Asset Prices and Business Cycles with Financial Frictions

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Abstract. Existing dynamic general equilibrium models have failed to explain the high volatility of asset prices that we observe in the data. We construct a general equilibrium model with heterogenous firms and financial frictions that addresses this failure. In each period only a fraction of firms can start new projects, which cannot be fully financed externally due to a financial constraint. We allow the tightness of the financial constraint to vary over time. Fluctuations in the tightness of the financial constraint result in fluctuations in the supply of equity and consequently in the price of equity. We calibrate the model to the U.S. data to assess the quantitative importance of fluctuations in the tightness of the financial constraint. The model generates a volatility in the price of equity comparable to the aggregate stock market while also fitting key aspects of the behavior of aggregate quantities.

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1. Introduction

The excess volatility puzzle (Shiller, 1981, and LeRoy and Porter, 1981) and the equity premium puzzle (Mehra and Prescott, 1985) are two fundamental challenges to theoretical models that have been developed in the finance and macroeconomics literature. Building a production economy model that would satisfactorily account for both high aggregate stock market volatility and the behavior of aggregate quantities has proven to be difficult and no consensus model has arisen. In this paper we build a model in which variations in firms' ability to raise external capital to take profitable projects lead to asset price volatility. We calibrate the model to the U.S. data and find that it generates about 80% of the observed aggregate stock market volatility. At the same time, the model generates time-series properties of aggregate quantities that match the macroeconomic data.

Our model closely resembles the model described in Kiyotaki and Moore (2008). It is a dynamic stochastic general equilibrium model with heterogeneous entrepreneurs, who face a real and a financial friction. The real friction restricts entrepreneurs' access to new projects. In every period only a fraction of entrepreneurs find new profitable projects. Following the literature, we assume that the arrival of profitable projects is i.i.d. over time and over entrepreneurs, see e.g. Angeletos (2007) and Kocherlakota (2009). We model an entrepreneur's ability to start a profitable project as his ability to produce new capital goods one-to-one from the general consumption good. Entrepreneurs who cannot produce capital are willing to buy claims to returns of other entrepreneurs' projects to replace their depreciated capital. We call these claims equity. Markets are incomplete and equity is the only financial asset that is traded in the economy. The financial friction restricts new issuance of equity. We assume that entrepreneurs can only leverage a fraction of the returns of the newly produced capital, i.e. sell only a fraction of the new project as equity. On its own, this friction is standard in the literature. The novel feature of our model is that the ratio of outside to total financing of projects changes over time.

The interactions between these two frictions and the time variation in the financial friction play an important role in the ability of our model to explain the asset price volatility. Assuming that all entrepreneurs in the economy can produce new capital goods would imply that the price of equity is constant at the cost at which capital is produced, i.e. at price one. No entrepreneur would be willing to pay a higher price. Assuming heterogeneity in entrepreneurs' ability to produce new capital in the absence of the financial friction would imply that the price of equity is always one as well. If the price was higher, an entrepreneur with the ability to produce new capital would find it profitable to increase his investment in the project. He then would sell equity to the newly installed capital at a price that exceeds the costs. However, if the fraction of entrepreneurs that can produce new capital goods and the leverage ratio are relatively low, the price of equity will be greater than one. In that case, fluctuations in the leverage ratio result in fluctuations in the price of equity. The intuition behind this result is as follows. If the leverage ratio decreases, entrepreneurs with the ability to produce new capital goods decrease their investment and their supply of new equity. This decrease in supply increases the value of existing assets and therefore the price of equity will increase. A similar logic applies for an increase in the leverage ratio. Consequently, as the leverage ratio fluctuates over time so does the price of equity.

We calibrate the model and find that it generates about 80% of the quarterly volatility in asset prices relative to the Dow Jones Total Stock Market Index. On the annual basis, our benchmark model generates about 85% of the asset return volatility relative to the value weighted market return. We construct a shadow risk free rate and find that our model generates an annual equity premium of 1.6%. Finally, we find that time variation in the financial friction contributes significantly to the volatility of investment, but not to the volatility of output.

2. Related Literature

We build on Kiyotaki and Moore (2008), but our paper is different from theirs in the questions of interest and several modeling features. They are interested in the existence of money in a general equilibrium model and the optimal monetary policy responses to liquidity shocks. We abstract from both money and liquidity shocks. In their model, entrepreneurs can only sell a fraction of their asset holdings in a given time period. In our model, entrepreneurs are able to sell all their financial asset holdings. Finally, entrepreneurs' access to outside capital is constant in Kiyotaki and Moore's model while in our model it is time varying.

Theoretically, it has been argued that frictions in financial markets are important for explaining the fluctuations of the aggregate macroeconomic quantities, see for instance Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and the review paper of Bernanke, Gertler and Gilchrist (1999). In our model, financial frictions are important for explaining not only the behavior of aggregate quantities but also the behavior of asset prices. Our paper contributes to a growing literature that analyzes the effects of exogenous financial shocks. See for example Benk, Gillman and Kejak (2005), Christiano, Motto and Rostagno (2007) and Jermann and Quadrini (2009), whose results suggest that financial shocks play an important role for macroeconomic fluctuations. While these papers are mainly focused on macroeconomic quantities, we are interested in asset prices as well.

Our results are in contrast with the findings of Gomes, Yaron and Zhang (2003), who analyze a model in which financing frictions arise endogenously as an outcome of a private information problem with costly monitoring. The only primitive shocks in their model are total factor productivity (TFP) fluctuations. They find that the model generates only modest asset return volatility. Other attempts to build models with financial frictions that would generate a strong propagation of TFP shocks into the real economy and asset prices have not been very successful either[1]. Therefore, a departing assumption of our model is that fluctuations in productivity are not the only source of uncertainty in the economy. The second source of uncertainty in our model are fluctuations in the fraction of a project an entrepreneur can finance with outside capital[2].

Our paper explains the volatility of asset prices by introducing financial frictions into a dynamic general equilibrium model. However, other approaches have been taken to reconcile the asset price behavior with the predictions of consumption based asset pricing models. In endowment economy models, introducing habit formation or Epstein-Zin recursive preferences and changing the structure of the stochastic processes defining the consumption stream have been shown to be able to explain the high volatility of asset prices. Examples of this approach are Campbell and Cochrane (1999), who assume that agents have preferences with habit persistence and Bansal and Yaron (2004), who assume that agents have recursive Epstein-Zin preferences and there is a long run risk component in the consumption process. However, endowment economy models are silent about the behavior of aggregate macroeconomic quantities.

Explaining the volatility of asset prices in production economies has proved more challenging. Following the success of habit formation preferences in endowment economy models, Lettau and Uhlig (2000) incorporate the Campbell and Cochrane (1999) habit formation structure into a production economy[3]. They argue that these preferences make the households locally very risk averse and find that consumption volatility in the model is by an

[1] See Kocherlakota (2000), Arias (2003), and Cordoba and Ripoll (2004) .

[2] We find that our model generates an asset return volatility very similar to Gomes, Yaron and Zhang (2003) if we assume that this fraction is constant and fluctuations in TFP are the only source of uncertainty.

[3] Production economy models with Epstein-Zin preferences have so far not been successful in generating asset price volatility, see e.g. Croce (2009) and Tallarini (2000).

order of magnitude smaller than in the data. This result should not come as a surprise. In the standard one-sector growth model without frictions, firms can adjust their capital to reduce fluctuations in households' consumption. This motive is further enforced by habit persistence. To address this shortcoming, Jermann (1998) develops a production economy model with habit persistence, capital adjustment costs and fixed labor. His model generates high asset price volatility and a high equity premium, but it also generates a counterfactually high risk free rate volatility. This is a common problem of general equilibrium models with habit persistence. Further, as documented by Boldrin, Christiano and Fisher (2001), output is counterfactually smooth and negatively autocorrelated. In addition, dropping the assumption that labor supply is fixed makes labor supply countercyclical. In our model output is positively autocorrelated and volatile, and labor supply is procyclical.

Boldrin, Christiano and Fisher (2001) develop a model with habit persistence and limited mobility of labor and capital across the consumption good and the investment good sector. As in Jermann (1998) their model generates high asset price volatility at the cost of counterfactually high volatility of the risk free rate. Moreover, their model cannot explain the volatility of labor and investment. In contrast, our model generates the investment volatility observed in the data. There is no risk free asset in our model. Therefore we construct a shadow risk free rate and find that its volatility is about 50% of what Boldrin, Christiano and Fisher get[4].

The rest of our paper is organized as follows. Section 3 presents the model and section 4 characterizes the solution of the model. Section 5 describes our calibration procedure and section 6 discusses the quantitative implications of the model. Section 7 presents a summary of our sensitivity results and section 8 concludes.

[4]Christiano and Fisher (2003) add sector specific productivity shocks and adjustment costs to the Boldrin, Christiano and Fisher model. Their model still generates counterfactually high risk free rate volatility and counterfactually low investment volatility.

3. The Model

Time is discrete and infinite. There are two types of agents: a unit measure of ex-ante identical entrepreneurs who consume, produce and hold capital, but do not work, and a unit measure of identical hand-to-mouth workers who work and consume, but do not hold capital. There are two types of goods and two production technologies: a consumption good and a capital good, and a technology to produce the consumption good and a technology to produce the capital good. There is one type of financial asset traded: claims to returns of capital. Each period is divided into two subperiods. In the first subperiod consumption good is produced. In the second subperiod, capital good is produced and consumption and asset trading take place.

We first describe the details of the two production technologies. Then we describe the asset trading structure and the financial friction. Then we state the entrepreneurs' and workers' optimization problems and define the competitive equilibrium.

3.1. **Technology.**

In the first subperiod of each time period t consumption good production takes place. All entrepreneurs have access to the consumption good production technology. Entrepreneurs face a stochastic productivity shock A_t which is common to all of them. An entrepreneur who enters period t with capital k_t and hires labor l_t produces y_t with the technology:

$$y_t = A_t k_t^{\gamma} l_t^{1-\gamma}$$

where, y_t is the consumption good produced by the entrepreneur, $A_t > 0$ is the stochastic productivity shock common to all entrepreneurs, k_t is the capital of the entrepreneur, l_t is the labor hired by the entrepreneur, and γ is the capital share in the production of the consumption good. Capital depreciates at rate δ during the consumption good production, i.e. the entrepreneur enters the second subperiod with capital $(1-\delta)k_t$.

In the second subperiod, only a fraction π of entrepreneurs have the opportunity to start new profitable projects. We model this 'investment opportunity' as the entrepreneurs' ability to access the capital good production technology. This technology enables them to produce new capital one-to-one from the consumption good. The arrival of the opportunity to access the capital good production technology is i.i.d. over time and over entrepreneurs. We call entrepreneurs with access to the capital good production technology investing entrepreneurs and entrepreneurs without this access non-investing entrepreneurs.

3.2. **Trading and Financial Frictions.**
In the second subperiod, consumption, capital good production and asset trading take place. There is one type of financial asset traded: claims to capital returns (we refer to these simply as assets or equity). Before we proceed with the discussion of the asset trading structure, we want to emphasize that the return per unit of capital is equal across entrepreneurs independent of their capital holdings and independent of their opportunity to access the capital good production technology. Therefore entrepreneurs are indifferent as to whose equity they hold. To see this, consider the entrepreneur Toyoda with capital k_t^T. In the first subperiod he hires labor on a competitive labor market at wage w_t to maximize his profit $\mathit{Profit}(k_t^T; A_t, w_t) := A_t \left(k_t^T\right)^\gamma \left(l_t^T\right)^{1-\gamma} - w_t l_t^T$. The optimal behavior of Toyoda implies that he hires labor $l_t^T = \left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}} k_t^T$. This amount of labor equalizes the wage rate with the marginal product of labor, i.e. $w_t = MPL_t = (1-\gamma)A_t \left(k_t^T\right)^\gamma (l_t^T)^{-\gamma}$. Therefore, $\mathit{Profit}(k_t^T; A_t, w_t) = \gamma A_t \left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1-\gamma}{\gamma}} \cdot k_t^T = r_t k_t^T$, where $r_t = \gamma A_t \left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1-\gamma}{\gamma}}$ denotes the return per unit of capital. Since all entrepreneurs face the same stochastic productivity shock A_t and hire labor at the same wage w_t (determined by aggregate market clearing), the return on capital r_t is the same for all entrepreneurs.

To understand the trading structure in our economy we first describe the asset holdings of the entrepreneurs. Entrepreneurs can hold two types of assets: physical capital and equity to other entrepreneurs' capital returns. We define the individual state of the entrepreneur T by (k_t^T, e_t^T, s_t^T), where k_t^T is the physical capital held by the entrepreneur, e_t^T is equity to other entrepreneurs' capital and s_t^T is equity to entrepreneur T's own capital sold to other entrepreneurs.

Physical capital k_t^T is used by the entrepreneur T in the consumption good production and it depreciates at rate δ. We assume that physical capital is not traded in the economy. Equity e_t^T entitles the entrepreneur T to the stream of returns of e_t^T units of other entrepreneurs' capital. Since the underlying capital depreciates at rate δ, e_t^T depreciates at rate δ as well. As we discussed above, entrepreneur Toyoda is indifferent between holding equity of entrepreneur Ford and entrepreneur Durant, as they entitle Toyoda to the same stream of returns per unit of this asset. s_t^T, which denotes claims to own capital returns sold by entrepreneur T depreciates at rate δ as well. Therefore an entrepreneur with the individual state (k_t^T, e_t^T, s_t^T) is entitled to returns from $k_t^T - s_t^T + e_t^T$ units of capital.

In the second subperiod, entrepreneurs are facing a financial constraint, which restricts the amount of external financing. An investing entrepreneur that produces i_t units of new capital can at most sell θ_t fraction of returns from i_t. On the other hand we assume that claims to already installed capital can be traded without restrictions. This implies that the total amount of equity sold by period t (denoted as s_{t+1}^T) can be at most the sum of a fraction θ_t of period t investment i_t^T and the depreciated period t capital holdings $(1-\delta)k_t^T$:

$$s_{t+1}^T \leq \theta_t i_t^T + (1-\delta)k_t^T \tag{3.1}$$

To understand this constraint, we define $k_{t+1}^T = (1-\delta)k_t^T + i_t^T$ and rewrite inequality (3.1) as:

$$k_{t+1}^T - s_{t+1}^T \geq (1-\theta_t)i_t^T \tag{3.2}$$

The left hand side of inequality (3.2) captures the net amount of returns to the entrepreneur T's own capital that he must carry into period $t+1$. Since he can sell at most $\theta_t i_t^T$ of 'new' equity he must keep at least $(1-\theta_t)i_t^T$ of the newly produced capital unsold, which is captured in the right hand side of inequality (3.2). θ_t is assumed to be a stochastic process which is common to all entrepreneurs.

3.3. **Entrepreneurs' Maximization Problem.**

There is a unit measure of ex-ante identical entrepreneurs, who hold capital, trade assets and consume, but do not work. Ex-post, entrepreneurs will differ in their capital and asset holdings. The budget constraint of an entrepreneur with capital and asset holdings (k_t^T, e_t^T, s_t^T) can be written as:

$$c_t^T + i_t^T + q_t[k_{t+1}^T - s_{t+1}^T + e_{t+1}^T] \quad \leq \quad r_t[k_t^T - s_t^T + e_t^T] + (1-\delta)q_t[k_t^T - s_t^T + e_t^T] + q_t i_t^T$$

where r_t is the return on capital. Therefore the first term on the right hand side is the return that the entrepreneur T is entitled to. The second term is the market value of his depreciated unsold capital and asset holdings. The third term is the market value of equity to his newly installed capital at the market price q_t. The left hand side sums up his expenditure. He can consume c_t^T, invest i_t^T with investment being generated one-to-one from the consumption good and carry unsold capital $k_{t+1}^T - s_{t+1}^T$ or equity e_{t+1}^T into period $t+1$. These are traded at market price q_t. The maximization problem of this entrepreneur therefore is (we drop the T superscripts for simplicity):

$$
\begin{aligned}
& \max E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \qquad \text{s.t.} \\
(BC) \qquad & c_t + i_t + q_t[k_{t+1} - s_{t+1} + e_{t+1}] \;\leq\; [k_t - s_t + e_t][r_t + (1-\delta)q_t] + q_t i_t \\
(FC1) \qquad & k_{t+1} - s_{t+1} \;\geq\; (1-\theta_t) i_t \\
(FC2) \qquad & e_{t+1} \;\geq\; 0
\end{aligned}
$$

In this problem expectations are taken over the stochastic processes for θ_t and A_t, equilibrium processes for prices (taken as given and correctly forecasted by the entrepreneur) and the arrival of the investment opportunity. If the entrepreneur happens not to have an investment opportunity he must set i_t to zero. Note that the returns of the unsold capital $k_{t+1} - s_{t+1}$ and claims to returns of other entrepreneurs' capital e_{t+1} are the same state by state. Moreover trades in these assets in period $t+1$ are not subject to any restrictions. Therefore $k_{t+1} - s_{t+1}$ and outside equity e_{t+1} are perfect substitutes and (FC1) binding is equivalent to the no-short-sales (FC2) binding and we can sum them up without loss. The intuition for the equivalence of (FC1) and (FC2) is quite straightforward: an entrepreneur who has the investment opportunity and whose (FC1) is binding will sell all his other assets e_t to take advantage of this profitable opportunity. Therefore, we can simplify the maximization problem by defining net asset holdings $n_t := k_t - s_t + e_t$ and writing:

$$
\begin{aligned}
& \max E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \qquad \text{s.t.} \\
(BC) \qquad & c_t + i_t + q_t n_{t+1} \;\leq\; n_t[r_t + (1-\delta)q_t] + q_t i_t \\
(FC) \qquad & n_{t+1} \;\geq\; (1-\theta_t) i_t
\end{aligned}
$$

Having stated the maximization problem we can analyze the role of the real friction (only a fraction of entrepreneurs can start a new project) and the financial friction (they can only finance a fraction θ of new investment externally) in our model. Assuming that all entrepreneurs in the economy have the ability to start new projects would imply that $q_t = 1$ as no entrepreneur would be willing to pay more given that he can produce new capital at price one. Assuming that investing entrepreneurs can finance all their new investment externally, i.e. $\theta_t = 1$, would lead to $q_t = 1$ as well. If q_t was larger than one then an investing entrepreneur would be able to decrease his consumption by one unit, increase investment by one unit and sell claims to the newly produced capital at $q_t > 1$. Then he could increase his consumption by one unit back to the original level and he would end up with a net profit of $q_t - 1 > 0$. Therefore this cannot be an equilibrium and $q_t = 1$ at all times. We conclude that we need both these frictions to generate asset price volatility in our model. In fact, we need the financial constraint (FC) to bind otherwise $q_t = 1$ by a reasoning similar to the one for $\theta_t = 1$.

3.4. **Workers' Maximization Problem.**

There is a unit measure of identical workers, i.e. agents who do not have access to consumption good and capital good production technologies. In each period, a worker decides how much to consume and how much labor to provide. For simplicity we assume that workers do not participate in asset trading. A worker maximizes the expected lifetime utility subject to a period-by-period budget constraint. His maximization problem is therefore static and can be written as:

$$\max U\left(c_t' - \frac{\omega}{1+\eta}(l_t')^{1+\eta}\right) \text{ s.t. } c_t' \leq w_t l_t'$$

where c_t' is the consumption of the worker in period t, l_t' is the labor provided by the worker in period t. $U[.]$ is increasing and strictly concave function, $\omega > 0$ and $\eta > 0$.

3.5. **Equilibrium.**

A competitive equilibrium is quantities for entrepreneurs $[\{c_t^j, i_t^j, n_{t+1}^j\}_{t=0}^{\infty}]_{j\in[0,1]}$, quantities for workers $[\{c_t^{'j}, l_t^{'j}, \}_{t=0}^{\infty}]_{j\in[0,1]}$, and prices $(\{q_t, r_t, w_t\}_{t=0}^{\infty})$, such that quantities solve workers' and entrepreneurs' problems given prices, input prices w_t, r_t are determined competitively, and markets clear.

3.6. **Comparison with Kiyotaki and Moore (2008).**

In this subsection we discuss the differences between our model and Kiyotaki and Moore's. In their model entrepreneurs can hold equity n_t and fiat money m_t. The price of money in terms of the general consumption good is p_t. They assume that the leverage ratio θ is constant over time. An entrepreneur can sell all his money holdings but he can only sell a fraction ϕ_t of his equity holdings. ϕ_t is a stochastic process common to all entrepreneurs. The maximization problem of an entrepreneur in Kiyotaki and Moore's model is:

$$
\begin{aligned}
& \max E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \qquad \text{s.t.} \\
(BC) \qquad & c_t + i_t + q_t n_{t+1} + p_t m_{t+1} \leq n_t[r_t + (1-\delta)q_t] + q_t i_t + p_t m_t \\
(FC) \qquad & n_{t+1} \geq (1-\theta)i_t + (1-\phi_t)(1-\delta)n_t
\end{aligned}
$$

In the real world equity trades happen continuously. It is hard to document a restriction that puts a limit on the amount of equity an entrepreneur can sell in a given time period (in our model a time period is a quarter). Therefore in our model entrepreneurs are able to sell all their equity holdings, i.e. $\phi_t = 1$ in every period. The focus of our work in not monetary policy, therefore we have abstracted from fiat money in our model. Finally, we assume that θ varies over time.

4. Characterization

In this section we solve the model and characterize the solution. We show that the solution is determined by a single equation in the price of equity q_t. This enables us to do a comparative statics exercise in the exogenous shocks A_t and θ_t. Finally, to provide a better understanding of the role of other exogenous parameters, namely δ and π, we derive conditions under which the financial constraint binds in steady state.

4.1. Solving the Model.

We begin this section with a proof of a lemma that links the financial constraint to the price of equity q_t.

Lemma 4.1. *Suppose that $\theta_t < 1$. Then the financial constraint binds for all investing entrepreneurs if and only if $q_t > 1$.*

Proof: The problem of an entrepreneur with asset holdings n_t in this economy is:

$$
\begin{aligned}
& \max E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \qquad \text{s.t.} \\
(BC) \qquad & c_t + i_t + q_t n_{t+1} \leq n_t[r_t + (1-\delta)q_t] + q_t i_t \\
(FC) \qquad & n_{t+1} \geq (1-\theta_t)i_t
\end{aligned}
$$

If an entrepreneur does not have an investment opportunity at time t, he must set $i_t = 0$. If he has an investment opportunity, we can derive the above stated result using the first order condition with respect to i_t. We will denote the Lagrange multiplier on the budget constraint by λ_t and the Lagrange multiplier on the financial constraint by μ_t. The budget constraint always binds and therefore $\lambda_t > 0$. The necessary first order condition with respect to i_t is:

$$(q_t - 1)\lambda_t = (1 - \theta_t)\mu_t$$

This equation makes it clear that $q_t > 1 \implies \mu_t > 0$, the financial constraint binds and also $\mu_t > 0 \implies q_t > 1$. The result does not depend on the initial asset holdings n_t and therefore applies to all investing entrepreneurs. □

The intuition for the sufficient part is as follows. If $q_t > 1$ and the financial constraint does not bind then the solution to the problem does not exist, because there will be arbitrage opportunities for investing entrepreneurs. At any allocation an investing entrepreneur will find it profitable to increase i_t by Δ and consumption by $(q_t - 1)\Delta$.

4.1.1. *Simplifying the Workers' Problem.* In this section we simplify the workers' problem. We make use of this simplification in our quantitative analysis. We will show that output does not depend on the current realization of θ_t and derive the relationships between labor and consumption and aggregate output.

We can simplify the workers' problems as their decisions do not directly depend on the stochastic processes for A_t and θ_t. The representative worker solves:

$$\max U\left(c'_t - \frac{\omega}{1+\eta}(l'_t)^{1+\eta}\right) \text{ s.t. } c'_t \le w_t l'_t$$

Therefore:

$$l'_t = \left(\frac{w_t}{\omega}\right)^{1/\eta} \tag{4.1}$$

Equation (4.1) holds for each worker. Therefore the aggregate labor supply L'_t can be written as:

$$L'_t = \left(\frac{w_t}{\omega}\right)^{1/\eta} \tag{4.2}$$

The aggregate labor demand by the entrepreneurs L_t is determined by:

$$w_t = A_t(1-\gamma)K_t^{\gamma}L_t^{-\gamma}$$

In equilibrium supply equals demand, i.e. $L'_t = L_t$ and hence:

$$\begin{aligned} w_t &= \omega^{\frac{\gamma}{\eta+\gamma}} \left[(1-\gamma)A_t\right]^{\frac{\eta}{\eta+\gamma}} K_t^{\frac{\eta\gamma}{\eta+\gamma}} \\ L_t &= \left[\frac{A_t(1-\gamma)}{\omega}\right]^{\frac{1}{\gamma+\eta}} K_t^{\frac{\gamma}{\gamma+\eta}} \end{aligned}$$

For the return on capital r_t we get:

$$\begin{aligned} r_t &= A_t \gamma K_t^{\gamma-1} L_t^{1-\gamma} = A_t \gamma K_t^{\gamma-1} \left\{ \left[\frac{A_t(1-\gamma)}{\omega}\right]^{\frac{1}{\gamma+\eta}} K_t^{\frac{\gamma}{\gamma+\eta}} \right\}^{1-\gamma} = \\ &= A_t^{\frac{1+\eta}{\gamma+\eta}} \gamma \left[\frac{1-\gamma}{\omega}\right]^{\frac{1-\gamma}{\gamma+\eta}} K_t^{\frac{\eta(\gamma-1)}{\gamma+\eta}} \end{aligned}$$

Thus we can express L_t, w_t, r_t as functions of parameters and aggregate states K_t, A_t only. Note that L_t, w_t, r_t do not depend on the financial constraint parameter θ_t. Therefore in period t, output Y_t is not a function of θ_t. We can rewrite (4.2) as:

$$\begin{aligned} L_t &= \left(\frac{w_t}{\omega}\right)^{1/\eta} = \left(\frac{MPL_t}{\omega}\right)^{1/\eta} = \left(\frac{(1-\gamma)Y_t}{\omega L_t}\right)^{1/\eta} \Longrightarrow \\ L_t^{1+\eta} &= \frac{(1-\gamma)Y_t}{\omega} \Longrightarrow \\ (1+\eta)\log L_t &= \log Y_t + \log \frac{1-\gamma}{\omega} \end{aligned}$$

The implications for the dynamics of labor with respect to output are:

$$\begin{aligned} corr(\log L_t, \log Y_t) &= 1 \\ (1+\eta)^2 var(\log L_t) &= var(\log Y_t) \end{aligned}$$

Since workers cannot save, aggregate workers' consumption equals labor's share in output $C'_t = (1-\gamma)Y_t$. Thus:

$$\begin{aligned} corr(\log C'_t, \log Y_t) &= 1 \\ var(\log C'_t) &= var(\log Y_t) \end{aligned}$$

Since workers consume a large fraction of total consumption in the economy (including entrepreneurs' consumption), this will affect the dynamics of total consumption relative to output.

4.1.2. *Solving the Entrepreneurs' Problem.* The problem of an entrepreneur is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \qquad \text{s.t.}$$

$$\begin{array}{lrcl} (BC) & c_t + i_t + q_t n_{t+1} & \leq & n_t[r_t + (1-\delta)q_t] + q_t i_t \\ (FC) & n_{t+1} & \geq & (1-\theta_t) i_t \end{array}$$

We can rewrite the budget constraint of an investing entrepreneur (denoted with a superscript i) by plugging in for i_t from the financial constraint:

$$c_t^i + q_t^R n_{t+1}^i \leq n_t[r_t + (1-\delta)q_t]$$

where q_t^R is the replacement cost of capital defined as:

$$q_t^R := \frac{1-\theta_t q_t}{1-\theta_t}$$

If $q_t = 1$ and the financial constraint does not bind[5], the problem of an investing entrepreneur is the same as the problem of a non-investing entrepreneur. Note that in any equilibrium it must be[6] $q_t < \frac{1}{\theta_t}$. Finally note that the no-short-sale constraint $n_{t+1}^s \geq 0$, which is essentially the financial constraint of the non-investing entrepreneur[7], does not bind. To see this we have to consider two cases. If $q_t = 1$ both types of entrepreneurs are solving the same problem. The financial constraint of an investing entrepreneur with asset holdings n_t is not binding. This implies that the same is true for a non-investing entrepreneur with asset holdings n_t (right hand side of his financial constraint is 0 and therefore lower than for the investing entrepreneur while the left hand sides are the same). If $q_t > 1$ the financial constraint for non-investing entrepreneurs cannot bind. If it did bind then they would be selling equity as their financial constraint is $n_{t+1} \geq 0$ and their equity holdings at the beginning of the trading subperiod are $(1-\delta)n_t$. This would imply that on aggregate investing entrepreneurs are buying equity at price $q_t > 1$, which they will not do since they can produce capital at price one.

[5]We will ignore cases in which $q_t < 1$. This is only possible if the level of capital is so high that aggregate investment is 0. This will not happen in our quantitative exercises.

[6]To see that suppose $q_t \geq \frac{1}{\theta_t}$ and consider the following strategy of an entrepreneur with an investment opportunity: take one unit of consumption good, convert it into capital, keep fraction $1-\theta_t$ and sell fraction θ_t of this capital as equity, get $\theta_t q_t \geq 1$ units of consumption good (because of the price assumption). Convert this into capital etc. This strategy makes it possible to increase one's capital holdings beyond bounds, which is inconsistent with equilibrium. This along with $q_t \geq 1$ implies that $0 < q_t^R \leq 1$.

[7]We denote their allocations with a superscript ^{s}.

Log utility and linearity of the right hand side of the budget constraint in wealth guarantee that the decision rules are linear. In the appendix, we prove the following lemma, which verifies that this well-known result[8] carries over into our environment with idiosyncratic investment opportunity risk and the possibility of switching between regimes $q_t > 1$ and $q_t = 1$.

Lemma 4.2. *Individual policy functions are linear:*

$$\begin{aligned} c_t^i &= (1-\beta)n_t[r_t + (1-\delta)q_t] \\ q_t^R n_{t+1}^i &= \beta n_t[r_t + (1-\delta)q_t] \\ c_t^s &= (1-\beta)n_t[r_t + q_t(1-\delta)] \\ q_t n_{t+1}^s &= \beta n_t[r_t + q_t(1-\delta)] \end{aligned}$$

where n_t denotes the initial asset holdings of an entrepreneur. Superscript i denotes the state in which this entrepreneur has an investment opportunity in period t and superscript s denotes the state in which he does not have an investment opportunity in period t. With linear policy rules, prices are functions of aggregate quantities only. Without linear policy rules one would have to keep track of the whole asset distribution.

We will denote the aggregate quantities with capital letters and use the fact that the arrival of the investment opportunity is i.i.d. This implies that entrepreneurs with an investment opportunity hold fraction π of the total asset holdings in the economy at the beginning of period t and investors without an investment opportunity hold fraction $1-\pi$ of all assets at the beginning of period t. Integrating over individual policies thus yields:

$$C_t^i = (1-\beta)\pi N_t[r_t + (1-\delta)q_t] \tag{4.3}$$

$$q_t^R N_{t+1}^i = \beta\pi N_t[r_t + (1-\delta)q_t] \tag{4.4}$$

$$C_t^s = (1-\beta)(1-\pi)N_t[r_t + (1-\delta)q_t] \tag{4.5}$$

$$q_t N_{t+1}^s = \beta(1-\pi)N_t[r_t + (1-\delta)q_t] \tag{4.6}$$

4.1.3. *Equilibrium.* By definition, aggregate capital in the economy is equal to the aggregate amount of equity N_t. Therefore the dynamics of aggregate capital is determined by aggregate equity holdings of investing and non-investing entrepreneurs: $N_{t+1} = N_{t+1}^i + N_{t+1}^s$.

[8]See Samuelson (1969).

If $q_t = 1$ the equilibrium aggregate quantities will be determined by the aggregate policy function for capital (one can get the equation below by adding equations (4.4) and (4.6)):

$$\begin{aligned} N_{t+1} &= \beta N_t[r_t + (1-\delta)] \\ r_t &= A_t^{\frac{1+\eta}{\gamma+\eta}} \gamma \left[\frac{1-\gamma}{\omega}\right]^{\frac{1-\gamma}{\gamma+\eta}} N_t^{\frac{\eta(\gamma-1)}{\gamma+\eta}} \end{aligned}$$

These two equations fully describe the aggregate behavior of the model. The second equation determines r_t through the workers' problem. The rest of the variables are determined using the derived policy functions.

If $q_t > 1$, the dynamics of the model is determined by the aggregate policies for N^i_{t+1}, N^s_{t+1}, market clearing conditions and the financial constraint aggregated over investing entrepreneurs. Therefore the behavior of the model is determined by the following equations:

$$\begin{aligned} q^R_t N^i_{t+1} &= \beta\pi N_t[r_t + (1-\delta)q_t] \\ q_t N^s_{t+1} &= \beta(1-\pi)N_t[r_t + (1-\delta)q_t] \\ N^i_{t+1} &= (1-\theta_t)I_t \\ N^s_{t+1} + N^i_{t+1} &= (1-\delta)N_t + I_t \\ r_t &= A_t^{\frac{1+\eta}{\gamma+\eta}} \gamma \left[\frac{1-\gamma}{\omega}\right]^{\frac{1-\gamma}{\gamma+\eta}} N_t^{\frac{\eta(\gamma-1)}{\gamma+\eta}} \\ q^R_t &:= \frac{1-\theta_t q_t}{1-\theta} \end{aligned}$$

Plugging in for N^s_{t+1}, N^i_{t+1} and I_t from the first three into the fourth one we get:

$$(1-\delta) = \frac{\beta(1-\pi)}{q_t}[r_t + q_t(1-\delta)] - \frac{\theta_t}{(1-\theta_t)}\frac{\beta\pi}{q^R_t}[r_t + (1-\delta)q_t] \tag{4.7}$$

Since r_t is a function of states N_t, A_t, θ_t only, we can solve for q_t as a function of these states and then use (4.4) and (4.6) to compute $N_{t+1}(N_t, A_t, \theta_t)$.

4.2. **Properties of the Solution.**

4.2.1. *Comparative statics in A and θ.* In this subsection we study the properties of the solution of our model when $q_t > 1$. We analyze the effects of changes in A_t and θ_y on the price of equity q_t. We can write the net demand for equity by non-investing entrepreneurs as:

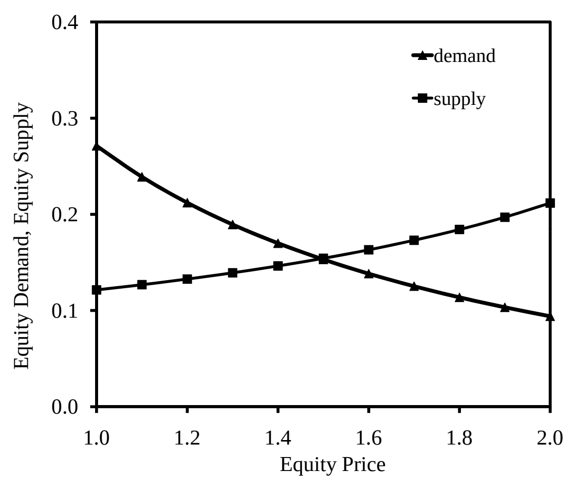
$$D^e := N^s_{t+1} - (1-\delta)(1-\pi)N_t = \beta(1-\pi)N_t[\frac{r_t}{q_t} + 1 - \delta] - (1-\delta)(1-\pi)N_t$$

D^e is a downward sloping demand function since $\frac{\partial D^e}{\partial q} < 0$. Net supply of equity by investing entrepreneurs is given by:

$$\begin{aligned} S^e : &= \pi(1-\delta)N_t + I_t - N^i_{t+1} = \pi(1-\delta)N_t + \frac{\theta_t}{(1-\theta_t)}N^i_{t+1} = \\ &= \pi(1-\delta)N_t + \frac{\theta_t}{(1-\theta_t)}\frac{1}{q^R_t}\beta\pi N_t[r_t + (1-\delta)q_t] \end{aligned}$$

S^e is an upward sloping supply function since $\frac{\partial S^e}{\partial q} > 0$. In equilibrium $S^e = D^e$, which is equivalent to equation (4.7). Figure 4.1 shows the supply and demand functions for a numerically computed example. In this example, we set $N = 8.6, \beta = 0.99, \eta = 1, \omega = 7.14, \delta = 0.0226, A = 1, \gamma = 0.36, \pi = 0.01, \theta = .2$.

Figure 4.1: Demand and supply of equity as a function of the price of equity



Next we analyze what happens when A_t or θ_t change. This is a comparative statics exercise. We fix states N_t, A_t, θ_t, derive the asset supply and demand and the equilibrium price q_t. Then we redo the exercise for a different value of θ_t or A_t.

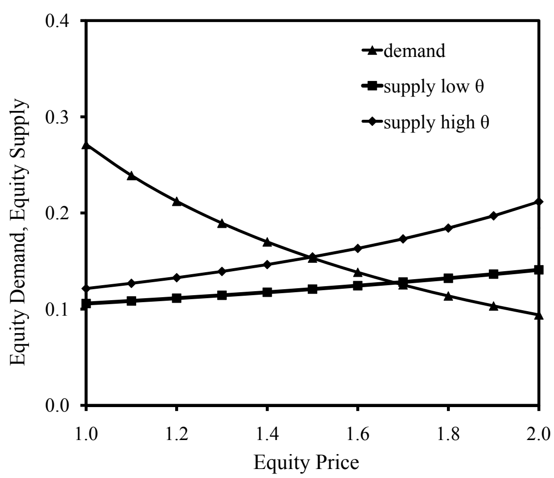
(1) $\Delta\theta_t$. The equity demand curve D^e does not move. We can simplify S^e to get:

$$S^e = \pi(1-\delta)N_t + \frac{\theta_t}{(1-\theta_t q_t)}\beta\pi N_t[r_t + (1-\delta)q_t]$$

$$\frac{\partial S^e}{\partial \theta_t} = \frac{1}{(1-\theta_t q_t)^2}\beta\pi N_t[r_t + (1-\delta)q_t] > 0$$

Then if $\Delta\theta_t > 0$, the equity supply curve moves up and the equity price decreases and the quantity of equity traded increases. Figure 4.2 presents this argument through a numerically computed example. In this example, we set $N = 8.6, \beta = 0.99, \eta = 1, \omega = 7.14, \delta = 0.0226, A = 1, \gamma = 0.36, \pi = 0.01, \theta_{low} = 0.2, \theta_{high} = 0.3$.

Figure 4.2: Demand and supply of equity for various levels of θ



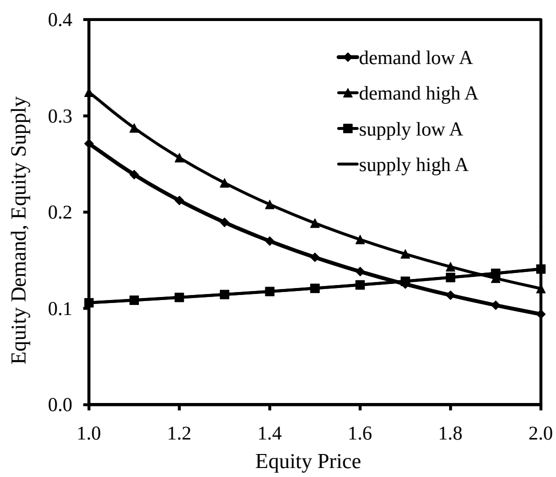
(2) ΔA_t. The demand curve and the supply curve move up with $\Delta A_t > 0$ because:

$$\frac{\partial D^e}{\partial A_t} = \frac{\beta(1-\pi)N_t\frac{\partial r_t}{\partial A_t}}{q_t} > 0 \tag{4.8}$$

$$\frac{\partial S^e}{\partial A_t} = \frac{\theta_t}{(1-\theta_t q_t)}\beta\pi N_t \cdot \frac{\partial r_t}{\partial A_t} > 0 \tag{4.9}$$

These claims are true since $\frac{\partial r_t}{\partial A_t} > 0$. Thus the volume of equity traded increases unambiguously with A_t. As for the price of equity, equations (4.8) and (4.9) imply that as long as $1-\pi-\theta_t q_t > 0$, the demand curve moves more than the supply curve implying an increase in price. Numerically, we find this to be the case around the equilibrium for small values of π. Figure 4.3 shows the effects of changes in A_t. The shift of the supply curve is very small and the two supply curves are not distinguishable. In this example, we set $N = 8.6, \beta = 0.99, \eta = 1, \omega = 7.14, \delta = 0.0226, A_{low} = 1, A_{high} = 1.1, \gamma = 0.36, \pi = 0.01, \theta = 0.2$.

Figure 4.3: Demand and supply of equity for various levels of A

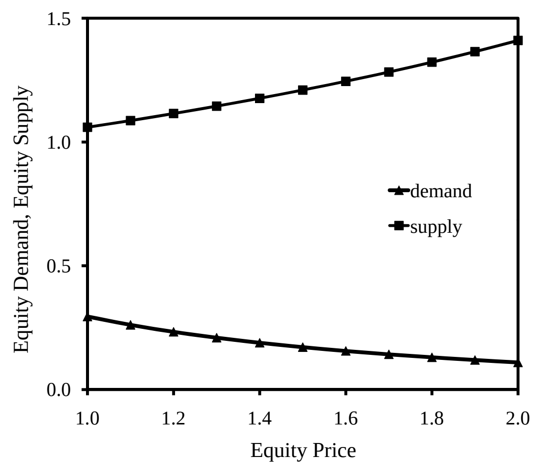


4.2.2. *Characterization of Steady State Equilibria.* There are two types of steady state equilibria: (1) equilibria in which the financial constraint binds and the price of equity is greater than one, and (2) equilibria in which the financial constraint does not bind and the price of equity is equal to one. Theorem 4.3 summarizes the conditions under which each of these equilibria exists. We prove this theorem in the appendix, section A.1.

Theorem 4.3. *In steady state the financial constraint binds and the price of equity is greater than one if and only if* $\theta < \frac{\delta-\pi}{\delta}$.

An example of a steady state, in which $q = 1$ is shown in Figure 4.4. It shows that at any price $q > 1$ supply of equity exceeds demand. At price $q = 1$ investing entrepreneurs are willing to supply any amount of equity that will not violate their financial constraint (any amount less or equal to the amount defined by the intersection of the supply curve with the y axis). Supply is indeterminate and asset trades are determined by demand. In this example we set $N = 8.6, \beta = 0.99, \eta = 1, \omega = 7.14, \delta = 0.0226, A = 1, \gamma = 0.36, \pi = 0.1, \theta = 0.2$.

Figure 4.4: Demand and supply of equity as a function of the price of equity



If θ is small then in steady state $q > 1$ and the financial constraint binds. If $q = 1$ investing entrepreneurs would not be willing to produce enough new capital without violating the financial constraint to cover the demand for equity by the non-investing entrepreneurs. This can be seen in Figure 4.1. At price one investing entrepreneurs are willing to supply any amount less or equal to the amount defined by the intersection of the supply curve with the y axis. Any larger amount would violate their financial constraint. Since at price one the demand for equity exceeds supply the price of equity must increase. Therefore $q > 1$ and the financial constraint binds.

5. Data and Model Specification

The time period for our data is 1964-2008. We obtain quarterly data from the Current Employment Statistics provided by the Bureau of Labor Statistics, National Income and Product Accounts and Fixed Asset Tables provided by the Bureau of Economic Analysis, COMPUSTAT, Flow of Funds, CRSP and Global Financial Data. Details of the construction of the time series can be found in appendix B.

5.1. **Model Specification.**

We divide parameters and stochastic processes in the model into two groups. The first group consists of utility and technology parameters. The second group consists of the parameter π capturing the fraction of firms with access to capital production technology, the process for the ratio of outside to total financing of investment projects θ_t, and the process for the total factor productivity A_t.

5.1.1. *Utility and Technology Parameters.* We divide utility and technology parameters into two groups: 1) parameters that we take from the literature: share of capital in output production $\alpha = 0.36$, subjective discount factor $\beta = 0.99$ (adjusted for quarterly analysis), and the labor supply elasticity parameter $\eta = 1$[9]. 2) parameters that we choose so that our model in steady state matches chosen moments in the data. The average annual nominal investment to nominal capital ratio from 1964 to 2008 is 9.35%. To match this ratio in the steady state of our model we set quarterly depreciation $\delta = 2.26\%$. We set the scaling parameter of the workers' utility function ω so that the labor supply in steady state is equal to $l_s = 0.3$. Table 5.1 summarizes our benchmark parameters.

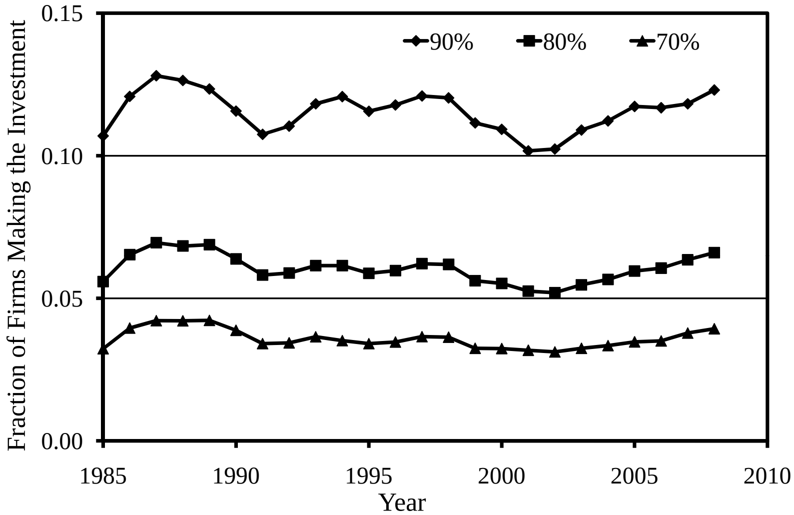
Table 5.1: Benchmark Parameters

Parameter	α	β	η	δ	ω
Value	0.36	0.99	1	0.0226	8.15

5.1.2. *Parameter π and The Processes for θ_t and A_t.* We estimate the fraction of firms that have access to capital good production technology (i.e. π) as follows. Using annual firm level data on net capital expenditures (variable *capexv* in the COMPUSTAT database), we construct a time series for corporate investment. We then compute the smallest percentage of firms who have done a certain percentage of total corporate investment. Figure 5.1 shows the smallest percentage of firms who have done 70% to 90% of total corporate investment. This figure shows that the majority of the corporate investment in done by a small percentage of firms. Moreover these ratios have been stable over the last 25 years. 80% of total investment has been done by about 6% of firms. Therefore we use annual $\pi = 0.06$ and perform a sensitivity analysis in section 7.

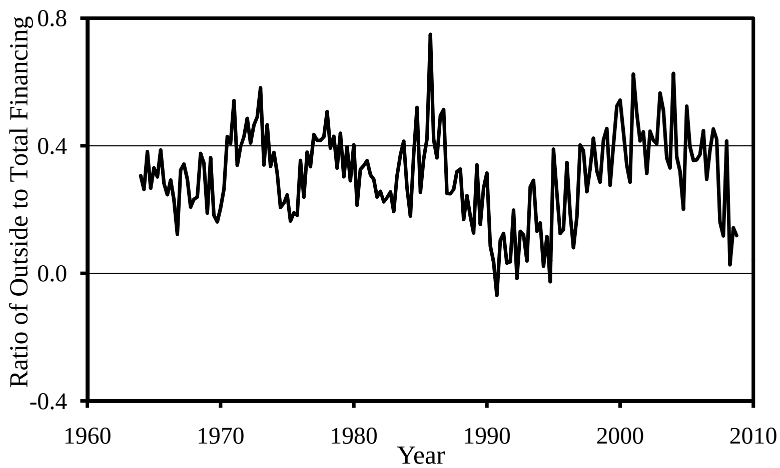
[9]We will perform sensitivity analysis on β and η to check whether our results are affected by our choice of these parameters.

Figure 5.1: Concentration of firms' investment



This figure shows the smallest fraction of firms that accounts for 90%, 80% and 70% of total investment by nonfinancial corporate sector.

Figure 5.2: External financing as a fraction of investment



This figure shows the ratio of outside financing to total investment of the nonfinancial corporate sector.

We construct the series for θ_t from the data as follows. θ in the model stands for the fraction of investment that is financed externally. Using Flow of Funds data we define for the nonfinancial corporate sector:

$$\theta = \frac{\text{Funds Raised in Markets}}{\text{Capital Expenditures}}$$

For definitions of these variables, see appendix B. The series is shown in Figure 5.2.

We construct the series for total factor productivity A_t using the time series of output, capital and labor assuming a Cobb-Douglas production technology with share of capital in output production $\alpha = 0.36$. We define $\hat{z}_t = \log(A_t)$, and use z_t, the linearly detrended version of $\hat{z}_t$ as a realization of the shock process for the consumption good production technology. Having constructed the series for z_t and θ_t we estimate the stochastic processes for z_t and θ_t as follows.

$$\begin{aligned} z_{t+1} &= \rho_z z_t + \varepsilon_{z,t} \\ \theta_{t+1} &= \mu_\theta + \rho_\theta(\theta_t - \mu_\theta) + \varepsilon_{\theta,t} \end{aligned}$$

$$E \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{\theta,t} \end{pmatrix}^2 = \begin{bmatrix} \sigma^2_{\varepsilon_z} & corr(\varepsilon_z, \varepsilon_x)\sigma_{\varepsilon_z}\sigma_{\varepsilon_\theta} \\ corr(\varepsilon_z, \varepsilon_x)\sigma_{\varepsilon_z}\sigma_{\varepsilon_\theta} & \sigma^2_{\varepsilon_\theta} \end{bmatrix}$$

Table 5.2 summarizes our estimation of the TFP[10] and θ processes. For the TFP process we find that $\rho_z = 0.95, \sigma^2_z = 0.00602^2$.

Table 5.2: Summary statistics for the TFP shock z and the θ processes

variable x	μ_x	ρ_x	σ_{ε_x}	$corr(\varepsilon_z, \varepsilon_x)$
θ	0.2844	0.6510	0.1679	-0.0736
z	0	0.9498	0.00602	1

[10]While the persistence parameter is standard in the literature, the standard deviation of the error term is slightly lower that those used in previous studies (see e.g. Prescott, 1986). This is consistent with the recent decrease in output volatility known as "the great moderation".

6. Benchmark Empirical Results

We solve our model and simulate it by generating random series of the primitive shocks A_t and θ_t using the estimated parameters for these processes[11]. Having simulated the model we compute a set of statistics and compare them to the data. We find that our model matches quite well the behavior of aggregate quantities and prices in the data. Our benchmark model generates about 80% of the volatility in asset prices. We find that most of the volatility in asset prices in our model comes from the volatility in the financial friction parameter θ_t.

Tables 6.1 and 6.2 summarize our benchmark results. In these two tables we present our results for 3 different models to highlight the role of financial frictions. The model in column (1) in Table 6.1 and Table 6.2 assumes that the financial constraint never binds. This is the case for example for large values of π. In this case the price of equity q_t is always one and fluctuations in θ are irrelevant for the dynamics of the model. Volatility in reported variables comes from the volatility in the TFP shock A_t. In column (2) in Table 6.1 and Table 6.2 we present our results for a version of the model in which the financial constraint binds, A_t is stochastic, but θ_t is constant at its mean level. This model highlights the role of the financial constraint as a propagator of TFP shocks. In column (3) in Table 6.1 and Table 6.2 we present our results for the model in which the financial constraint binds and both θ_t and A_t are stochastic. Relative to the model of column (2) this model highlights the role of fluctuations in θ_t.

6.1. **Standard Business Cycle Statistics.**

Table 6.1 summarizes our results for the standard business cycle statistics. We find that financial frictions do not affect output volatility and persistence. This indicates that the process for output in our model is determined by the process for the productivity shock (assumed to be the same in the 3 versions of the model). As discussed in section 4 labor and output are perfectly correlated and their relative volatility is determined by the parameter η. Therefore the properties of labor supply are not affected by the financial friction parameter θ_t either.

[11]We approximate the processes on a 25 point grid in the $z \times \theta$ space using the Tauchen approximation method, see Tauchen (1986), and then use $A_t = \exp(z_t)$.

Table 6.1: Standard Business-Cycle Statistics[a]

Statistic[c]	Data[b]	(1) FC not binding A stochastic	(2) FC binding θ constant A stochastic	(3) FC binding θ stochastic A stochastic
σ_Y	1.52	1.18	1.19	1.18
σ_I	5.00	1.70	1.32	5.12
σ_C	0.85	1.01	1.15	1.93
σ_L	1.73	0.59	0.60	0.59
ρ_Y	0.87	0.68	0.67	0.67
ρ_I	0.85	0.68	0.67	0.40
ρ_C	0.90	0.68	0.67	0.47
ρ_L	0.92	0.68	0.67	0.67
$\rho(Y, I)$	0.90	1.00	1.00	0.23
$\rho(Y, C)$	0.85	1.00	1.00	0.61
$\rho(Y, L)$	0.87	1.00	1.00	1.00

[a] Results for the models are based on 100 replications of size 180.

[b] σ_x is a standard deviation of variable x, ρ_x is the autocorrrelation of x and $\rho(x, y)$ is the correlation between x and y. All variables are logged and HP filtered before statistics are computed. Standard deviations are measured in percentage terms.

[c] This column contains quarterly statistics computed for the U.S. data in 1964:1 - 2008:4. Details of the construction of the series are in the appendix, section B.

Column (1) contains results for a version of the model in which the financial constraint is not binding and the process for TFP is estimated using U.S. data 1964:1 - 2008:4.

Column (2) contains results for a version of the model in which the financial constraint is binding, the process for TFP is estimated using U.S. data 1964:1 - 2008:4 and θ is constant at its mean level .2845.

Column (3) contains results for a version of the model in which the financial constraint is binding and both the process for TFP and θ is estimated using U.S. data 1964:1 - 2008:4.

In contrast to output the behavior of investment is significantly affected by financial frictions. In the model without financial frictions shown in column (1) of Table 6.1 investment is significantly less volatile than in the data. This seems to be at odds with the results for the standard one-sector growth model, in which TFP shocks generate investment volatility observed in the data. Our model of column (1) resembles the standard one sector growth model, but there is one important difference. In our model investment is determined by entrepreneurs only. Log utility implies that they save a fixed fraction of their income $r_t K_t + (1-\delta)K_t$, which is significantly less volatile than workers' income. This results in the low investment volatility in this version of our model. For the case of constant θ and a binding financial constraint shown in column (2) investment volatility is further decreased by endogenous changes in q. If A_t increases the asset demand increases as discussed in section 4. However the increase in the equilibrium quantity demanded will be smaller than if supply was infinitely elastics (column (1)). Therefore relatively less new capital will be produced and investment will be less volatile.

Adding volatility in θ_t increases the investment volatility significantly as shown in column (3) in Table 6.1. In fact investment volatility is slightly higher[12] than in the data. This result indicates that in our model shocks to θ play a more important role in investment fluctuations than shocks to A. This assertion is further supported by the relatively low persistence of investment coming from the lower persistence of θ relative to A. In contrast, we have argued above that output dynamics is driven by shocks to A only. The low correlation between θ and A that we estimated from the data therefore translates into the relatively low correlation between investment and output.

6.2. **Financial Statistics.**

Table 6.2 summarizes our results for quarterly asset prices and returns. The return on equity is defined as $r^e = \frac{r_t+(1-\delta)q_t}{q_{t-1}} - 1$. The corresponding counterpart in the data is the real value weighted stock return. We define the total market value in the model as $q_t N_t$. The corresponding counterpart in the data is the series *totval* from he CRSP database. We construct the model risk-free rate as follows. Shadow price of a risk free asset is:

$$p_t(s^t) = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \right]$$

[12] This is an improvement relative to models with habit persistence such as Boldrin, Christiano and Fisher (2001) or Christiano and Fisher (2003), whose models do not generate enough investment volatility.

Non-investing entrepreneurs are not constrained in their asset holdings whereas the investing entrepreneurs would like to sell more assets, but they cannot because of the financial constraint. Therefore, we construct the risk free rate as the shadow risk free rate of the unconstrained non-investing entrepreneurs[13]. We get:

$$r_t^f = \frac{1}{p_t(s^t)} = \frac{1}{q_t E_t\left[\frac{1}{r_{t+1}+(1-\delta)q_{t+1}}\right]}$$

We compare this model variable to the real return on 3-month T-Bills.

Table 6.2 shows that the model without financial frictions in column (1) generates very little volatility in equity returns and a very small equity premium. Recall that the price of equity $q = 1$ at all times in this model. Therefore there are no capital gains and all volatility in returns comes from the limited volatility of the return r_t. The model in column (2) has a constant θ and the financial constraint binding. It generates a standard deviation of the equity return of 0.77%. This result is similar to Gomes, Yaron and Zhang (2003), who get an equity return volatility of about 1% in a model with endogenous borrowing constraints and TFP shocks the only source of uncertainty. This result highlights the fact that in our model (and theirs as well) the financial constraint is not a strong enough propagator of TFP shocks[14]. With no shocks to θ, investment, output and asset prices are driven by shocks to TFP implying a high correlation between these variables as shown in the second panel in column (2). Finally this version of the model generates only a very modest equity premium.

The success of our benchmark model with a stochastic θ is documented in the first panel in column (3). Our model generates high asset return volatility comparable to the data. It generates over 80% of the observed volatility in asset prices and total market value. In addition our benchmark model generates a quarterly equity premium of 0.85%, which is over 70% of what we see in the data. This is result is of particular interest considering that entrepreneurs in our economy have logarithmic utility. As with investment, our results imply that the dynamics of asset prices and returns in our model are driven by the dynamics of θ_t. Therefore, we see a low persistence of q_t and a low correlation between q_t and Y_t.

[13] This is similar in spirit to the exercise that Gomes, Yaron and Zhang (2003) perform for their incomplete markets model. Alternatively, we could rationalize our choice by thinking about borrowing constrained entrepreneurs. The risk free rate would then be determined by the shadow risk free rate of the non-investing entrepreneurs, because investing entrepreneurs find investing and selling equity more profitable than buying the risk free asset.

[14] We find that our model is able to generate high asset price volatility even with constant θ. However, we would need to increase the volatility of TFP shocks significantly generating a counterfactually high volatility in investment and output.

Table 6.2: Quarterly Financial Statistics[a]

Statistic[c]	Data[b]	(1) FC not binding A stochastic	(2) FC binding θ constant A stochastic	(3) FC binding θ stochastic A stochastic
σ_q	11.85	0	0.93	9.69
σ_{r^e}	8.63	0.07	0.77	10.8
σ_{val}	10.72	0.13	0.92	9.74
ρ_q	0.74	1	0.67	0.38
$\rho(q, Y)$	0.39	0	1.00	0.11
$\rho(q, I)$	0.38	0	1.00	-0.94
$E(r^e)$	1.49	1.01	0.89	1.36
$E(r^f)$	0.30	1.01	0.88	0.51
σ_r^f	0.68	0.06	0.16	5.45
$E(r^e) - E(r^f)$	1.19	0.00	0.008	0.85

[a] Results for the models are based on 100 replications of size 180.

[b] σ_x is a standard deviation of variable x, ρ_x is the autocorrrelation of x and $\rho(x, y)$ is the correlation between x and y. All variables with the exception of the returns are logged and HP filtered before statistics are computed. Standard deviations and returns are measured in percentage terms.

[c] This column contains quarterly statistics computed for the U.S. data in 1964:1 - 2008:4. An exception is the measure for asset prices q, which was computed for 1974:1 - 2008:4. Details of the construction of the series are in the appendix, section B.

Column (1) contains results for a version of the model in which the financial constraint is not binding and the process for TFP is estimated using U.S. data 1964:1 - 2008:4.

Column (2) contains results for a version of the model in which the financial constraint is binding, the process for TFP is estimated using U.S. data 1964:1 - 2008:4 and θ is constant at its mean level .2845.

Column (3) contains results for a version of the model in which the financial constraint is binding and both the process for TFP and θ is estimated using U.S. data 1964:1 - 2008:4.

A shortcoming of our model is the counterfactual negative correlation between asset prices and investment. We have shown in section 4 and in this section that fluctuations in A will imply a positive correlation between investment and asset prices while fluctuations in θ will imply a negative correlation between investment and asset prices. In this section we have shown that the behavior of investment and asset prices is determined by shocks to θ rather than by shocks to A, which implies the large negative correlation.

A general lesson to be taken from this result is the following. In this class of models changes in the tightness of the financial constraint (parameter θ in our model) directly affect the amount of investment, but do not affect the productivity of existing capital in any way. A tighter constraint implies less investment and less new capital making old capital (and new capital as well) more valuable to agents in the economy. Therefore tighter constraints imply higher asset prices[15].

In the data we find the correlation between our measure of asset prices and the financial friction parameter θ_t to be 0.18. While this coefficient is not large and in fact not significant our model captures only part of the story. To bring the model closer to the data we would need to add e.g. a link between financial frictions and the productivity of current capital[16]. This is the logical next step in this line of research. Building a richer model of this kind would make it possible to determine when the 'investment channel' and when the 'current capital channel' plays a role for aggregate quantities and asset prices.

6.3. **Annual Returns Statistics.** In Table 6.3 we compare annual returns in our benchmark model to those reported in Boldrin, Christiano and Fisher (2003). Our model generates over 85% of the observed asset return volatility on the annual basis. This is somewhat less than in Boldrin, Christiano and Fisher. Our model also generates a smaller equity premium, that Boldrin, Christiano and Fisher's model was designed to match. On the other hand Boldrin, Christiano and Fisher's risk free rate volatility is way above what we observe in the data. Interestingly, our model is able to generate the volatility in asset returns with a much smaller volatility in the risk free rate.

[15]We have found this to be true in the original Koyotaki and Moore (2008) model in which the friction takes the form of limited resaleability. In this model an entrepreneur can only sell a fraction of his assets at a point in time to finance new investments. Tightening this constraint implies a decrease in investment and an increase in the asset price by the same logic.

[16]In a recent paper Jermann and Quadrini (2009) assume that firms need to borrow money in order to pay their workers, who have to be paid in advance. Tightening of this constraint results in less workers hired, which implies a decrease in productivity and the price of capital.

Table 6.3: Yearly Returns[a]

Statistic[c]	Data[b]	Benchmark	BCF (2003)
$E(r^e)$	6.03	4.30	7.82
$E(r^f)$	1.23	2.68	1.19
σ_r^e	17.6	15.2	19.4
σ_r^f	2.77	15.3	24.6
$E(r^e) - E(r^f)$	4.80	1.62	6.63

[a] Results for the Benchmark model are based on 100 replications of size 180.

[b] σ_x is a standard deviation of variable x. Returns are measured in percentage terms.

[c] This column contains quarterly statistics computed for the U.S. data in 1964:1 - 2008:4. Details of the construction of the series are in the appendix, section B.

Column 'Benchmark' contains results for the benchmark version of our model in which the financial constraint is binding and both the process for TFP and θ is estimated using U.S. data 1964:1 - 2008:4.

Column 'BCF (2003)' contains results from Boldrin, Christiano and Fisher (2003).

7. Sensitivity Analysis

In this section we provide a summary of our sensitivity analysis. All tables can be found in the appendix, section C. In terms of our main result, we find the following. 1) Asset price volatility is quite sensitive to the parameters of the θ process. 2) Asset price volatility is sensitive to π. 3) Asset price volatility is unaffected by changes in the labor supply elasticity parameter η. 4) Asset price volatility does not change significantly if we use standard balanced growth path preferences for the workers. 5) Asset price volatility increases with discount factor β. We discuss each of these results below.

7.1. **Sensitivity to the parameters of the θ process.** The parameters that define the stochastic process for θ are: mean $E[\theta]$, persistence ρ_θ, variance of innovations $\sigma^2_{\varepsilon_\theta}$ and correlation of innovations with innovations of the TFP process $corr(\varepsilon_z, \varepsilon_\theta)$. We find that changes in $corr(\varepsilon_z, \varepsilon_\theta)$ do not play any role for the asset price volatility. Increasing ρ_θ slightly increases the asset price volatility. We do not report these results in this paper and focus instead on changes in $\sigma^2_{\varepsilon_\theta}$ and $E[\theta]$. These results are reported in Table C.1. The asset price dynamics is driven by fluctuations in θ. Therefore, one would expect the asset price volatility to be increasing in θ. This is what we see in column (2) of Table C.1. We decrease the variance of innovations of the θ process to 50% of the benchmark value. Asset price volatility decreases, but the model still generates over 60% of the asset price volatility observed in the data. Volatility of investment decreases as well, but note that volatility and persistence of output remain unaffected.

Our identification strategy assumes that the financial constraint binds for all investing entrepreneurs at all times. If this was not the case it would not be possible to identify θ in the data. The true θ would be higher than the borrowing data indicates. Therefore we analyze to what extent our results depend on the chosen mean level of θ. We increase the mean θ by 50% and keep all other parameters unchanged. In this version of the model, the financial constraint binds if θ_t is small and it does not bind if θ_t is large. In the latter case no entrepreneurs are constrained[17]. Consequently, we find that the asset price volatility decreases to about 50% relative to the data. These results are reported in column (3) of Table C.1.

Finally, we change the mean θ and the volatility of innovations of the θ process at the same time. We decrease $\sigma^2_{\varepsilon_\theta}$ and increase $E[\theta]$ by 50%. These results are reported in column (4) in Table C.1. The model generates about a quarter of the observed volatility in asset prices and over 40% of the observed volatility in asset returns. This is much more than for example in Gomes, Yaron and Zhang's model. We conclude that in our model fluctuations in financial frictions explain a large fraction of the observed asset price volatility for a wide range of parameters of the θ process.

7.2. **Sensitivity to the probability of having an investment opportunity π.**

Table C.2 in the appendix, section C documents the importance of the parameter π for our model. In theorem 4.3 we showed that in steady state $\theta < \frac{\delta-\pi}{\delta} \iff q > 1$. Note that $\delta < \pi \implies q = 1$ in steady state independent of θ. Recall that investing entrepreneurs are able to sell all their current equity holdings at any point in time. If $\delta < \pi$ investing entrepreneurs hold a relatively large share of aggregate wealth and sales of current equity are enough to cover the demand for equity by non-investing entrepreneurs. Therefore $q = 1$ and the financial constraint does not bind. While we cannot directly map a steady state

[17]Because of linearity of entrepreneurs' decision rules, our model can never have the financial constraint binding for some investing entrepreneurs and not binding for others.

statement into our dynamic equilibrium analysis it is clear that if $\pi > \delta$ we should not expect the financial constraint to bind. If π is smaller than but close to δ, θ would have to be very small in order for the financial constraint to bind and $q > 1$. In a dynamic equilibrium as $\pi \longrightarrow \delta$ the financial constraint will bind ($q_t > 1$) or not bind ($q_t = 1$) depending on the realization of θ_t. Overall we should expect the volatility of asset prices and investment to decrease, which is exactly what we see in the table. The model still generates about 25% of the observed asset price volatility if $\pi = 0.02$ (this corresponds to annual $\pi \sim 8\%$). If $\pi = 0.025$ the financial constraint does not bind and $q_t = 1$ in every period.

This result should not be viewed as a shortcoming specific to our model. Other models with borrowing constraints have a similar feature (see e.g. Gomes, Yaron and Zhang, 2003). To avoid a first best solution and generate interesting dynamics they need to make sure that borrowing constrained entrepreneurs do not accumulate enough assets to escape the borrowing constraint. Therefore these models assume that entrepreneurs discount the future more heavily than households which guarantees that entrepreneurs will remain borrowing constrained. In this paper we have argued that it is reasonable to think about entrepreneurs being constrained in their ability to start new projects, but we have abstracted from several factors that would make the financial constraint tighter. Letting households save would decrease the fraction of wealth held by entrepreneurs and a larger fraction of them would need to have an investment opportunity in order to provide enough new capital for households and non-investing entrepreneurs keeping the financial constraint slack. Therefore we could increase π and still have the financial constraint not binding. Decreasing the discount factor β would enhance this effect by making the share of wealth held by entrepreneurs even smaller. Analyzing these changes to our basic environment remains a task for future research.

7.3. **Labor Supply Elasticity.**

Our choice of the benchmark η implies a Frisch labor supply elasticity[18] of one. While this value is in the range that has been considered in the literature, it is by no means a consensus in the profession. Therefore we consider both higher and lower values to see to what extent our results depend on our choice of η. The implications of changes in η are particularly easy to see for the relative volatility of output and labor. Recall from section 4 that labor supply and output are tightly connected: $corr(\log L_t, \log Y_t) = 1, (1+\eta)^2 var(\log L_t) = var(\log Y_t)$. Changing η therefore changes the relative volatility of labor and output. In fact, we find

[18]With these preferences $\eta = \frac{1}{Frisch\ elasticity}$.

that increasing (decreasing) η decreases (increases) both the volatility of output and labor. The same is true for consumption. This is intuitive since workers' consumption accounts for 64% of the aggregate output and even more of the aggregate consumption. However, the asset price behavior is unaffected by labor supply elasticity. These results are summarized in table C.3 in the appendix, section C.

7.4. **Standard Balanced Growth Preferences, Constant Labor Supply.**
In our model workers consume all their income period by period and adjust their labor supply depending on the current wage only. With balanced growth preferences workers' labor supply will be constant, because for these preferences the income and substitution effects associated with a change in the wage rate cancel out. Therefore this version of the model is equivalent to one in which workers' labor supply is fixed. We report our results for this version of the model in table C.3 in the appendix, section C. We find that while output and consumption volatility decrease other statistics remain unaffected. In particular the asset price behavior is almost identical to the benchmark version of our model with elastic labor supply.

7.5. **Sensitivity to the discount factor β.**
Other studies have used very high discount factors in order to account for both asset prices and macroeconomic quantities. Our results in table C.4 in the appendix, section C, show that our model generates a high asset price volatility for a range of β's. We consider $\beta \in \{0.98, 0.99, 0.999, 0.99999\}$. As β increases entrepreneurs become more patient postponing consumption into the future. Since their share in output is constant at 0.36, investment becomes more correlated with output. As a consequence investment becomes less volatile. On the other hand the share of workers' consumption in aggregate consumption increases as well, which increases the correlation between aggregate consumption and output (recall that the correlation between workers' consumption and output equals one). This is the logic behind the dynamics shown in the table. Importantly, asset price volatility is not affected very much. It increases with β, but even for $\beta = 0.98$ it is still at about $\frac{2}{3}$ of the level that we observe in the data.

8. Conclusion

In this paper we quantify the role of financial frictions in the behavior of asset prices and aggregate macroeconomic quantities. We build an asset pricing model with heterogeneous entrepreneurs, who face two frictions. First, in each period only a fraction of entrepreneurs can start new projects. Second, a new project cannot be fully financed externally due to a financial constraint. We allow the tightness of the financial constraint to vary over time. We calibrate the model to the U.S. data and find that the model generates about 80% of the asset price volatility relative to the aggregate stock market. At the same time the model also fits key aspects of the behavior of aggregate quantities. The model generates a strong propagation of financial shocks into asset prices and aggregate investment. Interestingly, financial shocks do not propagate into output and therefore play a decisively secondary role in the determination of output. Fluctuations in output are driven by TFP shocks.

Our paper contributes to the literature that has tried to reconcile the observed asset price behavior with production economy models. As far as we know this is the first paper in which financial frictions generate high asset price volatility. Financial frictions in the model propagate into the economy through investment only. A tightening of the financial constraint implies lower investment and an increase in the value of installed capital and in the price of equity. The next step in this research agenda is building a more general model of financial frictions. The model should include a second channel that would link financial frictions to current capital productivity. Such a model would allow us to disentangle the roles of these distinct channels in fluctuations in asset prices and aggregate quantities.

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Appendix A

A.1. **Proof of Theorem 4.3.** Theorem 4.3 says: $\theta < \frac{\delta-\pi}{\delta} \Longleftrightarrow$ in steady state FC binds and $q > 1$.

Proof: (i) $\Longrightarrow$ We will prove this part by contradiction. We will assume $q_t = 1, p_t = 0$ and show that the supposition implies that the FC must be violated for some investing entrepreneurs.

Denote by J the set of indexes identifying entrepreneurs with investment opportunity at time t and use capital letters for aggregate variables. Since the arrival of the investment opportunity is iid, investing entrepreneurs will hold a fraction π of the total equity holdings at the beginning of period $t : \int_{j\in J} n_t^j dj = \pi N_t$. Since they solve the same problem as non-investing entrepreneurs, they will hold a fraction π of total equity holdings in period $t+1$ as well: $\int_{j\in J} n_{t+1}^j = \pi N_{t+1}$. Now we will show that the financial constraint of some of these entrepreneurs is violated. Suppose it was satisfied. Integrating over the set J we get:

$$\begin{aligned}\int_{j\in J} n_{t+1}^j dj &\geq (1-\theta)\int_{j\in J} i_t^j dj \\ \pi N_{t+1} &\geq (1-\theta) I_t\end{aligned}$$

In steady state $N_t = N_{t+1} = N$ and $I_t = \delta N$. Thus we can rewrite the above as:

$$\pi - (1-\theta)\delta \geq 0$$

This contradicts our supposition. $\square$

The proof makes it clear that $\theta \geq \frac{\delta-\pi}{\delta} \Longleftrightarrow \exists$ an equilibrium with the FC slack and $q = 1$.

(ii) $\Longleftarrow$ Suppose in steady state the financial constraint binds and $q > 1$. The behavior of the model is then determined by the system of equations defined in section 4.1.3. Using the steady state conditions $N_{t+1} = N_t$ and $I_t = \delta N_t$ and dropping the time indexes, we can rewrite the system as:

$$\begin{aligned}q^R N^i &= \beta\pi N[r + (1-\delta)q] \\ qN^s &= \beta(1-\pi)N[r + (1-\delta)q] \\ N^i &= (1-\theta)\delta N \\ N^s + N^i &= N\end{aligned}$$

This can be simplified to:

$$\begin{aligned} q^R(1-\theta)\delta N &= \beta\pi N[r+(1-\delta)q] \\ q\left[1-\delta(1-\theta)\right]N &= \beta(1-\pi)N[r+(1-\delta)q] \end{aligned}$$

Plugging in for $\beta N[r+q(1-\delta)]$ from the first one into the second one and simplifying further we get:

$$\begin{aligned} \pi q\left[1-\delta(1-\theta)\right] &= (1-\pi)(1-\theta q)\delta \\ \pi q\left[1-\delta(1-\theta)\right] &= (1-\pi)\delta-\theta\delta(1-\pi)q \\ q\left[\pi-\pi\delta+\pi\delta\theta+\theta\delta-\pi\theta\delta\right] &= (1-\pi) \\ q &= \frac{(1-\pi)\delta}{\pi-\pi\delta+\theta\delta} \\ q &= \frac{(1-\pi)\delta}{(1-\pi)\delta+\pi-\delta+\theta\delta} \end{aligned}$$

Both the numerator and the denominator are positive and therefore $q>1 \Longrightarrow \pi-\delta+\theta\delta<0$, which is equivalent to $\theta<\frac{\delta-\pi}{\delta}$. $\square$

A.2. **Proof of Lemma 4.2: Linearity of the policy rules.** Lemma 4.2 states that the policy rules are linear and of the following form:

$$\begin{aligned} c_t^i &= (1-\beta)n_t[r_t+(1-\delta)q_t] \\ q_t^R n_{t+1}^i &= \beta n_t[r_t+(1-\delta)q_t] \\ c_t^s &= (1-\beta)n_t[r_t+(1-\delta)q_t] \\ q_t n_{t+1}^s &= \beta n_t[r_t+(1-\delta)q_t] \end{aligned}$$

Proof: For the proof we find it useful to define the aggregate state of the economy as $s_t=(A_t,\theta_t)$ and $s^t=(s_0,\ldots,s_t)$. We consider a problem of one particular entrepreneur indexed by his initial wealth n_0. We define a variable u_t that equals 1 if the entrepreneur has an investment opportunity and equals 0 if he does not have not an investment opportunity at time t. As before $u^t=(u_0,\ldots,u_t)$. We denote probabilities by $\pi(s^t),\pi(u^t)$. Fixing K_0 the problem of this entrepreneur is:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \sum_{u^t} \beta^t \pi(s^t)\pi(u^t) \log c_t(s^t,u^t) \qquad \text{s.t.}$$

$$u_t = 0: \qquad c_t(s^t,u^t) + q_t(s^t) n_{t+1}(s^t,u^t) \le n_t(s^{t-1},u^{t-1})[r_t(s^t) + q_t(s^t)(1-\delta)] \qquad (\lambda_t(s^t,u^t))$$

$$u_t = 1: \qquad c_t(s^t,u^t) + q_t^R(s^t) n_{t+1}(s^t,u^t) \le n_t(s^{t-1},u^{t-1})[r_t(s^t) + q_t(s^t)(1-\delta)] \qquad (\lambda_t(s^t,u^t))$$

The appropriate budget constraint applies depending on whether the entrepreneur has or has not the investment opportunity. To make the notation clearer we will denote by a superscript i allocations and Lagrange multipliers in the state in which the entrepreneur has an investment opportunity at time t and by a superscript s allocations and Lagrange multipliers in a state in which the entrepreneur does not have an investment opportunity at time t. The first order conditions are:

$$\begin{aligned}
\frac{\beta^t \pi(u^t)\pi(s^t)}{c_t^s(s^t,u^t)} &= \lambda_t^s(s^t,u^t) \\
\lambda_t^s(s^t,u^t) q_t(s^t) &= \sum_{s^{t+1}} \big[\lambda_{t+1}^i(s^{t+1},u^{t+1})\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\} + \\
&+ \lambda_{t+1}^s(s^{t+1},u^{t+1})\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\}\big] \\
\frac{\beta^t \pi(u^t)\pi(s^t)}{c_t^i(s^t,u^t)} &= \lambda_t^i(s^t,u^t) \\
\lambda_t^i(s^t,u^t) q_t^R(s^t) &= \sum_{s^{t+1}} \big[\lambda_{t+1}^i(s^{t+1},u^{t+1})\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\} + \\
&+ \lambda_{t+1}^s(s^{t+1},u^{t+1})\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\}\big]
\end{aligned}$$

Now we verify that the proposed policies satisfy the FOC (which is enough by their sufficiency). Plugging in for the Lagrange multipliers and using the iid assumption on the arrival of the investment opportunity, we get:

$$\begin{aligned}
\frac{q_t(s^t)}{c_t^s(s^t,u^t)} &= \beta \sum_{s^{t+1}} \big[\frac{\pi(s^{t+1}|s^t)(1-\pi)}{c_{t+1}^i(s^{t+1},u^{t+1})}\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\} + \\
&+ \frac{\pi(s^{t+1}|s^t)\pi}{c_{t+1}^s(s^{t+1},u^{t+1})}\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\}\big] \\
\frac{q_t^R(s^t)}{c_t^i(s^t,u^t)} &= \beta \sum_{s^{t+1}} \big[\frac{\pi(s^{t+1}|s^t)(1-\pi)}{c_{t+1}^i(s^{t+1},u^{t+1})}\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\} + \\
&+ \frac{\pi(s^{t+1}|s^t)\pi}{c_{t+1}^s(s^{t+1},u^{t+1})}\{r_{t+1}(s^{t+1}) + (1-\delta)q_{t+1}(s^{t+1})\}\big]
\end{aligned}$$

Plugging in the proposed rules for $c^s_{t+1}(s^{t+1},u^{t+1})$ and $c^i_{t+1}(s^{t+1},u^{t+1})$, we get:

$$\begin{aligned}
\frac{q_t(s^t)}{c^s_t(s^t,u^t)} &= \beta\sum_{s^{t+1}}\left[\frac{\pi(s^{t+1}|s^t)(1-\pi)}{(1-\beta)n^s_{t+1}(s^t,u^t)}+\frac{\pi(s^{t+1}|s^t)\pi}{(1-\beta)n^s_{t+1}(s^t,u^t)}\right] \\
\frac{q^R_t(s^t)}{c^i_t(s^t,u^t)} &= \beta\sum_{s^{t+1}}\left[\frac{\pi(s^{t+1}|s^t)(1-\pi)}{(1-\beta)n^i_{t+1}(s^t,u^t)}+\frac{\pi(s^{t+1}|s^t)\pi}{(1-\beta)n^i_{t+1}(s^t,u^t)}\right]
\end{aligned}$$

These can be rewritten as:

$$\begin{aligned}
(1-\beta)q_t(s^t)n^s_{t+1}(s^t,u^t) &= \beta c^s_t(s^t,u^t) \\
(1-\beta)q^R_t(s^t)n^i_{t+1}(s^t,u^t) &= \beta c^i_t(s^t,u^t)
\end{aligned}$$

Given the budget constraints this confirms our initial guess independent of whether $q_t > 1$ or $q_t = 1$. □

Appendix B. Construction of Time Series

B.1. **Macroeconomic variables.**

Databases used for 1964q1 - 2008q4:

(1) CES-BLS: Current Employment Statistics survey published by the Bureau of Labor Statistics.
(2) FAT-BEA: Fixed Asset Tables published by the Bureau of Economic Analysis.
(3) NIPA-BEA: National Income and Product Accounts published by the Bureau of Economic Analysis.
(4) Flow of Funds.

Series generated:

(1) Hours L: from CES-BLS:
    - Hours = average weekly hours · average number of workers.
    - Average weekly hours: in private sector, series CES0500000036.
    - Average number of workers: average number of workers in private sector over a quarter computed using monthly data in series CES0500000001.
(2) Real capital K : we generate quarterly data by interpolating the yearly " Fixed assets and consumer durable goods", line 2 in table 1.2 in FAT-BEA.
(3) Output Y: real GDP, line 1 in table 1.1.6 in NIPA-BEA.

(4) TFP series A_t: generated from the capital and hours series as:

$$A_t = \frac{Y_t}{L_t^{.64} \cdot K_t^{.36}}$$

(5) Nominal capital NK : we generate quarterly data by interpolating the yearly " Fixed assets and consumer durable goods", line 2 in table 1.1 in FAT-BEA.

(6) Nominal Investment NI = nominal private fixed investment + nominal durable consumption good expenditure + nominal government gross investment.
   - nominal private fixed investment: line 7 in table 1.1.5 in NIPA-BEA.
   - nominal durable consumption good expenditure: line 4 in table 1.1.5 in NIPA-BEA.
   - nominal government gross investment: line 3 in table 3.9.5 in NIPA-BEA, does not include investment in inventories.

(7) Real investment I: Nonfarm nonfinancial corporate businesses fixed investment, line 12 in the Flow of Funds table F.102 deflated using the deflator for Gross private domestic investment constructed using line 7 in NIPA-BEA 1.1.5 and 1.1.6. We choose this series because we use it to estimate θ. The time series properties of various real investment measures are very similar. Excluding government, inventories and durable consumption makes the series slightly more volatile (standard deviation of 5.00% versus 4.41%).

(8) Real consumption C = Nondurable goods + Services.
   - Nondurable goods: line 4 in table 1.1.5 in NIPA-BEA.
   - Services: line 6 in table 1.1.5 in NIPA-BEA.

   The real counterparts of these nominal series are only reported starting in 1995. To generate the real series we deflated these nominal series by a Personal consumption expenditure deflator constructed from line 2 in tables 1.1.5 and 1.1.6. The correlation between the deflator for Personal consumption expenditure and Nondurable goods and Services from 1995 onwards is .991 and .997, respectively.

B.2. **Financial Variables.**

(1) Asset price q was constructed from the Dow Jones Total Stock Market Index (Wilshire 5000) for the priod 1974 - 2008. We have constructed the same series for the S&P 500 Composite Price Index for the 1964 - 2008 period. The time series properties of HP filtered logged versions of these indexes are very similar. Dow Jones Total

Stock Market Index is slightly more volatile. Both raw series were recovered from the Global Financial Data database and computed as averages over the given quarter.

(2) Asset return r^e: series *vwretd* from the CRSP database (Center for Research in Security Prices), value weighted returns including distributions from NYSE, AMEX and NASDAQ. We constructed quarterly data from monthly observations.

(3) Total market value *val*: series *totval* from he CRSP database. We constructed quarterly data as averages over monthly observations.

(4) Real risk free rate r^f is the 3-month T-bill as priced on the secondary market recovered from the Global Financial Data database deflated by CPI.

(5) CPI: nominal returns are deflated using the CPI series from the BLS database, series ID: CPI-U, BLS CUUR0000SA0.

B.3. **Construction of θ.**

We construct a measure of θ from the Flow of Funds data for the non-financial corporate sector:

$$\theta = \frac{\text{Funds Raised in Markets}}{\text{Capital Expenditures}}$$

The variables are:

- Net funds raised in markets: line 38 in table F.102, equals: net new equity issuance (line 39) plus credit market instruments (line 40) for non-farm nonfinancial corporate businesses
- Fixed investment: line 12 in table F.102, for non-farm nonfinancial corporate businesses

Appendix C. Sensitivity Analysis

Table C.1: Quarterly Statistics[a] - Sensitivity to Parameters of the θ Process

Statistic[c]	Data[b]	(1) $\sigma^2_{\varepsilon_\theta} = .17^2$ $E(\theta) = .28$	(2) $\sigma^2_{\varepsilon_\theta} = .12^2$ $E(\theta) = .28$	(3) $\sigma^2_{\varepsilon_\theta} = .17^2$ $E(\theta) = .43$	(4) $\sigma^2_{\varepsilon_\theta} = .12^2$ $E(\theta) = .43$
σ_Y	1.52	1.18	1.17	1.17	1.18
σ_I	5.00	5.12	3.77	2.93	2.07
σ_C	0.85	1.93	1.63	1.36	1.16
σ_L	1.73	0.59	0.58	0.59	0.59
ρ_Y	0.87	0.67	0.68	0.68	0.67
ρ_I	0.85	0.40	0.41	0.38	0.48
ρ_C	0.90	0.47	0.52	0.53	0.61
ρ_L	0.92	0.67	0.68	0.68	0.67
$\rho(Y,I)$	0.90	0.23	0.30	0.48	0.73
$\rho(Y,C)$	0.85	0.61	0.72	0.80	0.92
$\rho(Y,L)$	0.87	1.00	1.00	1.00	1.00
σ_q	11.85	9.69	7.39	5.36	3.08
σ_{r^e}	8.63	10.77	8.15	6.37	3.71
σ_{val}	10.72	9.74	7.42	5.38	3.10
ρ_q	0.74	0.38	0.39	0.28	0.27
$\rho(q,Y)$	0.39	0.11	0.16	0.11	0.13
$\rho(q,I)$	0.38	-0.94	-0.89	-0.81	-0.57
$E(r^e)$	1.49	1.36	1.18	1.13	1.04
$E(r^f)$	0.30	0.51	0.63	0.83	0.92
σ_r^f	0.68	5.45	4.08	3.54	2.19
$E(r^e) - E(r^f)$	1.19	0.85	0.55	0.30	0.11

[a] Results for the models are based on 100 replications of size 180.
[b] σ_x is a standard deviation of variable x, ρ_x is the autocorrrelation of x and $\rho(x,y)$ is the correlation between x and y. All variables with the exception of the returns are logged and HP filtered before statistics are computed. Standard deviations and returns are measured in percentage terms.
[c] This column contains quarterly statistics computed for the U.S. data in 1964:1 - 2008:4. An exception is the measure for asset prices q, which was computed for 1974:1 - 2008:4. Details of the construction of the series are in the appendix, section B.
Column (1) contains results for the benchmark model. In this model the financial constraint is binding and both the process for TFP and θ is estimated using U.S. data 1964:1 - 2008:4.
Columns (2), (3) and (4) contain results for versions of the model with various values for the parameters of the θ process. In column (2) the variance of innovations in θ is 50% of the benchmark value. In column (3) the mean value of θ is increased by 50% relative to benchmark. In column (4) both of these changes apply. Other than that the models are the same as the benchmark model.

Table C.2: Quarterly Statistics[a] - Sensitivity to π

Statistic[c]	Data[b]	(1) $\pi = .01$	(2) $\pi = .015$	(3) $\pi = .02$	(4) $\pi = .025$
σ_Y	1.52	1.21	1.18	1.19	1.19
σ_I	5.00	9.99	5.12	2.10	1.71
σ_C	0.85	3.12	1.93	1.16	1.02
σ_L	1.73	0.60	0.59	0.59	0.60
ρ_Y	0.87	0.68	0.67	0.68	0.67
ρ_I	0.85	0.42	0.40	0.48	0.67
ρ_C	0.90	0.47	0.47	0.59	0.67
ρ_L	0.92	0.68	0.67	0.68	0.67
$\rho(Y, I)$	0.90	0.03	0.23	0.74	1.00
$\rho(Y, C)$	0.85	0.47	0.61	0.92	1.00
$\rho(Y, L)$	0.87	1.00	1.00	1.00	1.00
σ_q	11.85	16.02	9.69	3.07	0
σ_{r^e}	8.63	17.31	10.77	3.84	0.07
σ_{val}	10.72	16.12	9.74	3.09	0.13
ρ_q	0.74	0.43	0.38	0.21	1
$\rho(q, Y)$	0.39	0.16	0.11	0.10	0
$\rho(q, I)$	0.38	-0.97	-0.94	-0.59	0
$E(r^e)$	1.49	1.88	1.36	1.05	1.01
$E(r^f)$	0.30	-0.26	0.51	0.93	1.01
σ_r^f	0.68	8.16	5.45	2.32	0.06
$E(r^e) - E(r^f)$	1.19	2.14	0.85	0.12	0.00

[a] Results for the models are based on 100 replications of size 180.
[b] σ_x is a standard deviation of variable x, ρ_x is the autocorrrelation of x and $\rho(x, y)$ is the correlation between x and y. All variables with the exception of the returns are logged and HP filtered before statistics are computed. Standard deviations and returns are measured in percentage terms.
[c] This column contains quarterly statistics computed for the U.S. data in 1964:1 - 2008:4. An exception is the measure for asset prices q, which was computed for 1974:1 - 2008:4. Details of the construction of the series are in the appendix, section B.
Column (2) contains results for the benchmark model with $\pi = .015$. In this model the financial constraint is binding and both the process for TFP and θ is estimated using U.S. data 1964:1 - 2008:4.
Columns (1), (3) and (4) contain results for versions of the model with various values of π. Other than that the models are the same as the benchmark model.

Table C.3: Quarterly Statistics[a] with Alternative Labor Supply Elasticity

Statistic[c]	Data[b]	(1) $\eta = .5$	(2) $\eta = 1$	(3) $\eta = 2$	(4) constant labor
σ_Y	1.52	1.41	1.18	1.03	0.79
σ_I	5.00	5.02	5.12	5.05	5.19
σ_C	0.85	2.75	1.93	1.55	1.79
σ_L	1.73	0.94	0.59	0.35	0
ρ_Y	0.87	0.68	0.67	0.67	0.67
ρ_I	0.85	0.41	0.40	0.40	0.39
ρ_C	0.90	0.46	0.47	0.51	0.43
ρ_L	0.92	0.68	0.67	0.68	1
$\rho(Y, I)$	0.90	0.26	0.23	0.16	0.13
$\rho(Y, C)$	0.85	0.52	0.61	0.69	0.46
$\rho(Y, L)$	0.87	1.00	1.00	1.00	0
σ_q	11.85	9.49	9.69	9.71	10.07
σ_{r^e}	8.63	10.45	10.77	10.71	11.10
σ_{val}	10.72	9.54	9.74	9.76	10.12
ρ_q	0.74	0.40	0.38	0.39	0.39
$\rho(q, Y)$	0.39	0.15	0.11	0.13	0.10
$\rho(q, I)$	0.38	-0.91	-0.94	-0.95	-0.97
$E(r^e)$	1.49	1.35	1.36	1.36	1.42
$E(r^f)$	0.30	0.66	0.51	0.48	0.62
σ_r^f	0.68	5.36	5.45	5.45	5.61
$E(r^e) - E(r^f)$	1.19	0.69	0.85	0.88	0.80

[a] Results for the models are based on 100 replications of size 180.
[b] σ_x is a standard deviation of variable x, ρ_x is the autocorrrelation of x and $\rho(x, y)$ is the correlation between x and y. All variables with the exception of the returns are logged and HP filtered before statistics are computed. Standard deviations and returns are measured in percentage terms.
[c] This column contains quarterly statistics computed for the U.S. data in 1964:1 - 2008:4. An exception is the measure for asset prices q, which was computed for 1974:1 - 2008:4. Details of the construction of the series are in the appendix, section B.
Column (2) contains results for the benchmark model with $\eta = 1$. In this model the financial constraint is binding and both the process for TFP and θ is estimated using U.S. data 1964:1 - 2008:4.
Columns (1) and (3) contain results for versions of the model with various values of η.
Column (4) contains results for a version of the model with constant labor supply. Other than that the models are the same as the benchmark model.

Table C.4: Quarterly Statistics[a] - Sensitivity to β

Statistic[c]	Data[b]	(1) $\beta = .98$	(2) $\beta = .99$	(3) $\beta = .999$	(4) $\beta = .9999$
σ_Y	1.52	1.21	1.18	1.18	1.20
σ_I	5.00	7.85	5.12	1.36	1.20
σ_C	0.85	2.07	1.93	1.23	1.20
σ_L	1.73	0.60	0.59	0.59	0.60
ρ_Y	0.87	0.68	0.67	0.68	0.68
ρ_I	0.85	0.40	0.40	0.61	0.68
ρ_C	0.90	0.48	0.47	0.64	0.68
ρ_L	0.92	0.68	0.67	0.68	0.68
$\rho(Y, I)$	0.90	0.13	0.23	0.87	1.00
$\rho(Y, C)$	0.85	0.60	0.61	0.96	1.00
$\rho(Y, L)$	0.87	1.00	1.00	1.00	1.00
σ_q	11.85	7.78	9.69	12.67	13.00
σ_{r^e}	8.63	8.54	10.77	14.27	14.68
σ_{val}	10.72	7.85	9.74	12.68	13.00
ρ_q	0.74	0.40	0.38	0.37	0.37
$\rho(q, Y)$	0.39	0.16	0.11	0.07	0.08
$\rho(q, I)$	0.38	-0.95	-0.94	-0.42	0.02
$E(r^e)$	1.49	2.21	1.36	0.85	0.80
$E(r^f)$	0.30	1.63	0.51	-0.51	-0.63
σ_r^f	0.68	4.42	5.45	7.20	7.44
$E(r^e) - E(r^f)$	1.19	0.58	0.85	1.36	1.44

[a] Results for the models are based on 100 replications of size 180.
[b] σ_x is a standard deviation of variable x, ρ_x is the autocorrrelation of x and $\rho(x, y)$ is the correlation between x and y. All variables with the exception of the returns are logged and HP filtered before statistics are computed. Standard deviations and returns are measured in percentage terms.
[c] This column contains quarterly statistics computed for the U.S. data in 1964:1 - 2008:4. An exception is the measure for asset prices q, which was computed for 1974:1 - 2008:4. Details of the construction of the series are in the appendix, section B.
Column (2) contains results for the benchmark model with $\beta = .99$. In this model the financial constraint is binding and both the process for TFP and θ is estimated using U.S. data 1964:1 - 2008:4.
Columns (1), (3) and (4) contain results for versions of the model with various values of β. Other than that the models are the same as the benchmark model.