distribution which might be hard to know in many real life applications. Hafalir and Krishna (2008) refine the multiplicity of equilibria in a single unit second-price asymmetric auction with resale by considering robust equilibrium. They show that the truth-telling is the unique robust equilibrium.

Robustness property requires that in the case of auctions with resale, the pricing strategy in the resale stage cannot depend on the distribution. This implies that the equilibrium bidding strategy has to be separating for the realizations of signals that may lead to losing the auction stage and buying in the resale market. Therefore, the resale outcome of an equilibrium will be efficient.

[6] In these auctions the losing bids are told to the winner since the losing bids determine the prices that she pays. On the other hand, announcing the winning bid reveals redundant information since the winner sets the price in the resale stage (see the description of resale stage).

[7] Take-it-or-leave-it offer by the winner of an auction is the optimal resale mechanism for the winner in our setting. Additionally, since the bidders learn all the bids after the auctions, the results will not change qualitatively if the resale offers are made sequentially.

3. Symmetric Locals

In this section we consider the case where the locals are symmetric in the sense that the signals of the locals are the same.

3.1. Vickrey Auctions with Separate Bidding

First we consider the mechanism where N second-price auctions are run simultaneously (one for each object). We analyze this situation when the resale is allowed and not allowed.

3.1.1. Vickrey Auction with Separate Bidding: Without Resale

The format of the auction stage is as described above. Post-trade among bidders is not allowed.

Let $b_{l_i}(s_l)$ be the bidding strategy of local *i* with signal s_l; and $b_{g_i}(s_g)$ be the bidding strategy of the global with signal s_g in auction *i*.

Proposition 1. *For Vickrey auction with separate bidding and without resale, when the locals are symmetric, there exists an efficient equilibrium where the locals bid their signals truthfully and the global bids truthfully her valuation per unit in the biggest package in each market, i.e. for any* $i \in \{1,...,N\}$, $s_l \in [\underline{s}, \overline{s}]$ *and* $s_g \in [\underline{s}, \overline{s}]$, $b_{l_i}(s_l) = s_l$, *and* $b_{g_i}(s_g) = \frac{v^N(s_g)}{N}$ *construct a symmetric equilibrium.*

Corollary 1. *The equilibrium strategies described in Proposition 1 is robust.*

If the resale markets are preventable and efficiency is the concern of the designer, then running separate second price auctions may serve the purpose. The equilibrium strategies described in Proposition 1 changes if we relax the assumption that $\frac{v^n(s)}{n}$ is increasing in n, nevertheless there exists an efficient robust equilibrium.

3.1.2. Vickrey Auction with Separate Bidding: With Resale

As described above, simultaneous auctions with resale is a two stage game where *N* simultaneous second-price auctions are run parallel (one for each object) in the first stage. After the auctions are completed, the winners of the auction may, if they wish, offer a price to unsuccessful bidders. If an offer is accepted then the object is exchanged and the winner of the auction is paid the amount she has asked. Obviously, at the resale stage, the winner can set the price conditional on the bid of the unsuccessful bidder.

Although the equilibrium of the simultaneous auctions without resale that is described in Proposition 1 allocates the goods efficiently, it is no longer an equilibrium when the resale markets are present. In particular, bidding truthfully may not be a best response for the locals when the global bids her valuation per unit, $\frac{v^N(s)}{N}$, in each auction. In order to see this, without loss of generality, let us consider *N=2* case. Assume that the global and one of the locals, say l_1, bid truthfully. Then l_2 with signal $\underline{s}$ would be better off by bidding $\frac{v^2(\overline{s})}{2}$ rather than $\underline{s}$. If l_2 with signal $\underline{s}$ bids $\underline{s}$, then her expected payoff is zero. Instead, if she bids $\frac{v^2(\overline{s})}{2}$ then she wins the auction in her market for sure and pays the bid of the global, $\frac{v^2(s_g)}{2}$, to the seller. After the auction, l_2 learns the signal of the global by just inverting the strategy of the global and can ask for a resale price equivalent to $v^2(s_g)-v^1(s_g)$ which is greater than $\frac{v^2(s_g)}{2}$. Therefore, l_2 with signal $\underline{s}$ would be better off by not bidding $\underline{s}$.

Next, we will show that inefficient allocation of the auction stage is indeed a property of any equilibrium of this game.

Proposition 2. *Vickrey auction with separate bidding and with resale where the local bidders are symmetric does not have any robust equilibrium where the auction stage allocates the objects efficiently.*

Unlike Propostion 2, in second-price single-unit asymmetric auction with resale, bidding truthfully is the unique robust equilibrium (Hafalir and Krishna, 2008). The intuition for not having this type of equilibrium in multi-unit case is that if the global wins all the objects except one in the auctions, the winning local can collect the additional surplus generated by complementarities in the resale stage when the efficient allocation is to sell everything to the global. This motivates the locals with low valuations to speculate in multi-unit environment. In order to prevent this type of exposure problem, the global needs to bid very aggressively (more than the valuation per unit).

In any robust equilibrium, the locals shade their bids and bid less than or equal to their signals (shown in Lemma A2 in the Appendix.) In particular, there exists equilibrium where the locals bid less than their signals (e.g. half of their signals) and the global bids a very high constant that makes her win all the time (e.g. $v^N(\bar{s})$). Such an equilibrium will not exist when all the signals are independent as in Section 4 and as in Hafalir and Krishna (2008) since the bidder who always loses will deviate to a smaller bid in order to get a better price in the resale stage. When the signals of the locals are symmetric, any unilateral deviation will be detected by the global via observing the bids of the other locals. In this type of robust equilibria, the locals always make zero payoff in other words the locals -independent of their signals- do not have any strict incentives in participating in the auction. Next, we show that if strict individual rationality is required, signal bidding is the unique robust equilibrium strategy for the locals.

Proposition 3. *Vickrey auction with separate bidding and with resale where the local bidders are symmetric, for any symmetric robust equilibrium such that no bidder except the one with the lowest signal expects to make zero payoff, the locals bid their signals and the global bids more than the unit value of the largest package in each market i.e ,* $b_l(s) = s$ *and* $b_g(s) \geq \frac{v^N(s)}{N}$ *for any* $s \in (\underline{s}, \bar{s}]$.

Proposition 3 states that the global bids aggressively in any robust and strictly individually rational equilibrium. The aggressive bidding will not hurt the global due to resale opportunity. In the resale market after the second-price auctions, the winner can infer the valuation of the loser from the auction price if the strategies are invertible. Indeed, the robustness requires invertible strategies when there is room for resale activity. In that case, the resale market will allocate the objects efficiently and if a resale takes place, the seller will extract the full surplus of the trade.

Given the aggressive bidding strategy of the global, locals do not speculate in the equilibrium. This is opposite to the intuition of speculators in single unit settings when resale is possible. In a single unit environment, when resale is allowed, the speculators, who aim to win the auction solely for making profit in the resale stage, may start bidding in the auction.

However, in our setup, the resale possibility makes the global bidder to bid aggressively to prevent the speculating behavior of the locals. In Proposition 3, we show that the allocation of the auction can be inefficient when the global wins the auctions, but it is efficient if the locals win.

The strict individual rationality refinement is related to Blume and Heidhues (2004). They show that having a reserve price in a second-price single unit auction eliminates all the equilibria except the value bidding. In that result, reserve price prevents bidders from bidding more than their value. Although strict individual rationality works similar to the reserve price in single unit environment, in multi unit auction with resale robustness prevents locals from overbidding and strict individual rationality prevents them from underbidding comparing to value bidding.

Proposition 3 characterizes equilibrium when it exists. Example 1 demonstrates that indeed robust and strictly individually rational equilibrium exists. Note that the auction may allocate the objects inefficiently.

Example 1. *There exists a symmetric, robust, and strictly individually rational equilibrium where* $b_l(s) = s$ *and* $b_g(s) = v^N(s)$ *for* $s \in [\underline{s}, \overline{s}]$.

This equilibrium is supported by the following beliefs on the signal of the opponent in resale stage: When the global wins all objects and observes identical bids from the locals in all markets, she infers the signal of locals by using the inverse of the equilibrium bidding function of locals; When the global wins all objects and observes different bids from the locals of each market, she believes that the signal of the locals is the inverse of the equilibrium bidding function of the locals evaluated at the highest of these different bids; When a local wins an auction, she infers the signal of the global by using the inverse of the equilibrium bidding function of the global evaluated at the bid of the global at that market. Moreover, in the resale stage the bidders play rationally, i.e. they offer the best price with respect to their beliefs at that outcome of the auction when they sell an object, they accept any offer that makes them better off, and reject the offer otherwise. The Appendix provides a proof that the strategies described above construct an equilibrium.

The equilibrium described in Example 1 may allocate the objects inefficiently in the auction stage because the global is bidding too aggressively. Since the locals are bidding their signals truthfully, the global prefers winning in the auction even when she is not the bidder with the highest valuation. This allocation will not hurt the global because she can always sell the objects to the locals at the auction price in the resale if she wants.

3.2. Vickrey Auction with Package Bidding: With and Without Resale

In the auction stage, a single auction is run to sell all the objects. Each bidder submits bids for all possible packages of the objects that she is interested. In our set up, it means that each l_i submits a bid for object *i* (denoted by b_{l_i}) and the global bidder submits bids for each package of size $n \in \{1,...,N\}$. Let b_{g_n} denotes the global's bid for the package with *n* elements.[8] The auction allocates the objects to the bidders who have the highest total bids for the objects. Each winner pays the price that is equivalent to the externality she exerts on other competing bidders (see Vickrey (1961)).

After Vickrey auction is completed, the bids are known by all the bidders and the resale market opens as described before.

It is a well known result that bidding truthfully in Vickrey auction without resale is an equilibrium (see Vickrey (1961)). The next proposition shows that this result carries over in the presence of resale markets.

Proposition 4. *In Vickrey auction with package bidding and with resale where the local bidders are symmetric, bidding the true valuation of a package for each type of bidder constructs an equilibrium, i.e.* $b_{l_i}(s) = s,$ *and* $b_{g_n}(s) = v^n(s)$ *for* $i \in \{1,...,N\}$ *and* $n = \{1,...,N\},$ *where* $b_{g_n}(.)$ *is the bidding strategy of the global for the package with n objects.*

Corollary 2. *The equilibrium strategy described in Proposition 4 is robust.*

Proposition 4 implies that the single auction with resale allocates the objects efficiently at the auction stage and the resale does not take place. Therefore, the resale opportunity does not affect the bidding strategies comparing to no-resale case. This suggests that a seller, who wants

[8] In our setup the value of a package for the global depends only on the signal and the number of the objects in it. Therefore, the bidding strategy of the global will also depend only on these parameters. However, the results of this subsection can be generalized if the objects were heterogeneous.

to allocate the objects efficiently and cannot prevent resale markets to develop although she wants to, should go for Vickrey auction with package bidding.

4. Independent Locals

In this section we assume that the signals of the locals are independently distributed.

4.1. Vickrey Auction with Separate Bidding

First we consider simultaneous auctions where *N* second-price auctions are run parallel (one for each object) with and without resale for the case that the signals of the locals are independently drawn.

4.1.1. Vickrey Auction with Separate Bidding: Without Resale

The format of the auction stage is as described before. Ex-post trade among bidders is not allowed.

First we will show that in any equilibrium (even if we do not require robustness and look at Bayesian Nash Equilibria), the allocation of the auction stage is inefficient.

Proposition 5. *Vickrey auction with separate bidding and without resale where the local bidders' signals are independent does not have any equilibrium where the auction stage allocates the objects efficiently.*

In this setup, there is always equilibria where only one type of bidder (either local or global) wins at any realization of the signals. One example of such equilibria is that the locals bid $\underline{s}$ and the global bids $v^N(\overline{s})$. This equilibrium is robust but it achieves minimum revenue. However, in this case there is no equilibrium such that it is robust and strictly individually rational. In order to see that, first observe that any robust and strictly individually rational equilibrium should be reached by iterated elimination of weakly dominated strategies. Otherwise, at least one of the strategies should be found via maximization of expected payoff of the corresponding bidder. Since the expected utility depends on the distribution (otherwise that bidder would be either always winning or always losing and that would contradict with strict individual rationality), the best response will depend on the distribution as well. Observe that, in this setup, the local bidders have a weakly dominant strategy which is bidding their signals truthfully. Then the equilibrium strategy of the global will be to best respond to that by expected utility maximization. This equilibrium will not be robust. This argument concludes the following result.

Proposition 6. *Vickrey auction with separate biding and without resale where the local bidders' signals are independent does not have any robust equilibrium which is strictly individually rational.*

However, this setup has non-robust equilibrium (see Krishna and Rosenthal (1996) for a special case of value functions that we allow here).

4.1.2. Vickrey Auction with Separate Bidding: With Resale

The format of the auction stage and the resale stage are as described before. First we will show that in any equilibrium, the allocation of the auction stage is inefficient as it was the case for the analysis without resale in Subsection 4.1.1.

Proposition 7. *Vickrey auction with separate bidding and with resale where the local bidders' signals are independent does not have any equilibrium where the auction stage allocates the objects efficiently.*

The proof of this statement is exactly the same as the proof of Proposition 5. Note that, under resale, it is still an equilibrium that one type of bidder (either the locals or the global) always wins the auctions, for example, the locals bid $\underline{s}$ and the global bids $v^N(\overline{s})$ construct an equilibrium. Although this type of equilibria was robust when the resale markets do not exist, it is not anymore. In any robust equilibrium, in the resale stage the winner should learn the signal of the loser by inverting the equilibrium bidding strategies otherwise the resale strategy would depend on the distribution. Since in the equilibrium that is described above, the losing type is pooling, the winner cannot know the loser's signal in the resale stage.

In subsection 3.1.2, even though there was no equilibrium with efficient allocation at the auction stage (see Proposition 2), there were inefficient robust equilibria (see Example 1). In that case, since the locals were symmetric, the global could detect a deviation of a single local by just observing that she behaved different than other locals. So, the locals cannot deviate unilaterally in the auction stage to get a better price in the resale stage. However, when the locals are independent the global cannot detect such deviations. Hence, there is more room for a local to deviate. Indeed, when the locals are independent, there is no robust equilibrium.

Proposition 8. *Vickrey auction with separate bidding and with resale where the local bidders' signals are independent does not have any robust equilibrium.*

4.2. Vickrey Auction with Package Bidding: With and Without Resale

In this subsection, we will analyze the Vickrey auction where all the objects are sold in a single auction and the global is allowed to submit package bids. The price of the auction is as the same as the one that is described in Subsection 3.2.

If the resale is not allowed after the auction, then bidding the value of the package for each package is an equilibrium (see Vickrey (1961)).

If the resale markets open after the auction as described before, then bidding the true valuation remains to be optimal equilibrium strategy for the auction.

Proposition 9. *For Vickrey auction with package bidding and with resale where the local bidders' signals are independent, bidding the true valuation for each type of bidder constructs an equilibrium, i.e.* $b_{l_i}(s) = s,$ *and* $b_{g_n}(s) = v^n(s)$ *for* $i \in \{1,\dots,N\}$ *and* $n = \{1,\dots,N\}$*, where* $b_{g_n}(.)$ *is the bidding strategy of the global for the package with n objects.*

Corollary 3. *The equilibrium strategy described in Proposition 9 is robust.*

Proposition 9 implies that Vickrey auction with package bidding and with resale allocates the objects efficiently at the auction stage and the resale does not take place even when the signals of the locals are independently drawn. Therefore, the resale opportunity does not affect the bidding strategies comparing to the no-resale case. This suggests that a seller, who wants to allocate the objects efficiently and cannot prevent resale markets to develop although she wants to, should go for Vickrey auction with package bidding when the signals of the locals either symmetric or independent.

5. Some Remarks on Revenue Comparisons

The main objective of this paper is to compare Vickrey auction with and without package bidding in terms of efficiency of their equilibrium allocations when resale markets are present. Nevertheless, one may want to see how revenue of the seller changes under different mechanisms even though we are not suggesting any of these mechanisms as the optimal one.

For the case of symmetric locals, Proposition 1 shows that Vickrey auction with separate bidding and without resale has a symmetric robust equilibrium where the locals bid their signals and the global bids the unit value of the largest package in each market. This equilibrium generates a revenue equivalent to

$$R^{SNR}_{symmetric} = N\int_{\underline{s}}^{\overline{s}}[1-F(x)][1-F((v^N)^{-1}(Nx))]dx$$

where SNR stands for separate bidding and no-resale. When resale is allowed for this mechanism, we showed in Proposition 3 that in any strictly individually rational and robust equilibrium the locals still bid their signals but the global bids more aggressively. Therefore, the revenue increases comparing to the no-resale case, i.e. $R^{SR}_{symmetric} \geq R^{SNR}_{symmetric}$ where *SR* stands for separate bidding with resale. On the other hand, without the strict individual rationality requirement it is possible to find robust equilibria where the revenue decreases with resale possibility.

For the case of symmetric locals, we concluded that Vickrey auction with package bidding is efficient with or without resale possibility. However, the truthtelling equilibrium of Vickrey auction with package bidding with and without resale generates lower revenue than Vickrey auction with separate bidding and without resale (as described by Proposition 4), i.e. $R^{SNR}_{symmetric} \geq R^{P}_{symmetric}$ where *P* stands for package bidding.

6. Further Extensions

More than one global bidder: One can easily show that the package bidding results will still hold when there are more than one global bidder. When the locals are independent, auction with separate bidding does not allocate the objects efficiently whether resale is allowed or not. This impossibility result cannot get any better if we have a more complicated model with more globals or locals. When the locals are symmetric, in the symmetric equilibrium, either global buys all the goods or locals buy all the goods. When there are more than one global bidder, again in the symmetric equilibrium, all of the goods should be in the hands of one type of bidder. However, in that case the off-equilibrium beliefs need to be carefully defined depending on resale protocols.

Other auction formats: The aim of this paper is to analyze how efficient auction formats are affected from the presence of efficient resale markets. The same results can be obtained if the open auction counterparts of the sealed bid auctions that we consider in this paper are used. Besides efficiency, revenue is another concern of the seller. Clearly, providing an optimal mechanism design analysis is not in the scope of the current study. However, one can show that for certain distributions some other formats, for example first price auctions, may lead to higher revenue than Vickrey auction with separate bidding when resale is allowed.

Experiment: All the predictions of the theory built in this paper can be studied experimentally. Because of the extensive use of multi-unit auctions in reality (for example spectrum auctions), it is important to understand the behavior of bidders and its relation with the theoretical results in the controlled environment of the laboratory. Especially, it will be interesting to investigate the behavior in Vickrey auction with separate bidding when resale is present. Chernomaz and Levin (2009) experimentally study allocating multi-units with first price auctions without resale possibility. Experimental extension of the current study would contribute to this literature.

7. Conclusion

In this paper, we show that in multi-unit auctions the presence of resale markets may lead to resale activity. Even though the outcome of an auction is efficient when the resale is prohibited, it may become inefficient when the resale is allowed as it is the case for Vickrey auction with separate bidding when the locals are symmetric.

If the seller's main objective is to allocate the objects efficiently in the auction as in governmental auctions, Vickrey auction with package bidding should be run especially when the resale markets are unavoidable. Moreover, the equilibrium bidding strategy (value-bidding) of

Vickrey auction is simple and robust. On the other hand, in Vickrey auction with separate bidding, there is no equilibrium that guarantees the efficiency in the auction stage when the bidders know that there are resale markets.

When the locals' signals are independently drawn, the auction stage does not allocate the objects efficiently if Vickrey auction with separate bidding is used. Moreover, if the resale opportunity exists, this auction format does not have any robust equilibrium besides the ones that generate minimum revenue.

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Appendix

Proof of Proposition 1. First observe that in each market, the locals are participating in a second price sealed bid auction where value bidding is the weakly dominant strategy.

Next, we show that the global bidder's best response to the locals' value bidding strategy is $\frac{v^N(s_g)}{N}$.

If $s_l < \frac{v^N(s_g)}{N}$ then winning k objects achieves a payoff of $v^k(s_g) - ks_l$ and it is maximized when k=N. Hence, any bid that wins all the auctions including the bid $\frac{v^N(s_g)}{N}$ will give the highest payoff.

If $s_l \geq \frac{v^N(s_g)}{N}$ then winning any k objects will give non-positive payoff to the global since $v^k(s_g) - ks_l \leq 0$ for any k. Hence, any losing bid including the bid $\frac{v^N(s_g)}{N}$ will give the highest payoff.

QED

Proof of Example 1: First, we will show that under the strategies described in Example 1, a local does not deviate to a bid below her signal.

If a local with signal s_l wins by bidding s_l (i.e. $s_l > v^N(s_g)$), bidding below s_l will either keep her winning position with the same payment in the auction and the same prices in resale (if it occurs) or she will lose. If she loses and tries to buy the object from the resale market, the global will charge s_l because the global will learn the signal of the locals from the bids of the other locals and this is worse than winning the auction with bid s_l.

If the local loses by bidding s_l then bidding less than s_l will cause her to lose again and she will see the same price in the resale if she goes to the resale market. Therefore, bidding below s_l is not beneficial for the local.

Second, we will show that a local does not deviate to a bid above her signal.

If the local with signal, s_l, wins by bidding s_l, then she will keep winning the auction after deviating to a larger bid. If she makes any resale in either case, she can charge the same prices in both.

If the local loses by bidding s_l (i.e. $v^N(s_g) > s_l$) then she will either lose or win with a bid above s_l. If she loses after the deviation then in the resale market she sees a higher price than the case that she loses with bid s_l if resale takes place. If she wins after the deviation then she pays more than her valuation in the auction (she pays $v^N(s_g) > s_l$). If she goes to the resale, the price she may offer is at most $v^N(s_g) - v^{N-1}(s_g)$ and this is less than $v^N(s_g)$ (which is her payment in the auction). Therefore bidding more than s_l is not beneficial for the local.

Finally, we will show that the global does not deviate from the strategy described in the example. If the global wins by bidding $v^N(s_g)$ in all the markets and if this is efficient allocation then observe that she cannot buy the objects at a cheaper price in the auction or in the resale by changing her bids. With the equilibrium bids, the global is paying s_l per object and in resale the locals will not sell at a price less than s_l. If it is not efficient to win everything by the global then she will sell the objects in the resale market at price s_l which is the amount that she paid for those objects in the auction. By deviating, the global cannot get the objects at a cheaper price than s_l in the auction or in the resale or she cannot sell them at a higher price than s_l. Therefore, no deviation is beneficial for the global.

QED

The following lemma is going to be used in the proof of Proposition 2.

Lemma A1. For simultaneous auctions with resale where the local bidders are symmetric, in any efficient equilibrium (if it exists), $b_{g_i}(s) = b_{l_i}\left(\frac{v^N(s)}{N}\right)$ for any $i \in \{1, ..., N\}$ and for any s such that $\frac{v^N(s)}{N} \in [\underline{s}, \overline{s}]$.

Proof of Lemma A1. Let $b_{g_i}(s) > b_{l_i}\left(\frac{v^N(s)}{N}\right)$ for some signal s and market i. Then for any $s' \in \left(\frac{v^N(s)}{N}, \frac{v^N(s)}{N} + \varepsilon\right)$ for $\varepsilon > 0$, $b_{g_i}(s) > b_{l_i}(s')$ from right continuity of the bidding functions. This means that the global with signal s' wins in market i against local i with the signal in $\left(\frac{v^N(s)}{N}, \frac{v^N(s)}{N} + \varepsilon\right)$ interval. However, on this interval $Ns' > v^N(s)$ and therefore the auction allocation is not efficient.

Let $b_{g_i}(s) < b_{l_i}\left(\frac{v^N(s)}{N}\right)$ for some signal s and market i. Then for any $s' \in \left(\frac{v^N(s)}{N} - \varepsilon, \frac{v^N(s)}{N}\right)$ for $\varepsilon > 0$, $b_{g_i}(s) < b_{l_i}(s')$ from right continuity of the bidding functions. This means that the global with signal s' wins in market i against local i with the signal in $\left(\frac{v^N(s)}{N} - \varepsilon, \frac{v^N(s)}{N}\right)$ interval. However, on this interval $Ns' < v^N(s)$ and therefore the auction allocation is not efficient.

QED

Lemma A2. In any symmetric WIIE, $b_l(s) \le s$ for any $s \in [\underline{s}, \overline{s}]$.

Proof of Lemma A2. Let $b_l(s)$ be a symmetric WIIE. For contradiction assume that there is a signal s' such that $b_l(s') > s'$. From right continuity, there exists a constant δ_1 such that $\forall s \in [s', s' + \delta_1] = I$, $b_l(s) > s$. Define $\delta_2 := \inf_{s \in I} \{b_l(s) - s\} > 0$ and define $\delta := \min\{\delta_1, \delta_2\}$.

Consider the global bidder with signal s_g such that $s' + \delta > \frac{v^N(s_g)}{N} > s'$. Observe that such a global bidder makes negative payoff whenever he wins against locals who have signal in I and if the global loses, his utility is zero. This is because for $s \in I$, when $b_g(s_g) > b_l(s)$, the utility of the global from bidding $b_g(s_g)$ in all the auctions is $\max\{v^N(s_g) - Nb_l(s), Ns - Nb_l(s)\}$. Note that

$$v^N(s_g) - Nb_l(s) < Ns' + N\delta - Nb_l(s) \le Ns' + N\delta_2 - Nb_l(s)$$
$$\le Ns' + N\left[b_l(s) - s\right] - Nb_l(s) = Ns' - Ns \le 0 \text{ and}$$
$$Ns - Nb_l(s) < 0$$

If $\exists \tilde{s} \in I$ such that $b_g(s_g) > b_l(\tilde{s})$ then from right continuity, $\exists\ \tilde{\delta}$ such that $\forall s \in [\tilde{s}, \tilde{s} + \tilde{\delta}] \subset I$, $b_g(s_g) > b_l(s)$, i.e. the global wins against local bidders who have signal in $[\tilde{s}, \tilde{s} + \tilde{\delta}]$. Because of the argument above, in such situations the global makes negative payoff. Then for a distribution that assigns probability close to 1 to interval $[\tilde{s}, \tilde{s} + \tilde{\delta}]$, the global bidder with signal s_g is better off by bidding $b_l(\underline{s})$.

If $\forall s \in I$, $b_g(s_g) \le b_l(s)$ then consider $\tilde{s}_g$ such that $s' + \delta > \frac{v^N(s_g)}{N} > \frac{v^N(\tilde{s}_g)}{N} > s'$. Observe that the global bidder with signal $\tilde{s}_g$ makes negative payoff whenever she wins against locals who have signal in I and if the global loses, her utility is zero.

If $\exists \hat{s} \in I$, $b_g(\tilde{s}_g) > b_l(\hat{s})$ then similar to the argument stated two paragraphs above, the global with signal $\tilde{s}_g$ would be better off by bidding $b_l(\underline{s})$ for some distributions.

If $\forall s \in I$, $b_g(\tilde{s}_g) \le b_l(s)$ then the global with signal s_g is better off by bidding less than $b_g(\tilde{s}_g)$ because then when she loses, she can buy the goods in the resale market at a cheaper price when she wants to buy.

QED

Proof of Proposition 2. Assume for contradiction that there is an equilibrium where the auction outcome is efficient. Note that it has to be true that for all i

$$E_{s_g}[v^N(s_g) - v^{N-1}(s_g) - b_{g_i}(s_g)] \le 0$$

otherwise, local i with signal $\underline{s}$ will be better off if she bids the highest bid of the global in that market, $b_{g_i}(\overline{s}_g)$. By bidding so aggressively, the local i will always win her market and sell in the resale stage at the price $v^N(s_g) - v^{N-1}(s_g)$.

Case 1: If there exists a market *i* such that for any $s_g \in [\underline{s}, \overline{s}]$, $v^N(s_g) - v^{N-1}(s_g) - b_{g_i}(\overline{s}_g) \le 0$, then take signal s_g such that $\frac{v^N(s_g)}{N} \in [\underline{s}, \overline{s}]$. Then observe that

$$v^N(s_g) - v^{N-1}(s_g) \le b_{g_i}(s_g) = b_{l_i}\left(\frac{v^N(s_g)}{N}\right) \le \frac{v^N(s_g)}{N}$$

The equality follows from Lemma A1 and the last inequality is from Lemma 1. By rearranging the term one can observe that the observation above contradicts with the assumption that $\frac{v^n(.)}{n}$ increases with *n*.

Case 2: If for any market *i* there exists $s_g \in [\underline{s}, \overline{s}]$ such that $v^N(s_g) - v^{N-1}(s_g) - b_{g_i}(s_g) > 0$ then from right continuity of $b_{g_i}(.)$ and continuity of $v^n(.)$ for any *n* one can find $\varepsilon > 0$ such that for any $s_g^{'} \in (s_g, s_g + \varepsilon)$, $v^N(s_g^{'}) - v^{N-1}(s_g^{'}) - b_{g_i}(s_g^{'}) > 0$. Then local *i* with signal $\underline{s}$ can bid $\max_{s_g^{'} \in (s_g, s_g+\varepsilon)} b_{g_i}(s_g^{'})$ and win against the global with signal $\tilde{s}_g \in (s_g, s_g + \varepsilon)$ and sell at the resale stage at price $v^N(\tilde{s}_g) - v^{N-1}(\tilde{s}_g)$ and make positive profit on this interval. If the probability that the global has a signal on $(s_g, s_g + \varepsilon)$ is close to 1, then this deviation is profitable.

QED

Proof of Proposition 3. From Lemma A2, we know that $b_l(s) \le s$. Assume for contradiction that $\exists s \in (\underline{s}, \overline{s}]$ such that $b_l(s) < s$.

If $\exists s_g$ such that $b_l(s) < b_g(s_g) < s$ then from right continuity of $b_g(.)$, $\exists \delta > 0$ such that $\forall s_g' \in \left[s_g, s_g + \delta\right] = I$, $b_l(s) < b_g(s_g') < s$

Define $b := \max_{s_g' \in I} b_g(s_g') + \varepsilon$ for some $\varepsilon > 0$, then if a local, say l_1, with signal *s* bids b rather than $b_l(s)$ and if the distribution assigns high probability to I, then the expected payoff of l_1 from this deviation is close to her expected payoff on interval I which is greater than or equal to

$s - E_I(b_g(s_g))$. This is higher than the expected payoff of l_1 from following the equilibrium strategy which is close to *0*.

If $\exists s_g$ s.t. $b_g(s_g) < b_l(s) < s$ then from right continuity of b_l, $\exists \delta > 0$ such that $\forall \tilde{s} \in [s, s+\delta] = I$, $b_g(s_g) < b_l(\tilde{s}) < \tilde{s}$.

Define $b := \max_{\tilde{s} \in I} b_l(\tilde{s}) + \varepsilon$ for some $\varepsilon > 0$, then the global with signal s_g wins against all $\tilde{s} \in I$ if she bids *b*. If the distribution assigns high probability to I, then bidding *b* is better than following the equilibrium strategy $b_g(s_g)$ which pays off zero. To see this, note that

- If $\frac{v^N(s_g)}{N} < \tilde{s}$ then after bidding b, the global wins and sells in the resale market at $p_g = \tilde{s}$ and her utility is $N\tilde{s} - Nb_l(\tilde{s}) > 0$.
- If $\frac{v^N(s_g)}{N} > \tilde{s}$ then after bidding b, the global wins and keeps the goods and her utility is $v^N(s_g) - Nb_l(\tilde{s}) > 0$.

If $\exists s_g$ such that $b_l(s) = b_g(s_g) < s$ then from right continuity there is a $\delta > 0$ such that for the interval $I = [s_g, s_g + \delta]$ there are three possibilities:

- Either $\exists\ \tilde{s}_g \in I$ such that $b_l(s) < b_g(\tilde{s}_g) < s$ then we showed above that one of the locals with signal *s* would deviate if the distribution is concentrated on *I*,
- or $\exists\ \tilde{s}_g \in I$ such that $b_g(\tilde{s}_g) < b_l(s) < s$ then we showed above that the global with signal s_g would deviate if the distribution is concentrated on $[s, s+\delta']$ for some δ',
- or $\forall \tilde{s}_g \in I \quad b_l(s) = b_g(\tilde{s}_g) < s$ then for the distribution that is concentrated on I, a local, say l_1, with signal *s* deviates to $b_l(s) + \varepsilon$.

Finally, if $\forall s_g \quad b_l(s) < s \le b_g(s_g)$ then the local with signal *s* always loses the auction. This contradicts with strict individual rationality.

Therefore, for any $s \in (\underline{s}, \overline{s}]$, $b_l(s) = s$.

Next, we will show that for any $s \in [\underline{s}, \overline{s}]$ $b_g(s) \geq \frac{v^N(s)}{N}$. Assume that $\exists s'$ such that $b_g(s') < \frac{v^N(s')}{N}$.

If $\frac{v^N(s')}{N} \geq \overline{s}$, then from right continuity $\exists \delta$>0 such that $\forall \tilde{s} \in [s', s'+\delta] = I$ $b_g(\tilde{s}) < \frac{v^N(\tilde{s})}{N}$. If the distribution is concentrated on *I* then a local, say l_1, with signal $\underline{s}$ deviates to $v^N(\overline{s})$ rather than bidding $b_l(\underline{s})$. To see this without loss generality assume $\underline{s} = 0$. From Lemma 1 $b_l(0) = 0$ and therefore the expected payoff of the local with signal zero is 0. However bidding $v^N(\overline{s})$ pays off close to $E_I(p_l(.) - b_g(\tilde{s}))$ where p_l is the price l_1 may charge from the global. Then observe that

$$E_I(p_l(.) - b_g(\tilde{s})) > E_I\left(v^N(\tilde{s}) - v^{N-i}(\tilde{s}) - (i-1)\overline{s} - \frac{v^N(\tilde{s})}{N}\right) \text{ for } N \geq i \geq 1$$

$$= E_I\left(\frac{1}{N}\Big((N-1)v^N(\tilde{s}) - Nv^{N-i}(\tilde{s})\Big) - (i-1)\overline{s}\right)$$

$$> E_I\left(\frac{1}{N}\Big((N-1)v^N(\tilde{s}) - (N-i)v^N(\tilde{s})\Big) - (i-1)\overline{s}\right)$$

$$= E_I\left((i-1)\left(\frac{v^N(\tilde{s})}{N} - \overline{s}\right)\right)$$

$$\geq 0$$

If $\frac{v^N(s')}{N} < \overline{s}$ then $\exists s$ such that $b_g(s') < s < \frac{v^N(s')}{N}$. Then $\exists \delta$ such that $\forall \tilde{s} \in (s-\delta, s+\delta) = I$ the global with signal s' loses against the locals with signal $\tilde{s}$ since those locals are bidding their signals as shown above. If the distribution is concentrated on *I* then the global with signal

s' deviates to $\frac{v^N(s')}{N}$ rather than bidding $b_g(s')$ since bidding $b_g(s')$ pays off close to *0* but bidding $\frac{v^N(s')}{N}$ pays off close to

$E_I(v^N(s') - N\tilde{s}) > 0$.

QED

Proof of Proposition 4.[9] Define $j := \arg\max_i \left(v^i(s_g) + (N-i)s_l \right)$.

Let $b_{l_i}(s) = s,\ b_{g_n}(s) = v^n(s_g)$ for all n and $i \neq 1$, we will first show that for any realization of s_l and s_g, the best action for l_1 is to bid her signal.

Case 1: $j \neq N$ and $j \neq 0$

If l_1 bids s_l, her expected payoff is *0*. If she bids less than s_l, she loses and her payoff is *0*. If she bids more than s_l, she wins but needs to pay s_l, and hence her payoff is 0.

Case 2: $j = N$

If l_1 bids s_l, her expected payoff is *0*. If she bids less than s_l, she loses and her payoff is *0*. If she bids more than s_l, even if she wins her payment is $v^N(s_g) - v^{N-1}(s_g) < s_l$ since $j = N$, and hence her payoff is less than *0*.

Case 3: $j = 0$

If l_1 bids s_l, she wins and her payoff is $s_l - v^N(s_g) + v^{N-1}(s_g) \geq 0$ since *j=0* . By changing her bid, if she loses her payoff is 0, if she keeps winning then her payment will not change.

[9] Here, we provide the proof without the increasing per unit value of a package assumption since it does not bring any notational simplification.

Let locals bid their signal. Next, we will show that for any realization of s_l and s_g, the best action for the global bidder is to bid $b_{g_n}(s_g) = v^n(s_g)$.

If the global bids $b_{g_n}(s_g) = v^n(s_g)$, her payoff is $v^j(s_g) - js_l$. If the global deviates to a strategy which makes her win *i<j* objects, then she will buy *(j-i)* objects in the resale market. Since the locals will charge $p \geq s_l$, then her deviation payoff is not profitable: $v^j(s_g) - is_l + (i-j)p \leq v^j(s_g) - js_l$.

If the global deviates to a strategy which makes her win *i>j* objects, then she will sell *(j-i)* objects in the resale market. Since the locals will not accept any price, $p > s_l$, then her deviation payoff is not profitable: $v^j(s_g) - is_l - (j-i)p \leq v^j(s_g) - js_l$.

QED

Proof of Proposition 5. To simplify the notation we will provide the proof for *N=2*, but it can be easily generalized for arbitrary *N*. Assume for contradiction that there is an equilibrium, $(b_{l_i}(.), b_{g_i}(.))_{i\in\{1,2\}}$. Let s_g, s_{l_1}, s_{l_2}, and $\tilde{s}_{l_2}$ be realizations of signals such that

$$\begin{aligned}
&s_{l_1} + s_{l_2} < v^2(s_g) \\
&s_{l_i} + v^1(s_g) < v^2(s_g) \text{ for } i \in \{1,2\} \\
&s_{l_1} + \tilde{s}_{l_2} > v^2(s_g) \text{ and} \\
&s_{l_1} + \tilde{s}_{l_2} > s_{l_i} + v^1(s_g) \text{ for } i \in \{1,2\}
\end{aligned}$$

Hence, when the signal of local 2 is s_{l_2}, it is efficient to allocate both objects to the global, and when the signal of local 2 is $\tilde{s}_{l_2}$, it is efficient to allocate objects to the locals. However, if the equilibrium is efficient then $b_{l_1}(s_{l_1}) < b_{g_1}(s_g)$ for the first case and $b_{l_1}(s_{l_1}) > b_{g_1}(s_g)$ for the second case. This leads to a contradiction.

QED

Proof of Proposition 8. Since there is no efficient equilibrium by Proposition 7, if a robust equilibrium exists, for some realizations of signals $(s_{l_i}, s_g)_{i\in\{1,\ldots,N\}}$, a resale opportunity will occur.

Case 1: Let local *i* wins the auction in market *i*, i.e. $b_{l_i}(s_{l_i}) > b_{g_i}(s_g)$, but the efficient allocation for signals $(s_{l_i}, s_g)_{i\in\{1,\dots,N\}}$ requires the global should have the object. From right continuity, there exists a $\delta > 0$, such that for any $\tilde{s}_{l_i} \in (s_{l_i}, s_{l_i} + \delta)$, $b_{l_i}(\tilde{s}_{l_i}) > b_{g_i}(s_g)$ and still the efficient allocation for signals $(\tilde{s}_{l_i}, s_{l_{-i}}, s_g)_{i\in\{1,\dots,N\}}$ is to give the object *i* to the global.

Aside, from right continuity of $b_{g_i}(.)$, there exists $\varepsilon > 0$, such that for any $\tilde{s}_{l_i} \in (s_{l_i}, s_{l_i} + \delta)$, $b_{l_i}(\tilde{s}_{l_i}) > b_{g_i}(s_g + \varepsilon)$ and still the efficient allocation for signals $(\tilde{s}_{l_i}, s_{l_{-i}}, s_g + \varepsilon)_{i\in\{1,\dots,N\}}$ is to give the object *i* to the global. If the probability that the signal of the local *i* will be in interval $(s_{l_i}, s_{l_i} + \delta)$ is close to 1 then the global with signal $s_g + \varepsilon$ is better off if she bids $b_{g_i}(s_g)$ in market *i* rather than $b_{g_i}(s_g + \varepsilon)$. By doing that she will still lose the auction *i* but will get a cheaper price from local *i* in resale and this is a contradiction. Therefore, there is no equilibrium outcome where the locals sell to the global in the resale after an inefficient allocation of the objects in the auction stage.

Case 2: Let the global bidder wins the auction in market *i*, i.e. $b_{l_i}(s_{l_i}) < b_{g_i}(s_g)$, but the efficient allocation for signals $(s_{l_i}, s_g)_{i\in\{1,\dots,N\}}$ requires local *i* should have the object. From right continuity, there exists a $\delta > 0$, such that for any $\tilde{s}_g \in (s_g, s_g + \delta)$, $b_{l_i}(s_{l_i}) < b_{g_i}(\tilde{s}_g)$ and still the efficient allocation for signals $(s_{l_i}, \tilde{s}_g)_{i\in\{1,\dots,N\}}$ is to give the object *i* to local *i*.

Aside, from right continuity of $b_{l_i}(.)$, there exists $\varepsilon > 0$, such that for any $\tilde{s}_g \in (s_g, s_g + \delta)$, $b_{l_i}(s_{l_i} + \varepsilon) < b_{g_i}(\tilde{s}_g)$ and still the efficient allocation for signals $(s_{l_i} + \varepsilon, s_{l_{-i}}, \tilde{s}_g)_{i\in\{1,\dots,N\}}$ is to give the object *i* to local *i*. If the probability that the signal of the global will be in interval $(s_g, s_g + \delta)$ is close to 1 then local *i* with signal $s_{l_i} + \varepsilon$ is better off if she bids $b_{l_i}(s_{l_i})$ in market *i* rather than $b_{l_i}(s_{l_i} + \varepsilon)$. By doing that she will still lose auction *i* but will get a cheaper price from the global bidder in resale and this is a contradiction. Therefore, there is no equilibrium outcome where the locals buy from the global in the resale after an inefficient allocation of the objects in the auction stage.

Case 1 and Case 2 conclude that the auction outcome should be efficient and this contradicts with Proposition 7. **QED**

Proof of Proposition 9. Assume all the local bidders except local *i* bids their true signals and the global bids her valuation for each package. We will first show that it is a best response for local i with signal s_{l_i} to bid her signal truthfully. Let local *i* be one of the winners when she bids her signal truthfully and let her payment in that event be *p*. Then bidding anything more than s_{l_i} will not change the outcome and her payoff. If she bids less than her signal and wins the auction, then she will pay the same amount, *p*, in the auction and will not sell the object in resale since the auction allocation will be efficient. If she bids less than her signal and loses the auction then there are two possibilities: (i) another local, say s_{l_j}, who would not have won the auction if local *i* had bid truthfully will win the object, or (ii) the global win one more object than the number of objects that he would have won if local *i* had bid truthfully. If it is case (i), then $p = s_{l_j}$ and the payoff of local *i* after the resale will be less than or equal to $s_{l_i} - s_{l_j}$. If it is case (ii), then $p = v^{n+1}(s_g) - v^n(s_g)$ where s_g is the signal of the global, and *n* is the number of objects the global wins if everyone bids true valuations. The payoff of local *i* after the resale will be less than or equal to $s_{l_i} - (v^{n+1}(s_g) - v^n(s_g))$ since the global will not sell the object less than its marginal value to the global. Therefore, no deviation is profitable for local *i* who would win the auction if she bid her signal truthfully.

Let local *i* lose the auction when she bids her signal truthfully. Then bidding anything less than s_{l_i} will not change the outcome and her payoff. If she bids more than her signal and still loses the auction, then her payoff would be the same. If she bids more than her signal and wins the auction then there are two possibilities: (i) another local, say s_{l_j}, who would have won the auction if local *i* had bid truthfully will lose the auction, or (ii) the global win one less object than the number of objects that he would have won if local *i* had bid truthfully. If it is case (i), then $p = s_{l_j}$ and the payoff of local *i* after the resale will be equal to $s_{l_j} - p$. If it is case (ii), then $p = v^n(s_g) - v^{n-1}(s_g)$ where s_g is the signal of the global, and *n* is the number of objects the global wins if everyone bids true valuations. The payoff of local *i* after the resale will be $(v^n(s_g) - v^{n-1}(s_g)) - p$ since the global will not pay for the object more than its marginal value to the global. Therefore, no deviation is profitable for local *i* who would win the auction if she bid her signal truthfully.

We will next show that it is a best response for the global with signal s_g to bid her valuation for each package truthfully if all the locals are bidding their signals. Assume that if she follows this strategy, the global wins *n* objects in the auction. If she deviates to another strategy and wins $n' < n$ objects, then she will buy $n - n'$ objects in the resale stage. For each object she buys from local *i* she will pay at least s_{l_i} which is what she pays for that object if she had bid her true valuation for the packages. If she deviates to another strategy and wins $n' > n$ objects, then she will sell $n' - n$ objects in the resale stage at a price less than or equal to the signals of the buying locals. However, those signals are exactly the prices of those objects that the global would have paid if she had bid her valuations for the packages truthfully. Therefore, no deviation from bidding truthfully is profitable for the global.