A Theory of Market Microstructure and Reputation*

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Abstract

The paper investigates the properties of the equilibrium outcomes and the microstructure of the markets where the sellers announce their demands but do not necessarily commit, while the buyers move back and forth between the sellers to negotiate for a better deal by extending the analysis of the multilateral bargaining problem introduced in Ozyurt (2009). When the sellers are constrained to announce their prices before the buyer's arrival at their stores, the unique equilibrium outcome is in Bertrand fashion. However, this uniqueness result is not robust to perturbation on higher order beliefs regarding the players' commitment types. If the sellers compete in the spirit of Diamond (1971), then a continuum of prices can be supported in equilibrium. However, independent of the sellers' announcements, the payoff to the seller (visited by the buyer first) is always the same, and the buyer's payoff approaches a unique limit as the initial priors vanish.

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1. Introduction

This paper aims to investigate the properties of the equilibrium outcomes and the microstructure of the markets where there is more than one potential seller (competing with each other over the buyers) and each seller (if he chooses) suggests or posts his price so that the buyers can pay the posted price, buy the good or the service, and finalize the trade; however, the general practice is that the buyers negotiate with the sellers with the hope of getting a deal better than the solicited "*buy it now*" prices. Examples of such markets range from old bazaars to online trade environments such as E-bay.

The baseline game theoretic model that I use for the analysis is introduced and investigated in detail in Ozyurt (2009): A single (generic) buyer facing two spatially-separated sellers can negotiate with only one potential seller at a time. However, with some delay, the buyer can move back and forth between the two sellers. Ozyurt (2009) shows that the introduction of behavioral types that are completely inflexible in their demands and offers, even with low probabilities, makes equilibrium of this multilateral bargaining problem essentially unique, and the unique equilibrium has a war of attrition structure that engenders inefficiency due to possible delay in reaching an agreement.

Ozyurt (2009) takes the behavioral types and demands as (exogenously) given. Under this simplifying assumption, the study characterizes the equilibrium strategies and investigates the properties of the players' expected payoffs.[1] On the other hand, in this paper I extend the analysis of this multilateral bargaining problem by allowing the players to choose their behavioral demands.

However, this extension is not a straightforward application. In terms of the equilibrium strategies and outcomes, it matters whether the buyer can learn the sellers' demands before or after his arrival at the stores. For example, price undercutting is not a dominant strategy in the price competition game I consider in Section 2, where the buyer can learn the sellers' demands before visiting the stores (in that game, thus, the sellers compete in the spirit of Bertrand (1883)). In equilibrium, each seller strictly prefers to be visited by the buyer first (whenever initial priors are small) and the buyer may visit the seller who posts the higher price first. On the other hand, increasing the posted prices is not always the best response strategy for the sellers even if the buyer can learn the prices

[1] Section 2 discusses these issues in more detail.

when he visits the stores (i.e., the sellers compete in the spirit of Diamond (1971)). This is true because relative to the expected payoff he would get by increasing his demand, a seller can improve his expected payoff by decreasing it so that it is sufficiently close to the buyer's behavioral demand (even if the buyer visits the seller first in either case).

Section 2 considers the case where both sellers are constrained to announce their demands (or post their prices) before the buyer arrives at their stores, as in the case of Bertrand price competition. In this section, I show that as the initial priors about the players' commitment types are sufficiently small, in the unique equilibrium, each seller posts the price of zero. However, Section 3 demonstrates that this uniqueness result is not robust to perturbation on higher order beliefs about the initial priors. To show this point, I assume that each seller is confident (believes with certainty) that, relative to himself, the other seller is more likely to commit on the price he initially posts, or equivalently is less likely to bargain with the buyer. With this perturbation, the common knowledge assumption retains. However, the sellers do not agree on the content of the knowledge that they believe is common. I then show that we can support a large set of prices as the equilibrium of the price competition game even when players' beliefs about their opponents' types are arbitrarily close to zero or the perturbation on higher order beliefs vanishes. However, the expected payoff to each seller is uniquely determined and is equal to α_b (the buyer's behavioral demand).

Section 4 searches for an answer to the question of what prices (or behavioral demands) the sellers post in equilibrium given that the sellers cannot announce their demands before the buyer's arrival at their stores, as in the case of Diamond's (1971) search model. The analyzes in this section suggests that a continuum of prices can be supported in equilibrium. However, independent of the sellers' announcements, the payoff to the seller (that is visited by the buyer first) is always the same and the payoff to the buyer approaches a unique limit as the initial priors vanish.

Regarding the market's microstructure, Section 5 aims to understand whether the sellers want to announce their prices before or after the buyer's arrival at their stores, if the sellers have chance to make such public announcements. I show that if the sellers agree on the prior probabilities of all the players' commitment types, which is very possible especially when the two sellers operate in the market for a long time and have chance to learn about one another through their experiences, then each seller has incentive to announce his demand before the buyer arrives at his store. In this case, marginal cost (that is zero) pricing is the unique equilibrium outcome. However, if we perturb the higher order beliefs as I do in Section 3, i.e. if the sellers' priors are not common (which could be the case when the sellers do not have enough experience about their opponents), then the sellers may not announce their prices to the buyer before he makes his visits to their stores. This case leads to a multiplicity in equilibrium prices while each seller expects to receive the payoff of α_b.

Finally in Section 6, I consider the case where the buyer chooses his behavioral demand and announces it before the sellers make their decisions. I show that as the prior beliefs converge to zero, in equilibrium of the price search game (where the sellers cannot post and announce their prices before the buyer's arrival), the share to the buyer, which is 1/2, is equal to his share in the standard Rubinstein alternating offer bargaining game (between a buyer and a single seller) with no commitment types. However, if the buyer can learn the prices before visiting the stores, then in equilibrium the buyer gets the whole surplus.

2. Price Competition Game

There are two spatially-separated stores selling an indivisible homogeneous good to a single (generic) buyer who wants to consume only one unit. The sellers are located at opposite ends of a street, and the buyer's position is midway in between the two stores. The sellers' locations are fixed at all times. To purchase the good, the buyer has to visit a seller's store. However, moving from one store to another takes time. Each player is impatient, i.e. time matters for everyone.

The valuation of the good is one for the buyer and zero for the sellers. There is no informational asymmetry regarding the valuations of the good and the players' time preferences. The buyer can negotiate with one potential seller at a time. However, he can move back and forth between the two sellers with some delay (since the stores are physically apart it takes time to move from one store into the other).[2]

[2]By this assumption, I aim to capture the search cost involved finding a better deal.

***Behavioral Types*:** The game I introduce above is a multilateral bargaining game between a buyer and two sellers. There are two main challenges of incorporating this market setup into a price competition game between two sellers. The first one is that this multilateral bargaining problem has a continuum of equilibria, and this set depends on the fine details of the bargaining protocol. Ultimately, the relevant question is which bargaining protocol we should pick to model the multilateral bargaining game and when we pick a particular one, how we can rationalize it over the others.

The second challenge is incorporating the posted (announced buy-it-now) prices. Posting a price has an unambiguous meaning in Bertrand price competition or in Diamond's search model because in these models, the buyer knows that he cannot attain a price lower than the posted prices. Thus, he is not inclined to bargain with the sellers. However, the main motivation of the analysis in this paper is to understand what might happen when the buyer believes that he can actually get a better price through haggling with the sellers. Therefore, the challenge is the interpretation of the posted prices when the sellers post price and when there is a common belief that the sellers may not commit to these prices.

Ozyurt(2009) provides a simple approach that can be seen as a remedy to these two challenges. Here is what the study suggests. The main starting point of its motivation is the following observation. When the buyer enters a store and sees the posted price, it might cross his mind that the seller may not be willing to negotiate with the buyer, and that the seller instead will insist on the price he posts. The buyer does not have to believe in this firmly, but ignoring the existence of a slight belief of this sort of behavior is unintuitive.

Thus, parallel to Kreps and Wilson (1982), Milgrom and Roberts (1982), each player suspects that his opponents might have some kind of behavioral commitment forcing them to insist on a specific allocation. To be more specific, there is a small but strictly positive probability, z_i, that seller i is a commitment type implementing the following strategy: He always offers his posted price (announced *buy it now* price) $\alpha_i \in (0, 1)$, rejects any price offer strictly below it and accepts any price offer weakly above it. Similarly, there is a small but strictly positive probability, z_b, that the buyer is a commitment type executing the following strategy: He always offers $\alpha_b \in (0, 1)$ to sellers, accepts any price offer less than or equal to α_b and rejects any price offer strictly above it. The timing and location decisions of the behavioral type buyer are the same as those of the rational buyer.[3]

Analogous to Abreu and Gul(2000), Ozyurt (2009) generalizes Rubinstein's alternating offer bargaining protocol so that it accommodates non-stationary and non-alternating protocols. The study, then, shows that in the presence of these simple behavioral types, the equilibrium outcomes of a discrete-time bargaining game converge to a unique limit (independent of the exogenously given bargaining protocols) as players make increasingly frequent offers. It also shows that this limit is equivalent to the unique outcome of the continuous-time bargaining problem (a modified war of attrition problem between a buyer and two sellers) that I will define in detail next.

***The Price Competition Game* G:** Before the game starts, nature makes its move and determines whether a player is a commitment type or not, and then the players privately learn their types. For simplicity, I assume that the demand of the commitment type buyer is some $\alpha_b \in (0, 1)$ that is commonly known.[4]

The price competition game G consists of two stages. In the first stage, the sellers simultaneously choose and declare their behavioral demands (posted prices), so the buyer can learn the posted price in each store at no cost. Then, the buyer decides which store to go to first. In the second stage, the buyer and the sellers play the modified war of attrition game as follows: Upon arrival at store $i \in \{1, 2\}$ at time 0, the beginning of the second stage, the buyer and seller i instantaneously begin to play the following concession game: At any given time, a player either accepts his opponent's behavioral demand or waits for a concession.[5] At the same time, the buyer decides whether to stay or leave store i. Concession of the buyer or seller i, while the buyer is in the store, marks the completion of the game. In case of simultaneous concession, surplus is split equally.[6]

While the buyer plays the concession game with seller i, if either one accepts his opponent's behavioral demand at time t, then the game ends with an agreement $x \in$

[3] Note that, as Ozyurt (2009) and Abreu and Gul (2000) point out, introducing these behavioral types only for one side (only for the sellers or the buyer) is not sufficient to get the convergence to the war of attrition game result.

[4] Section 6 considers the case where the buyer can choose his behavioral demand before the sellers choose theirs.

[5] If the buyer's behavioral demand is weakly higher than seller i's demand, then the game ends upon the buyer's arrival at store i by the buyer's acceptance of the posted price.

[6] This assumption is not binding since this event will occur with probability zero in equilibrium.

$\{\alpha_b, \alpha_i, \frac{\alpha_b+\alpha_i}{2}\}$ and the associated payoffs to the players are as follows; $u_i(x,t,i) = xe^{-tr_i}$ for seller i; $u_j(x,t,i) = 0$ for seller $j \in \{1,2\}$ where $j \neq i$; and $u_b(x,t,i) = (1-x)e^{-tr_b}$ for the buyer, where r_k is the interest rate for player $k \in \{1,2,b\}$. If no player makes any acceptance until the time ∞, each player receives the payoff of 0.

If the buyer leaves store i and goes to store j, the buyer and seller j start playing the concession game upon the buyer's arrival at that store. Both sellers can perfectly observe the buyer's moves throughout the game. Thus, the players' actual types are the only source of uncertainty in the game. After leaving store i and traveling part way to store j, the buyer could, if he wished, turn back and enter store i again.

***Why is it the War of Attrition Game?*:** The multilateral bargaining problem arising in stage 2 is modeled as a continuous-time war of attrition game where the strategies of the players (in the concession games) are effectively either to stick with the initial behavioral demand or accept the opponent's behavioral demand. The selection of this particular game is not arbitrary. As Ozyurt (2009) shows, in the presence of these simple behavioral types, equilibrium outcomes of the discrete-time bargaining problem converge in distribution to the unique equilibrium outcome of its continuous-time counterpart if players can make increasingly frequent offers. This convergence result implies that the bargaining game is actually reduced to the war of attrition game, wherein each player just needs to choose whether to accept his opponent's behavioral demand or wait for a concession.

It is striking that the players' strategies in a possible multilateral bargaining game reduce to these simple forms of strategies. This convergence is solely due to the nature of these simple behavioral types. While the buyer negotiates with one of the sellers, if the buyer, for example, wants to offer something other than his behavioral demand at some point, the sellers will conclude that the buyer is indeed rational. Moreover, in any sequential equilibrium after a history where the buyer is known to be rational but sellers are not, the buyer's expected payoff cannot be higher than $1-\alpha$ (supposing for the moment that both sellers' behavioral demands are equal to some $\alpha \in (0,1)$ such that $\alpha > \alpha_b$). So, in equilibrium, if the buyer reveals his type, he does it by accepting a seller's behavioral demand. Similarly, in any sequential equilibrium after a history that a seller is known to be rational while the buyer is not, the payoff to the buyer cannot be lower than $1-\alpha_b$. Thus, in equilibrium, if a seller reveals his type, he would do it by accepting the buyer's behavioral demand.

Therefore, by pretending to be the behavioral type, a player can force his opponent to offer his behavioral demand or to reveal his rationality. Thus, in any equilibrium, if the buyer accepts a seller's behavioral demand, his payoff is $1-\alpha$. If he waits for a seller to reveal his rationality, the buyer will get the expected payoff of $1-\alpha_b$. So, the highest payoff the buyer can attain is $1-\alpha_b$ and the lowest payoff is $1-\alpha$. Similarly, if a seller accepts the buyer's behavioral demand, his payoff is α_b. However, if he waits for the buyer to reveal his rationality, he can attain α. Thus, in equilibrium the highest payoff a seller can get is α and the lowest payoff (in case he makes the deal with the buyer) is α_b. Hence, in the limit (as players can make their offers arbitrarily frequent) the bargaining game turns into a concession game where each player chooses a time to concede.[7]

***Strategies in the Price Competition Game*:** In stage 1, each seller i chooses his behavioral demand (posted price) $\alpha_i \in [0,1)$. If a seller posts a price less than or equal the buyer's behavioral demand α_b, the game ends upon the buyer's arrival at this store by the buyer's acceptance of the posted price.

For the sake of simplicity in presentation and notation, I will present here the players' equilibrium strategies in stage 2. For the detailed characterization and the analysis, readers may refer to Ozyurt (2009). Equilibrium of the continuous-time bargaining problem in stage 2 is unique up to the buyer's selection of store to visit first at time 0.[8] A short descriptive summary of the equilibrium strategy is as follows. At time 0, the beginning of stage 2, the buyer enters store 1, for example, and starts playing the concession game with the seller until time T_1^d. At this time the buyer leaves store 1 and goes directly to store 2 if neither seller 1 nor the buyer concedes before this time. Once the buyer arrives at store 2, the buyer and seller 2 play the concession game until T_2^e, when both players' reputations reach 1. That is, by time T_2^e the game ends with certainty if one of the players is rational. So, in equilibrium, the buyer visits each store at most once.[9]

The departure time of the buyer from store i, T_i^d depends on the buyer's initial

[7]See Section 4 of Ozyurt (2009) for more elaborate discussion of this convergence result.

[8]For some parameter values, the buyer might be indifferent between the sellers to visit first at time 0.

[9]This is true for all possible values of the behavioral demands as long as $\alpha_i > \alpha_b$ and $1-\alpha_i \geq \delta(1-\alpha_j)$, where $i, j \in \{1,2\}$ and $j \neq i$. If $1-\alpha_i < \delta(1-\alpha_j)$, then in equilibrium the buyer may visit store j twice.

reputation (probability that the buyer is the commitment type), time preference and behavioral demand relative to the sellers'. So, T_1^d can take values in the interval $[0, T_1^e]$. It is zero when the buyer's initial reputation is sufficiently high (or sellers' demands are close enough to the buyer's demand). In this case the buyer leaves store 1 immediately unless seller 1 accepts the buyer's behavioral demand upon the buyer's arrival at his store. If the buyer's initial reputation is not high enough, then the buyer spends a positive amount of time in store 1 to build a reputation for inflexibility before leaving to negotiate with the second seller. However, $T_1^d = T_1^e$ implies that the (rational) buyer never leaves store 1 until he reaches an agreement with seller 1. In equilibrium, the buyer may choose to negotiate only with one seller whenever the traveling cost is sufficiently high or his behavioral demand is close to the sellers' demand, making traveling to bargain with the other seller not worth his time.

On the other hand, each player's equilibrium strategy in the concession game is a continuous and strictly increasing distribution function. That is, in equilibrium both the buyer and the sellers concede by choosing the timing of acceptance randomly with a constant hazard rate. Therefore, at any moment, the players are indifferent to either accepting the opponent's behavioral demand or waiting. The hazard rate of a player depends only on the behavioral demands and his opponent's time preferences.

More formally, the buyer's strategy in the second stage has two parts. The first part, σ_b, determines the buyer's location as a function of history. Assume that in equilibrium, without loss of generality, the buyer visits store 1 first and then store 2. Let T_1^d denote the time that the buyer leaves store 1 if no agreement has been reached yet. Denote by ω_i the time that the buyer starts negotiating with seller i (if agreement has not been reached yet). That is, $\omega_1 = 0$ and $\omega_2 = T_1^d + \Delta$ where Δ is the travel time between the stores. For notational simplicity, I manipulate the subsequent notation and denote ω_2 by 0. That is, I reset the clock once the buyer arrives in store 2 (but not the players' reputations).[10]

The second part is a pair of right continuous distribution functions $F_b^i(t) = 1 - c_b^i e^{-\lambda_b^i t}$, for $t \geq 0$ and $i = 1, 2$. Thus, for each t, $F_b^i(t)$ is the probability that the buyer concedes to seller i by time t (inclusive). Similarly, seller i's strategy in the second stage is a right

[10]Thus, with some manipulation of the notation, I define each player's distribution function as if the concession game in each store starts at time 0.

continuous distribution function $F_i(t) = 1 - c_i e^{-\lambda_i t}$, for $t \geq 0$, such that for all $t \geq 0$, $F_i(t)$ denotes the probability that seller i concedes to the buyer by time t (inclusive). Let δ denote the discount factor for the buyer that occurs due to the time required to travel from one store into the other, i.e. $\delta = e^{-r_b \Delta}$ where r_b is the interest rate for the buyer.

Suppose that the sellers chooses (α_1, α_2) in stage 1 such that, without loss of generality, $\alpha_1 \geq \alpha_2$ and $(1-\alpha_1) \geq \delta(1-\alpha_2)$.[11] The following result, which is borrowed from Ozyurt (2009), summarizes the unique equilibrium strategy of the players in stage 2 of the price competition game G.

Fact A. *In the unique equilibrium of the continuous-time bargaining problem in stage 2 of the price competition game G, where the sellers choose (α_1, α_2) in stage 1 satisfying $(1-\alpha_1) \geq \delta(1-\alpha_2)$ and the buyer arrives at store 1 at time 0, the strategies of the players are as follows: Consider*

(a) $A_1 = \frac{1-\alpha_b - \frac{1-\alpha_1}{\delta}}{\alpha_2 - \alpha_b} > 0$, *and*

(b) $X_1 = \left(\frac{z_2}{A_1}\right)^{\lambda_b^2/\lambda_2} \in (0, 1]$

1. *If both (a) and (b) hold, then $F_b^2(t) = 1 - e^{-\lambda_b^2 t}$ and $F_b^1(t) = 1 - c_b^1 e^{-\lambda_b^1 t}$ where*
$$c_b^1 = \begin{cases} \frac{z_b}{X_1} e^{\lambda_b^1 T_1^d}, & \text{if } z_b < X_1 \\ 1, & \text{otherwise,} \end{cases}$$
and the optimal time for the buyer to leave store 1 is
$$T_1^d = \begin{cases} \min\{-\frac{\log z_1}{\lambda_1}, -\frac{\log(z_b/X_1)}{\lambda_b^1}\}, & \text{if } z_b < X_1 \\ 0, & \text{otherwise,} \end{cases}$$
whereas $F_1(t) = 1 - z_1 e^{\lambda_1 (T_1^d - t)}$ and $F_2(t) = 1 - z_2 e^{\lambda_2 (T_2^e - t)}$ where the concession game ends in store 2 at time
$$T_2^e = \min\left\{-\frac{\log X_1}{\lambda_b^2}, -\frac{\log z_b}{\lambda_b^2}\right\}$$
and for each $i \in \{1, 2\}$, $\lambda_i = \frac{(1-\alpha_i) r_b}{\alpha_i - \alpha_b}$, $\lambda_b^i = \frac{\alpha_b r_i}{\alpha_i - \alpha_b}$.

2. *If either (a) or (b) fails to hold, then the buyer never goes to store 2 and the concession game in store 1 ends by the time $T_1^e = \min\left\{-\frac{\log z_1}{\lambda_1}, -\frac{\log z_b}{\lambda_b^1}\right\}$. The concession game strategies are; $F_b^1(t) = 1 - z_b e^{\lambda_b^1 (T_1^e - t)}$ and $F_1(t) = 1 - z_1 e^{\lambda_1 (T_1^e - t)}$.*

[11] Supposing that $\alpha_2 > \alpha_b$.

So, if the buyer's initial reputation, z_b, is sufficiently high, i.e. $z_b \geq X_1$, then the buyer and seller 1 does not play the concession game at all. The buyer makes a take it or leave it ultimatum to seller 1, and if he is rational, seller 1 accepts the buyer's behavioral demand immediately at time 0. However, if $z_b < X_1$, then the buyer and seller 1 plays the concession game until the time T_1^d, and depending on the value of z_b relative to $z_1^{\lambda_b^1/\lambda_1} X_1$, both the buyer's and the seller 1's strategies change with the coefficient of the exponential term, determining which player makes an immediate concession at time 0 as well as the magnitude of this probabilistic gift.

On the other hand, if the sellers choose (α_1, α_2) in stage 1, that are far from one another relative to the location of the stores, i.e. $(1-\alpha_1) < \delta(1-\alpha_2)$, then the buyer never negotiates with seller 1 (but he may want to visit this seller to make a take it or leave it offer), and he may visit seller 2 twice. The following result borrowed from Ozyurt (2009) summarizes the equilibrium strategies in the second stage when the sellers choose their posted prices in stage 1 such that $(1-\alpha_1) < \delta(1-\alpha_2)$ holds.[12]

Fact B. *In the unique equilibrium of the continuous-time bargaining problem in stage 2 of the price competition game G, where the sellers choose (α_1, α_2) in stage 1 satisfying $(1-\alpha_1) < \delta(1-\alpha_2)$, the strategies of the players are as follows:*

If the buyer enters store 1 at time 0 first, then rational seller 1 immediately accepts the buyer's behavioral demand and finishes the game at time 0 with probability 1. In case seller 1 does not concede to the buyer, the buyer infers that seller 1 is the behavioral type, so he immediately leaves store 1 and never comes back to this store again. The buyer directly goes to store 2 to play the concession game with the second seller. The concession game in store 2 continues until the time $T_2^e = \min\{-\frac{\log z_b}{\lambda_b^2}, -\frac{\log z_2}{\lambda_2}\}$ and players concede according to the following strategies: $F_2(t) = 1 - z_2 e^{\lambda_2(T_2^e - t)}$ and $F_b^2(t) = 1 - z_b e^{\lambda_b^2(T_2^e - t)}$ for all $t \geq 0$.[13]

However, if the buyer arrives at store 2 at time 0 and if seller 1's initial reputation z_1 is strictly higher than $z_1^* = \frac{1-\alpha_b-\frac{1-\alpha_2}{\delta}}{1-\alpha_b-\delta(1-\alpha_2)}$, then the buyer never leaves store 2. The buyer and seller 2 play the concession game in store 2 that lasts until the time T_2^e, and concede according to the distribution functions given in the last paragraph. If however, seller 1's initial reputation z_1 is strictly less than z_1^*, then seller 2 immediately accepts the buyer's

[12] Once again, supposing that $\alpha_1 \geq \alpha_2 > \alpha_b$.

[13] Note that with some manipulation of the notation, I reset the clock once the buyer enters store 2.

behavioral demand upon his arrival. Otherwise, the buyer leaves store 2 *immediately at time* 0 *(knowing that seller 2 is the behavioral type), and goes directly to store* 1*. Rational seller* 1 *instantly accepts the buyer's behavioral demand with probability* 1 *upon the buyer's arrival. In case seller* 1 *does not concede, the buyer immediately leaves his store. He directly returns to store* 2*, accepts the seller's behavioral demand* α_2 *with probability* 1*, and finalizes the game.*

***Expected Payoffs in the Concession Game*:** Since the buyer is indifferent between conceding and waiting at all times during the concession game with seller i, his expected payoff (during the concession game with seller i), v_b^i, is equal to what he can achieve at time 0, i.e. at the time the buyer enters to store i, which implies that $v_b^i = F_i(0)(1 - \alpha_b) + (1 - F_i(0))(1 - \alpha_i)$. Seller i concedes to the buyer with probability $F_i(0)$ at time 0, in which case the buyer attains the payoff of $1 - \alpha_b$. With probability $1 - F_i(0)$ seller i does not make a concession, but the buyer can immediately concede to seller i (at time 0^+) and guarantees himself a payoff of $1 - \alpha_i$. Similarly, seller i's expected payoff in the concession game is $v_i = F_b^i(0)\alpha_i + (1 - F_b^i(0))\alpha_b$.

For the sake of notational simplicity, I assume the following throughout the paper.

Assumption 0: Players' time preferences, i.e. interest rates, are the same and normalized to 1. That is, $r_i = 1$ for all $i \in \{1, 2, b\}$.

Equilibrium Prices of the Price Competition Game

Notice that the first stage of the price competition game G is similar to the Bertrand competition: The buyer can learn the posted prices before visiting the stores. However, the main difference between the two is that the buyer can negotiate with the sellers in the second stage of the game G. In this section, I characterize the equilibrium price(s) in stage 1 for small values of the prior beliefs on players' types (or initial reputations) and investigate the structure of the set of equilibrium prices as prior beliefs get values sufficiently close to zero. For ease of analysis in this section, I assume that each player shares a common initial reputation. That is, the prior probabilities of being the commitment type is the same across players and equal to some $z \in (0, 1)$.

Note that $(0,0)$ pricing in stage 1 can be supported as an equilibrium for all values of z. However, the following result shows that it is the unique equilibrium of the price competition game G in stage 1 when the uncertainty regarding the players' commitment types is sufficiently small. We can attain the same result when we lose the assumption that each player's initial reputation is the same, but it just adds extra complication in the proof with no additional insight.

Proposition 1. *For any $\alpha_b \in (0,1)$ and sufficiently low z, the unique equilibrium of the price competition game in stage 1 is that both sellers choose 0.*

I defer the proofs of all the results to Appendix. Proposition 1 implies that as the existence of commitment types becomes increasingly less likely, the price cutting strategy rules in the first stage of the price competition game G, and thus the unique equilibrium of this game coincides with the Bertrand's prediction of marginal cost pricing.

However, the next section shows that this uniqueness result is not robust to even the slightest perturbation on higher order beliefs. It indicates that the common knowledge assumption plays a crucial role to get this uniqueness result.

3. Robustness of the Uniqueness Result

In this section, I perturb the sellers' first order beliefs and show that we can support a continuum of prices as the equilibrium of the price competition game G in stage 1 even when players' beliefs about their opponents' types are arbitrarily close to zero. Therefore, suppose that seller 1 believes that it is common knowledge (with certainty) that $z_1 = \underline{z}$ and $z_2 = z_b = \bar{z}$, whereas seller 2 believes that it is common knowledge (with certainty) that $z_2 = \underline{z}$ while $z_1 = z_b = \bar{z}$ where $\bar{z}, \underline{z} \in (0,1)$ and $\bar{z} = \underline{z} + \epsilon$ for some $\epsilon > 0$. Therefore for $i, j \in \{1,2\}$, seller i believes that it is common knowledge that seller j and the buyer are commitment types with probability z^i_j and z^i_b respectively, and according to our assumption, for each i, $z^i_b = \bar{z}$ and $z^i_j = \bar{z}$ if $j \neq i$ or else (i.e., $j = i$) $z^i_j = \underline{z}$.

Therefore, each seller is confident (believes with certainty) that, relative to himself, the other seller is more likely to commit on the price he posts in stage 1, or equivalently is less likely to bargain with the buyer. With this perturbation in first order beliefs, the common knowledge assumption retains. However, the sellers do not agree on the content of the knowledge that they believe common. However, since each seller believes with certainty that his belief is common knowledge, no seller knows that his opponent indeed disagrees on the content of the knowledge.

I do not make any assumption regarding what the buyer believes because the main concern of this section is to understand how this perturbation affects the sellers' choices in stage 1 of the pricing game G. In the rest of this section, I characterize the set of equilibrium prices (chosen by the sellers in stage 1) for sufficiently small values of $\bar{z}$ and $\underline{z}$, and show that the uniqueness result does not survive even when the perturbation in the first order beliefs (ϵ) vanishes or the initial priors ($\bar{z}, \underline{z}$) converge to zero.

Let $\theta = \langle \underline{z}, \bar{z}, \alpha_1, \alpha_2, \alpha_b, \delta \rangle$, in short, denote the vector of primitives of the price competition game G, where $\bar{z} > \underline{z}$, $\alpha_b \in (0, 1)$, $\alpha_b \leq \alpha_i < 1$ for $i = 1, 2$ and without loss of generality $\alpha_1 \geq \alpha_2$. For any given θ, let $\alpha_\theta = \langle \alpha_1, \alpha_2, \alpha_b \rangle$ and $z_\theta = \langle \bar{z}, \underline{z} \rangle$ denote the vector of behavioral demands and initial priors, respectively. I restrict our analysis on vector of primitives θ where z_θ is sufficiently small. So, for the rest of the paper, I assume that θ satisfies the following assumption:

Assumption 1: For each seller $i \in \{1, 2\}$; *(a)* $\bar{z} < A_i$ (whenever $A_i = \frac{1-\alpha_b-\frac{1-\alpha_i}{\delta}}{\alpha_j-\alpha_b} > 0$), and *(b)* $\bar{z} < \alpha_b$.

The vector of primitives $\theta = \langle \underline{z}, \bar{z}, \alpha_1, \alpha_2, \alpha_b, \delta \rangle$ can be supported as an equilibrium of the price competition game G means that there exists a sequential equilibrium of the game G in which the sellers choose their posted prices α_1 and α_2 in stage 1 given $\bar{z}, \underline{z}$, α_b and δ. Suppose that θ can be supported as an equilibrium of the price competition game G. Then, Facts A and B still characterize the equilibrium strategies of the game G in the second stage. This is the case because each seller is confident about his belief (each seller believes with certainty that his belief is common knowledge), and each seller will interpret a behavior that is not supposed to occur in equilibrium as a deviation from the equilibrium strategy.[14]

[14]For example, in an equilibrium seller 1 may believe that the buyer will visit his store first at time 0 (given the posted prices chosen in stage 1). However, depending on his belief, the buyer who is playing his equilibrium strategy may visit seller 2 at time 0. If such a case occurs, seller 1 will interpret this incident as an unprofitable deviation by the buyer. So, neither seller 1 nor seller 2 can infer anything from such out-of-equilibrium choices made by the buyer and learn something about the buyer's actual type.

Therefore, according to the unique equilibrium strategies in the concession games (the second stage of the game G, i.e. the continuous-time bargaining game) at most one player (either the buyer or the seller, but not both) can make an initial probabilistic concession. A player is ***strong*** if he receives this probabilistic gift from his opponent and ***weak*** if he does not.

Thus, according to Facts A and B, for any $i, j \in \{1, 2\}$, seller i believes that the buyer is ***weak in store*** j ***at*** θ if and only if $z_b^i \leq (z_j^i)^{\frac{\alpha_b}{1-\alpha_j}} X_j^i$ given that $0 < X_j^i \leq 1$ where $X_j^i = (\frac{z_i^i}{A_j})^{\frac{\alpha_b}{1-\alpha_i}}$ and $A_j = \frac{1-\alpha_b-\frac{1-\alpha_j}{\delta}}{\alpha_i-\alpha_b}$. If $A_j \leq 0$ or $X_j^i > 1$, then seller i believes that the buyer is weak in store j if and only if $z_b^i \leq (z_j^i)^{\frac{\alpha_b}{1-\alpha_j}}$, in which case seller i believes that the buyer is ***locally weak in store*** j ***at*** θ. Finally, seller i believes that the buyer is ***distance-corrected strong relative to seller*** j ***at*** θ whenever $z_b^i \geq X_i^i$.

According to the proof of Proposition 1, a vector of primitives θ such that $\alpha_i \geq \alpha_b$ for each $i = 1, 2$ can be supported as an equilibrium of the game G whenever each seller believes that the buyer visits his store first at time 0 with certainty. Suppose for a contradiction that θ can be supported as an equilibrium of the price competition game in which seller 1 believes that the buyer visits his store first at time 0 with probability $p_1 \in [0, 1)$. If, $p_1 = 0$, then seller 1's expected payoff in stage 2 must be less than $\delta\alpha_1 z_2^1$ which is less than α_b by the Assumption 1. Hence, seller 1 profitably deviates in stage 1 and posts the price of, for example, α_b and ensures a payoff of α_b in stage 2.

On the other hand, if $p_1 \in (0, 1)$, then each seller's expected payoff in stage 2 following the history that the buyer enters his store must be (strictly) higher than α_b. If not, then each seller has incentive to deviate to ensure the payoff of α_b in stage 2 with certainty by posting the price of α_b. Note that if $\alpha_2 = \alpha_b$ in stage 1, then there is still room for profitable deviation for seller 1 by posting a price slightly below α_b. However, if seller 1 believes that in equilibrium each seller gets the payoff higher than α_b at stage 2, then the buyer must get the payoff of $1 - \alpha_i$ in store i (sellers attaining above α_b implies that the buyer is weak in each store at θ). But, then seller 1 will expect that the buyer will choose seller i only when $\alpha_j > \alpha_i$.

Therefore, in equilibrium, we must have that (i) the buyer visits each store with a positive probability, (ii) the buyer is weak in each store at θ and (iii) each seller posts the same price in stage 1, i.e. $\alpha_1 = \alpha_2$. However, even when all these three conditions hold, there is opportunity for profitable deviation for each seller: Each seller would increase his expected payoff by posting a price slightly below his opponent's price such that the buyer is still weak in both stores at this new vector of primitives, so the buyer chooses to visit first the seller who posts the lowest price, and thus the deviating seller gets the buyer with certainty and increases his expected payoff. The following result formally summarizes the above discussion.

Proposition 2. *A vector of primitives $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ such that $\alpha_i \geq \alpha_b$ for each $i = 1, 2$ can be supported as an equilibrium of the price competition game G whenever each seller believes that the buyer visits his store first at time 0 with certainty.*

Proof. See the proof of Proposition 1 □

Remark that $(0, 0)$ is an equilibrium outcome for all values of the primitives of the game G, but in the rest of this section I will show that it is not the unique equilibrium outcome. The next result ensures that in equilibrium, supporting the vector of primitives θ, if a seller believes that he is weak in his store at θ, then the seller must believe that he will be weak in his store whatever price (above α_b) he chooses in stage 1. If the seller is not weak in his store but his payoff is α_b in the subgame that the buyer visits his store first at time 0,[15] the same condition is required to support θ as an equilibrium. More formally,

Proposition 3. *Suppose that the vector of primitives θ can be supported as an equilibrium of the price competition game G and seller $i \in \{1, 2\}$ believes that he is weak in his store at θ, or his expected payoff in stage 2 if the buyer visits his store first is α_b. Then it must be true that for all $\alpha_i \geq \alpha_b$, seller i believes that he is weak in his store at θ', which is equivalent to θ except α_i. That is, $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$.*

For presentational purposes and clarity, I characterize the equilibrium prices in stage 1 and analyze the limiting properties of the sets of the equilibrium prices case by case. These three exhaustive cases are *(i)* $\alpha_i = \alpha$ for $i = 1, 2$, *(ii)* $\alpha_1 \geq \alpha_2$ but $1-\alpha_1 < \delta(1-\alpha_2)$, and *(iii)* $\alpha_1 \geq \alpha_2$ but $1 - \alpha_1 \geq \delta(1 - \alpha_2)$.

[15] This is possible only when this seller chooses higher price in stage 1 and the prices are far apart relative to the location of the stores so the buyer never plays the concession game with that seller.

3.1. *Equilibrium Prices When $\alpha_1 = \alpha_2$*

I start with determining the buyer's expected payoff in stage 2 from the point of view of each seller. Take θ as given, we know through Fact A that there are two important cut-off points to evaluate the buyer's continuation payoff in stage 2 from the point of view of each seller.[16] So, seller $i \in \{1,2\}$ believes that these two cut-offs to evaluate the buyer's expected payoff of visiting store $j \in \{1,2\}$ first are: $X_j^i (z_j^i)^{\lambda_b/\lambda_j}$ and X_j^i.

If the buyer's initial reputation, z_b^i, is less than the first cut-off value, then seller i believes that the buyer is the weak player in store j (at θ). Thus (according to seller i), the buyer's continuation payoff evaluated at time 0, if he visits store j first, is

$$1 - \alpha_j \tag{1}$$

However, if z_b^i is higher than the second critical cut-off value, then seller i believes that the buyer is distance-corrected strong relative to himself (seller i) at θ, so the buyer's expected payoff evaluated at time 0 is given by

$$\left\{(1 - z_j^i) + \delta z_j^i \left(1 - \frac{z_i^i}{(z_b^i)^{\lambda_i/\lambda_b^i}}\right)\right\}(1 - \alpha_b) + \frac{\delta z_i^i z_j^i}{(z_b^i)^{\lambda_i/\lambda_b^i}}(1 - \alpha_i) \tag{2}$$

Finally, if the buyer is strong but not distance-corrected strong relative to seller i (from the point of view of seller i) at θ, seller i believes that the buyer's expected payoff is

$$\left[1 - z_j^i \left(\frac{X_j^i}{z_b^i}\right)^{\lambda_j/\lambda_b^j}\right](1 - \alpha_b) + z_j^i \left(\frac{X_j^i}{z_b^i}\right)^{\lambda_j/\lambda_b^j}(1 - \alpha_j) \tag{3}$$

The next result shows that the vector of primitives θ where $\alpha \geq \alpha_b$ can be supported as an equilibrium when each seller believes that the buyer is distance-corrected strong relative to himself and there is no price above α_b so that the seller can make himself locally strong in his store by asking this price in stage 1.

Proposition 4. *The vector of primitives θ, where $\alpha_i = \alpha$ for $i = 1, 2$, can be supported as an equilibrium of the price competition game G if and only if*

1) $\bar{z} \geq \left(\frac{\underline{z}}{A}\right)^{\frac{\alpha_b}{1-\alpha}}$ *whenever $A > 0$, and*

[16] Each seller will evaluate the buyer's expected payoff in stage 2 differently because their beliefs do not agree.

2) $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$ *hold.*

where $A = \frac{1-\alpha_b-\frac{1-\alpha}{\delta}}{\alpha-\alpha_b}$.

When A is non-positive, the buyer does not leave the store he visits at time 0. Hence, for such cases, the second condition (by itself) is both necessary and sufficient.

Proposition 5. *For any* $\alpha, \alpha_b \in (0,1)$ *where* $\alpha \geq \alpha_b$, *there exist some (small)* $\bar{z}, \underline{z} \in (0,1)$ *such that* α *is supported as the equilibrium price in stage 1 of the pricing game G.*

Take any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the pricing game G. By the statement "*the vector of prices* α_θ *is an equilibrium outcome of G as* $\underline{z}$ *converges to* $\bar{z}$", I mean the following: Take any sequence of prior beliefs $\{\underline{z}_n\}$ (where $\underline{z}_0 = \underline{z}$) converging to $\bar{z}$, i.e. $\forall \epsilon > 0$, $\exists M > 0$ such that $|\bar{z} - \underline{z}_m| < \epsilon$, $\forall m > M$. Then, the vector of primitives $\theta_m = \langle \underline{z}_m, \bar{z}, \alpha_\theta, \delta \rangle$ can be supported as an equilibrium of the pricing game G for all $m > M$.

The following result characterizes the set of equilibrium prices that can be supported as an equilibrium outcome of the pricing game G in stage 1 when the perturbation in first order beliefs vanishes.

Proposition 6. *Take any* $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ *that can be supported as an equilibrium of the pricing game G such that* $\alpha_i = \alpha$ *for* $i = 1, 2$. *The vector of prices* α_θ *is an equilibrium outcome of G as* $\underline{z}$ *converges to* $\bar{z}$ *if and only if*

1) $\frac{1}{2} \leq \alpha_b \leq \alpha \leq 1 - \delta(1-\alpha_b)$, *and*

2) $1 - \frac{1}{1+\bar{\epsilon}}\alpha_b \leq \alpha < 1$ *whenever* $0 < A$

where $\bar{\epsilon} = \frac{\ln A}{\ln \bar{z}}$

Therefore, as the perturbation on higher order beliefs vanishes, we can support a large set of prices in stage 1 whenever $\alpha_b \geq 1/2$. In the limit where both sellers' beliefs coincide, i.e. $\underline{z} = \bar{z}$, the unique equilibrium strategy in stage 1 is to post the price of 0 for both sellers (Proposition 1). The reason for this "*discontinuity*" in the set of equilibrium outcomes is that, as long as $\underline{z} < \bar{z}$, each seller believes that he is the seller with lower reputation, and hence the buyer will choose to visit his store first. Therefore, price undercutting is not a dominant strategy regardless of how close $\underline{z}$ and $\bar{z}$ are.

Now, I would like to characterize the set of equilibrium prices that can be supported as an equilibrium of the price competition game G as $\underline{z}$ and $\bar{z}$ converge to zero at the same rate. Take any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the pricing game G. By the statement " *the vector of prices α_θ is an equilibrium outcome of G as both $\underline{z}$ and $\bar{z}$ converge to zero at the same rate*," I mean the following: Take any sequences $\{\bar{z}_n\}$ and $\{\underline{z}_n\}$ (where $\bar{z}_0 = \bar{z}$ and $\underline{z}_0 = \underline{z}$) of the prior beliefs converging to zero, i.e. $\forall \epsilon > 0$, $\exists M > 0$ such that $|\bar{z}_m - 0| < \epsilon$, $\forall m > M$, such that for all $n \geq 0$, $\bar{z}_n = K\underline{z}_n$ for some finite $K > 1$. Then, the vector of primitives $\theta_m = \langle \underline{z}_m, \bar{z}_m, \alpha_\theta, \delta \rangle$ can be supported as an equilibrium of the pricing game G for all $m > M$.

Proposition 7. *Take any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the pricing game G such that $\alpha_i = \alpha$, i=1,2. The vector of prices α_θ is an equilibrium outcome of G as $\underline{z}$ and $\bar{z}$ converge to zero at the same rate if and only if $\alpha \geq \alpha_b \geq \frac{1}{2}$ holds.*

What is The Secret of $\frac{1}{2}$: As it is indicated by the previous two results, in the limiting case of the set of equilibrium prices where the initial priors either converge to one another or both converge to zero, we have a necessary condition that $\alpha_b \geq 1/2$ must hold. Since, this threshold will sit on the center of our findings in this paper, I would like to underline its relationship with our assumptions and the Nash Bargaining share we obtain in "*standard*" bargaining literature.

This threshold is not arbitrary, but instead it is the relative ratio of the players' time discounts, evaluated for $r_b = r_i = 1$ for $i = 1, 2$, i.e. each player's interest rate is normalized to one. Recall that we have this assumption just for the tractability purposes. It simplifies the algebra involved in our analysis with a great deal. However, in the limiting case of the initial priors, the threshold we must have is

$$\frac{r_b}{r_b + r_i} \tag{4}$$

which yields the ratio $\frac{1}{2}$ by our assumption that $r_b = r_i = 1$ for $i = 1, 2$.

Remark that the ratio in (4) is equal to the buyer's share in the "usual" bargaining game (Rubinstein's alternating offer bargaining protocol with no behavioral types) between the buyer and a single seller in the limit where the players can make increasingly frequent offers.

When we do not normalize the interest rates to one, we get the ratio in (4) because the inequality $z_b \geq (z_i)^{\frac{\alpha_b}{1-\alpha_b}}$ for seller i (Proposition 3 explains why it must hold in equilibrium) actually equals to $z_b \geq (z_i)^{\frac{r_i \alpha_b}{r_b(1-\alpha_b)}}$. Thus, in the limit, we must have that $\alpha_b \geq \frac{r_b}{r_b+r_i}$.

3.2. *Equilibrium Prices When* $\alpha_1 \neq \alpha_2$

This section characterizes the vector of primitives θ satisfying $\alpha_1 \neq \alpha_2$ that can be supported as an equilibrium of the price competition game G. Without loss of generality, I suppose that $\alpha_1 > \alpha_2$. As it is indicated by Facts A and B, the structure of the unique equilibrium strategy in the second stage of the pricing game alters as the distance between the posted prices relative to the distance between the stores changes. First, I focus on the case where the sellers choose posted prices that are relatively distant compared to the location of the stores, so the buyer does not play the concession game with seller 1 in the second stage. Recall that in this case even the buyer does not accept α_1, he may visit store 1 first to make a take it leave it offer to seller 1.

3.2.1. *The Case of Distant Prices*

For any given θ, this section characterizes the necessary and sufficient conditions of the equilibrium price vector α_θ where $1 - \alpha_1 < \delta(1 - \alpha_2)$ for sufficiently small $\bar{z}$ and $\underline{z}$. I start by introducing the buyer's expected payoffs from the point of view of each seller $i \in \{1, 2\}$. For any $i, j \in \{1, 2\}$, recall that $v_b^j(i)$ denotes the expected payoff of the buyer from the point of view of seller i if the buyer visits store j first.

So, when

$$z_1^i > z_1^* = \frac{1 - \alpha_b - \frac{1-\alpha_2}{\delta}}{1 - \alpha_b - \delta(1 - \alpha_2)}$$

then from seller i's point of view the buyer's expected payoff of visiting store 1 first is

$$v_b^1(i) = (1 - z_1^i)(1 - \alpha_b) + \delta z_1^i u_b(i) \tag{5}$$

whereas, his expected payoff of visiting store 2 first is

$$v_b^2(i) = u_b(i) \tag{6}$$

where

$$u_b(i) = \frac{z_2^i}{(z_b^i)^{\lambda_2/\lambda_b^2}}(1-\alpha_2) + \left(1 - \frac{z_2^i}{(z_b^i)^{\lambda_2/\lambda_b^2}}\right)(1-\alpha_b) \tag{7}$$

whenever $z_b^i > (z_2^i)^{\lambda_b^2/\lambda^2}$ (the buyer is locally strong in store 2 at θ) and

$$u_b(i) = 1 - \alpha_2 \tag{8}$$

if $z_b^i \leq (z_2^i)^{\lambda_b^2/\lambda^2}$, i.e. the buyer is locally weak in store 2 (according to seller i's beliefs).

However, when $z_1^i < z_1^*$, then there are two cases we need to consider regarding the buyer's strength relative to seller 2 (locally). If the buyer is locally strong relative to seller 2 at θ, i.e. $z_b^i > (z_2^i)^{\lambda_b^2/\lambda^2}$, then the buyer's expected payoff of visiting store 1 first is

$$v_b^1(i) = (1 - z_1^i)(1-\alpha_b) + \delta z_1^i u_b(i) \tag{9}$$

where $u_b(i)$ is given in Equation (7) whereas the buyer's payoff of visiting store 2 first is

$$v_b^2(i) = (1 - z_2^i)(1-\alpha_b) + z_2^i \delta[(1 - z_1^i)(1-\alpha_b) + \delta z_1^i(1-\alpha_2)] \tag{10}$$

On the other hand, when the buyer is locally weak in store 2 at θ, i.e. $z_b^i \leq (z_2^i)^{\lambda_b^2/\lambda^2}$, then the buyer's expected payoff of visiting store 1 first is

$$v_b^1(i) = (1 - z_1^i)(1-\alpha_b) + \delta z_1^i(1-\alpha_2) \tag{11}$$

while is payoff of visiting store 2, $v_b^2(i)$, is as given in Equation (10).

We start our analysis with the case where $\lambda_b^2/\lambda_2 > 1$, i.e $\alpha_b + \alpha_2 \geq 1$. In this case, we have $\bar{z} > (\bar{z})^{\lambda_b^2/\lambda^2}$ and $\bar{z} > (\underline{z})^{\lambda_b^2/\lambda^2}$, i.e. both sellers believe that the buyer is locally weak in store 2 at θ. Hence, in any equilibrium, both sellers' expected payoff is equal to α_b. To support prices in θ falling in this category, we must have that both sellers believe that the buyer will visit his store first with certainty at time 0 (Proposition 2). Otherwise, the seller who cannot get the buyer at time 0 will deviate and post the price of α_b to get the buyer for sure.

Then in equilibrium we must have that $v_b^i(i) > v_b^j(i)$ for $i, j = 1, 2$ and $i \neq j$. The equilibrium strategies and therefore the expected payoff calculations in stage 2 depends on whether z_1^i is higher than z_1^* or not. However when we perturb the first order beliefs, it is important to clarify which player compares these prior beliefs. Seller 2 may believe that $z_1^i > z_1^*$ because $\bar{z} > z_1^*$ whereas seller 1 may believe the opposite because $\underline{z} < z_1^*$. Since the threshold z_1^* does not depend on the prior beliefs, both sellers will calculate it in the same way. So, there are three exhaustive cases we need to consider *(i)* $z_1^2 = \bar{z} > z_1^1 = \underline{z} > z_1^*$, *(ii)* $\bar{z} > z_1^* > \underline{z}$ and *(iii)* $z_1^* > \bar{z} > \underline{z}$.

Lemma 1. *The vector of primitives θ where $\alpha_b + \alpha_2 \geq 1$ and $\underline{z} > z_1^*$ can be supported as an equilibrium of the pricing game G if and only if the following inequalities hold:*

1) $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$
2) $\bar{z} > \left[\frac{(1-\alpha_b)\left(1-\frac{1-\underline{z}}{1-\delta\underline{z}}\right)}{(\alpha_2-\alpha_b)}\right]^{\frac{\alpha_b}{\alpha_2+\alpha_b-1}}$
3) $\underline{z} < \frac{(1-\alpha_b)\left[1-\frac{1-\bar{z}}{1-\delta\bar{z}}\right]}{(\alpha_2-\alpha_b)}(\bar{z})^{\frac{1-\alpha_2}{\alpha_b}}$

With this result, we characterize equilibrium prices such that α_2 is very close to α_b. As conditions 2 and 3 are satisfied, each seller believes that the buyer will visit his store first at time 0. No seller wants to deviate and post a higher price in stage 1 because if a seller does so, he will get the payoff of α_b at most (sellers are already weak relative to the buyer (condition 1) and increasing the posted price in stage 1 does not help them to reverse it). Likewise, there are no profitable deviation by posting a price lower than α because even if a seller does so, the buyer will continue to be locally strong (thanks to inequality 1) and hence there will be no improvement on the seller's expected payoff by such deviation.

Example 1. Consider the vector of primitives θ where $\alpha_b = 0.6$, $\alpha_2 = 0.62$, $\delta = 0.9$ and $\underline{z} = 0.0001$ while $\bar{z} = 0.02$. The vector θ can be supported as an equilibrium of the pricing game G for all $\alpha_1 > 0.658$. One can easily check that for these values, z_1^* is negative. Therefore, $\underline{z} > z_1^*$ holds for any $\underline{z} > 0$. One can also see that with the equilibrium strategies in stage 2, both sellers believe that the buyer will visits their stores first at time 0 with certainty.

Lemma 2. *The vector of primitives θ where $\alpha_b + \alpha_2 \geq 1$ and $\bar{z} > z_1^* > \underline{z}$ can be supported as an equilibrium of the pricing game G if and only if the following inequalities hold:*

1) *Inequalities* (1) *and* (3) *of Lemma 1, and*

2) $\bar{z} > \left[\frac{\delta \underline{z}\bar{z}(\alpha_2-\alpha_b)}{(\bar{z}-\underline{z})(1-\delta)(1-\alpha_b)+\delta\underline{z}\bar{z}(1-\alpha_b-\delta(1-\alpha_2))}\right]^{\frac{\alpha_b}{1-\alpha_2}}$

Lemma 2 characterizes the equilibrium set of α_2's that are close, but not as much as the ones that can be identified in Lemma 1, to α_b. The following result characterizes the case where α_2 is not very close to α_b because z_1^* is allowed to be higher than $\bar{z}$.

Lemma 3. *The vector of primitives θ where $\alpha_b + \alpha_2 \geq 1$ and $z_1^* > \bar{z}$ can be supported as an equilibrium of the pricing game G if and only if the following inequalities hold:*

1) *Inequality (1) of Lemma 1 and (2) of Lemma 2,*
2) $\left[\bar{z} \geq \frac{\underline{z}}{1-\delta\underline{z}(\frac{1-\alpha_2}{1-\alpha_b})}\right]$ *or* $\left[\underline{z} \leq \frac{(\bar{z}-\underline{z})(1-\delta)(1-\alpha_b)}{\delta\bar{z}(1-\alpha_b-\delta(1-\alpha_2))}\right]$

Example 2. Suppose that $\alpha_b = 0.4, \alpha_2 = 0.8$ and $\delta = 0.9$ (in which case $z_1^* = 0.899$) while $\underline{z} = 0.0001$ and $\bar{z} = 0.01$. With these values, one can check that each seller believes that the buyer will visit his store first at time 0. Also, there are no prices (above α_b) that a seller can make himself locally strong by deviating in stage 1.

Now suppose that $\lambda_b^2/\lambda_2 < 1$, i.e. $\alpha_b + \alpha_2 < 1$. In this case, $z_b^1 = \bar{z}$ is strictly less than $(z_2^1)^{\frac{\alpha_b}{1-\alpha_2}} = (\bar{z})^{\frac{\alpha_b}{1-\alpha_2}}$. That is, seller 1 believes that the buyer is locally weak in store 2 at θ. However, depending on the values of $\bar{z}, \underline{z}, \alpha_b$ and α_2, $\bar{z}$ may or may not be greater than $(z_2^2)^{\frac{\alpha_b}{1-\alpha_2}} = (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$, implying that seller 2 may or may not believe that the buyer is locally weak in his store (store 2) at θ. Therefore, we need to analyze the following four exhaustive cases: (i) $\bar{z} > (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$, where it is either (i_a) $z_1^2 > z_1 > z_1^*$, (i_b) $z_1^2 > z_1^* > z_1$, or (i_c) $z_1^* > z_1^2 > z_1$, and (ii) $\bar{z} \leq (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$. We now analyze each case in turn.

The following three Lemma analyze the cases where seller 1 believes that the buyer is locally weak in store 2 at θ while seller 2 believes the opposite. The following result characterizes the set of equilibrium α_2's that are very close to α_b so that z_1^* is either negative or sufficiently close to zero.

Lemma 4. *The vector of primitives θ where $\alpha_b + \alpha_2 \leq 1$, $\bar{z} > (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$ and $\underline{z} > z_1^*$ can be supported as an equilibrium of the pricing game G if and only if the following inequalities hold:*

1) *Inequalities (1) and (3) of Lemma 1, and*

2) $\underline{z} < \frac{\alpha_2 - \alpha_b}{1-\alpha_b-\delta(1-\alpha_2)}$

The next lemma considers the case where z_1^* can be positive but close to zero, i.e. α_2 is close but not so close to α_b.

Lemma 5. *The vector of primitives θ where $\alpha_b + \alpha_2 \leq 1$, $\bar{z} > (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$ and $\bar{z} > z_1^* > \underline{z} >$ can be supported as an equilibrium of the pricing game G if and only if the following inequalities hold:*

1) *Inequalities* (1) *and* (3) *of Lemma 1, and*

2) $\bar{z} > \frac{1-\alpha_b-[(1-\underline{z})(1-\alpha_b)+\delta\underline{z}(1-\alpha_2)]}{1-\alpha_b-\delta[(1-\underline{z})(1-\alpha_b)+\delta\underline{z}(1-\alpha_2)]}$

The following result characterizes the set of equilibrium α_2's that might be far from α_b so that z_1^* takes high values.

Lemma 6. *The vector of primitives θ where $\alpha_b + \alpha_2 \leq 1$, $\bar{z} > (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$ and $z_1^* > \bar{z}$ can be supported as an equilibrium of the pricing game G if and only if the following inequalities hold:*

1) *Inequality* (1) *of Lemma 1, and*

2) *Inequality* (2) *of Lemma 3 and Lemma 5.*

As it was the case in previous Lemmas, Inequality (1) in Lemma 1 guarantees that seller 1 (who is asking the higher posted price in stage 1) has no opportunity to make a profitable deviation by lowering his price and making the buyer visit his store first at time zero as well as making himself locally strong. Other conditions ensure that each seller believes that in equilibrium the buyer will visit his store first at time 0, and hence price undercutting is not a dominant strategy.

Example 3. Suppose that $\underline{z} = 10^{-6}$ and $\bar{z} = 10^{-3}$. Once can check that with these initial priors, we can support $\alpha_b = 0.4$ while $\alpha_2 = 0.6$, $\alpha_b = 0.1$ while $\alpha_2 = 0.9$, and so on.

The following result shows that there is no vector of primitives supported as an equilibrium of the game G where both sellers believe that seller 2 (the seller who posts the lower price in stage 1) is locally strong in his store at θ, i.e. $\bar{z} \leq (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$. The main intuition behind the proof of this result is as follows. If θ is supported as an equilibrium, seller 1's expected payoff in the game is α_b if the buyer visits his store first (since $1 - \alpha_1 < \delta(1 - \alpha_2)$). However, since $\bar{z} \leq (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$ holds, seller 1 believes that he will be locally strong if he posts a price sufficiently close to α_b (since $\bar{z} \leq (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$ implies that $\bar{z} \leq (\underline{z})^{\frac{\alpha_b}{1-\alpha_2'}}$ where $\alpha_2' < \alpha_2$), and thus seller 1 can ensure to be the first store the buyer visits at time 0. Therefore, seller 1 can guarantee a payoff (slightly) higher than α_b, which contradicts the assertion that θ is supported as an equilibrium.

Lemma 7. *There exists no θ that can be supported as an equilibrium of the pricing game G, where $\alpha_b + \alpha_2 \leq 1$ and $\bar{z} \leq (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$.*

The next two results characterizes the vector of primitives θ, satisfying $\alpha_1 > \alpha_2$ and $\delta(1 - \alpha_2) > 1 - \alpha_1$, that can be supported as an equilibrium of the pricing game G for the limiting cases of the prior beliefs.

Proposition 8. *There exists no $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$, that can be supported as an equilibrium of the pricing game G, satisfying $\alpha_1 > \alpha_2$ and $\delta(1 - \alpha_2) > 1 - \alpha_1$ such that the vector of prices α_θ is an equilibrium outcome of G as $\underline{z}$ converges to $\bar{z}$.*

Proposition 9. *Take any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the pricing game G such that $\alpha_1 > \alpha_2$ and $\delta(1 - \alpha_2) > 1 - \alpha_1$. The vector of prices α_θ is an equilibrium outcome of G as $\underline{z}$ and $\bar{z}$ converge to zero at the same rate if and only if $\alpha_2 > \alpha_b \geq \frac{1}{2}$ and $z_1^* > 0$ hold.*

Notice that the last inequality $z_1^* > 0$ implies that α_2 must be sufficiently bigger than α_b.

3.2.2. *The Case of Distant Stores*

In this section, I assume that $\delta(1 - \alpha_2) \leq 1 - \alpha_1$. Therefore, Fact A summarizes the equilibrium strategies of the players in the second stage. To characterize the equilibrium prices in the first stage, we first focus on the case where α_2's are not so close to α_b so that $A_i > 0$ for $i = 1, 2$. There are four threshold points that we need to consider from the point of view of seller 2. These are $X_1^2(z_1^2)^{\lambda_b^1/\lambda_1}$, X_1^2, $X_2^2(z_2^2)^{\lambda_b^2/\lambda_2}$, and X_2^2. Formally, these thresholds are as follows:

$$(w_1) \qquad X_1^2(z_1^2)^{\lambda_b^1/\lambda_1} = (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}(\bar{z})^{\frac{\alpha_b}{1-\alpha_1}}\left(\frac{1}{A_1}\right)^{\frac{\alpha_b}{1-\alpha_2}}$$

$$(w_2) \qquad X_1^2 = (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}\left(\frac{1}{A_1}\right)^{\frac{\alpha_b}{1-\alpha_2}}$$

$$(w_3) \qquad X_2^2(z_2^2)^{\lambda_b^2/\lambda_2} = (\bar{z})^{\frac{\alpha_b}{1-\alpha_1}}(\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}\left(\frac{1}{A_2}\right)^{\frac{\alpha_b}{1-\alpha_1}}$$

$$(w_4) \qquad X_2^2 = (\bar{z})^{\frac{\alpha_b}{1-\alpha_1}}\left(\frac{1}{A_2}\right)^{\frac{\alpha_b}{1-\alpha_1}}$$

Since $\alpha_1 > \alpha_2$ and $\frac{1-\alpha_1}{1-\alpha_2} \geq \delta$, we have $1 > A_1 > A_2$ and $\frac{\alpha_b}{1-\alpha_1} > \frac{\alpha_b}{1-\alpha_2}$, implying that $\left(\frac{1}{A_2}\right)^{\frac{\alpha_b}{1-\alpha_1}} > \left(\frac{1}{A_1}\right)^{\frac{\alpha_b}{1-\alpha_2}}$. Therefore, we have $w_1 < w_3$. Moreover, for all values of the parameters, we have $w_1 < w_2$ and $w_3 < w_4$.

Also note that seller 2 believes that the buyer's expected payoff of visiting store 1 first is calculated with Equation (1) (in this case $i = 2, j = 1$) if $\bar{z} \leq w_1$, with Equation (3) when $w_1 < \bar{z} \leq w_2$, and with Equation (2) if $w_2 < \bar{z}$. On the other hand, seller 2 believes that the buyer's expected payoff of visiting store 2 first is calculated with Equation (1) (in this case $i = 2, j = 2$) if $\bar{z} \leq w_3$, with Equation (3) when $w_3 < \bar{z} \leq w_4$, and with Equation (2) if $w_4 < \bar{z}$.

Moreover, from seller 1's point of view, the four thresholds that we need to consider are $X_1^1 {z_1}^{\lambda_b^1/\lambda_1}$, X_1^1, $X_2^1 {z_2}^{\lambda_b^2/\lambda_2}$ and X_2^1. More formally,

$$(\gamma_1) \qquad X_1^1(z_1^1)^{\frac{\lambda_b^1}{\lambda_1}} = (\bar{z})^{\frac{\alpha_b}{1-\alpha_2}}(\underline{z})^{\frac{\alpha_b}{1-\alpha_1}}\left(\frac{1}{A_1}\right)^{\frac{\alpha_b}{1-\alpha_2}}$$

$$(\gamma_2) \qquad X_1^1 = (\bar{z})^{\frac{\alpha_b}{1-\alpha_2}}\left(\frac{1}{A_1}\right)^{\frac{\alpha_b}{1-\alpha_2}}$$

$$(\gamma_3) \qquad X_2^1(z_2^1)^{\frac{\lambda_b^2}{\lambda_2}} = (\underline{z})^{\frac{\alpha_b}{1-\alpha_1}}(\bar{z})^{\frac{\alpha_b}{1-\alpha_2}}\left(\frac{1}{A_2}\right)^{\frac{\alpha_b}{1-\alpha_1}}$$

$$(\gamma_4) \qquad X_2^1 = (\underline{z})^{\frac{\alpha_b}{1-\alpha_1}}\left(\frac{1}{A_2}\right)^{\frac{\alpha_b}{1-\alpha_1}}$$

It is always the case that $\gamma_1 < \gamma_3$, $\gamma_1 < \gamma_2$ and $\gamma_3 < \gamma_4$. Seller 1 believes that the buyer's expected payoff of visiting store 1 first is calculated according to Equation (1) (in this case $i = 1, j = 1$) if $\bar{z} \leq \gamma_1$, Equation (3) when $\gamma_1 < \bar{z} \leq \gamma_2$, and Equation (2) if

$\gamma_2 < \bar{z}$. On the other hand, seller 1 believes that the buyer's expected payoff of visiting store 2 first is calculated according to Equation (1) (in this case $i = 1, j = 2$) if $\bar{z} \leq \gamma_3$, Equation (3) when $\gamma_3 < \bar{z} \leq \gamma_4$, and Equation (2) if $\gamma_4 < \bar{z}$.

Assumption 2: θ satisfies that $\alpha_1 > \alpha_2$ where $1 - \alpha_1 \geq \delta(1 - \alpha_2)$ and $A_i > 0$ for $i = 1, 2$.

For the rest of this subsection, I assume that θ satisfies Assumptions 1 and 2. Remark that by Assumption 1-(a), we have $0 < X_i^j < 1$ as long as $A_i > 0$ for each $i, j \in \{1, 2\}$.

Proposition 10. *Suppose that θ can be supported as an equilibrium of the pricing game G. Then, we must have $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$.*

Therefore, there is no θ that can be supported as an equilibrium of the pricing game G such that a seller believes that he is locally strong in his store at θ. However, Proposition 10 implies more than this: At any (equilibrium) θ, at least one seller must believe that he is locally weak in his store even if he deviates to a lower price that is very close to α_b. Namely, if θ is supported as an equilibrium, then at least one seller's expected payoff in stage 2 is α_b.

The following result is a direct implication of Proposition 10 and I present it with no formal proof.

Corollary 1. *For any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the pricing game G and that satisfies Assumptions 1 and 2, the vector of prices α_θ is an equilibrium outcome of G as $\underline{z}$ converges to $\bar{z}$, or as $\underline{z}$ and $\bar{z}$ converge to zero at the same rate whenever we have $\alpha_b \geq \frac{1}{2}$.*

Corollary 1 implies that there are no equilibrium where $\alpha_b + \alpha_2 < 1$ for sufficiently small $\bar{z}$ and $\underline{z}$, or when the difference between the two is sufficiently small. This implies that we need to focus on θ's satisfying that $\alpha_b + \alpha_2 \geq 1$.

Proposition 11. *For sufficiently small values of $\bar{z}$ and $\underline{z}$, θ satisfying $\alpha_b + \alpha_2 > 1$ can be supported as an equilibrium of the pricing game G if and only if $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$ and either*

1) $\bar{z} \leq \left(\frac{\alpha_2 - \alpha_b}{\alpha_1 - \alpha_b}\right)^{\frac{\alpha_b}{\alpha_1 - \alpha_2}}$ *and* $\underline{z} < \frac{(\bar{z} - \underline{z})(1-\delta)(1-\alpha_b)}{\delta \bar{z} R}$, *or*

2) $\bar{z} > \left(\frac{\alpha_2-\alpha_b}{\alpha_1-\alpha_b}\right)^{\frac{\alpha_b}{\alpha_1-\alpha_2}}$ *and* $\bar{z} < \frac{(\bar{z}-\underline{z})(1-\delta)(1-\alpha_b)}{\delta \underline{z} R}$

holds where $R = \left|\left((\alpha_1-\alpha_b)/(\bar{z})^{\frac{1-\alpha_1}{\alpha_b}}\right) - \left((\alpha_2-\alpha_b)/(\bar{z})^{\frac{1-\alpha_2}{\alpha_b}}\right)\right|$

The following result directly follows from the conditions 1 and 2 of Proposition 11, and its proof follows from Corollary 1 and the analysis similar to the analysis used in the proofs of Propositions 6 and 7. Therefore, I provide it with no formal proof.

Proposition 12. *There exists no* $\theta = \langle z_\theta, \alpha_\theta, \delta\rangle$ *that can be supported as an equilibrium of the pricing game* G *such that the vector of prices* α_θ *is an equilibrium outcome of* G *as* $\underline{z}$ *converges to* $\bar{z}$*.*

Proposition 13. *Take any* $\theta = \langle z_\theta, \alpha_\theta, \delta\rangle$ *that can be supported as an equilibrium of the pricing game* G *such that* $\bar{z} = \underline{z}K$ *for some* $K > 1$*. The vector of prices* α_θ *is an equilibrium outcome of* G *as* $\underline{z}$ *and* $\bar{z}$ *converge to zero at the same rate if and only if* $\alpha_b \geq \frac{1}{2}$ *and* $\frac{\alpha_b}{1-\alpha_2} > K$ *hold.*

Therefore, as z_θ^n (where $\bar{z}^n = K\underline{z}^n$) converges to zero, the set of prices that can be supported in this case depends on the relative ratio of the initial priors, K, and this set shrinks as K increases.

The Case Where α_2 and α_b are Very Close

I finalize the characterization by considering the last case where the posted price of the second seller, α_2, is sufficiently close to the buyer's demand α_b, so the buyer does not leave store 2 if he ever visits this store. So, for the rest of this section, I assume that θ satisfies Assumptions 1 and 3:

Assumption 3: θ satisfies that $\alpha_1 > \alpha_2$ where $1-\alpha_1 \geq \delta(1-\alpha_2)$ but $A_2 \leq 0$, i.e. $\delta(1-\alpha_b) \leq 1-\alpha_2$.

In this case, the buyer prefers not to leave store 2 if he ever visits there because it is not worth bearing the cost of traveling. Hence, the buyer and seller 2 play the concession game until they reach an agreement in store 2. There are two cases that we need to consider depending on the value of A_1.

Take any θ such that $A_1 \leq 0$. In this case there are two thresholds for each seller i we need to consider: $(\underline{z})^{\frac{\alpha_b}{1-\alpha_i}}$ and $(\bar{z})^{\frac{\alpha_b}{1-\alpha_j}}$ where $i \in \{1, 2\}$ and $j \neq i$. Among all four, the smallest threshold is $(\underline{z})^{\frac{\alpha_b}{1-\alpha_1}}$. That is, if $\bar{z}$ is above this threshold, seller 1 believes that the buyer is locally strong in his store at θ. Therefore, there cannot be an equilibrium in which $\bar{z} < (\underline{z})^{\frac{\alpha_b}{1-\alpha_1}}$ because for such values, both sellers believe that the buyer is locally weak in both stores at θ, and thus the buyer will visit store 2 first.[17]

On the other hand, if $A_1 > 0$, then there are three threshold for each seller $i \in \{1, 2\}$ that we need to consider. These are $X_1^i(z_1^i)^{\frac{\alpha_b}{1-\alpha_1}}$, $(z_2^i)^{\frac{\alpha_b}{1-\alpha_2}}$ and X_1^i. For sufficiently small values of $\bar{z}$, more formally if $\bar{z} < A_1$, then the second smallest threshold among these six is $X_1^1(z_1^1)^{\frac{\alpha_b}{1-\alpha_1}}$ and as long as $\bar{z}$ is smaller than this, there cannot be any equilibrium with the similar reasoning I use in the previous paragraph. Hence, if θ is supported as an equilibrium, then seller 1 must believe that the buyer is strong in store 1 at θ.

The arguments in the last two paragraphs along with Proposition 3 imply that in equilibrium we must have that $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$, and hence as $\underline{z}$ converges to $\bar{z}$, or as both converge to zero at the same rate, we must have that $\alpha_b \geq \frac{1}{2}$ in the limit. Thus, I present the following result with no formal proof.

Proposition 14. *Take any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the pricing game G and satisfies Assumptions 1 and 3. The vector of prices α_θ is an equilibrium outcome of G as $\underline{z}$ converges to $\bar{z}$ or as $\underline{z}$ and $\bar{z}$ converge to zero at the same rate whenever we have $\alpha_b \geq \frac{1}{2}$.*

The following proposition considers the θ's such that each seller posts his price sufficiently close to α_b, so it is not worth it for the buyer to leave a store and travel to the other one to play the concession game with the second seller.

Proposition 15. *The vector of primitives θ satisfying $\alpha_b + \alpha_2 \geq 1$ and $A_1 \leq 0$ can be supported as an equilibrium of the pricing game G if and only if the following inequalities hold:*

1) $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$,

2) $\underline{z} < (\bar{z})^{\frac{\alpha_b-\alpha_1+\alpha_2}{\alpha_b}} \left(\frac{\alpha_2-\alpha_b}{\alpha_1-\alpha_b}\right)$, *and*

[17] Therefore, by Proposition 2, θ can not be supported as an equilibrium.

3) $\underline{z} < (\bar{z})^{\frac{\alpha_b+\alpha_1-\alpha_2}{\alpha_b}} \left(\frac{\alpha_1-\alpha_b}{\alpha_2-\alpha_b}\right)$

Proposition 16. *The vector of primitives $\theta = \langle z_\theta, \alpha_\theta, \delta\rangle$ satisfying $\alpha_b + \alpha_2 > 1$ and $0 < A_1$ can be supported as an equilibrium of the pricing game G for sufficiently small values of z_θ if the following inequalities hold:*

1) $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$,

2) $\underline{z} < \frac{(1-\delta)(1-\alpha_b)}{(1-\delta\bar{z})(\alpha_2-\alpha_b)}(\bar{z})^{\frac{1+\alpha_b-\alpha_2}{\alpha_b}}$, *and*

3) $\bar{z} > \left[\frac{\underline{z}(1-\delta)(1-\alpha_b)}{(1-\delta\underline{z})(\alpha_2-\alpha_b)}\right]^{\frac{\alpha_b}{\alpha_b+\alpha_2-1}}$

Following two results characterize the set of θ's, satisfying Assumptions 1 and 3, that can be supported as an equilibrium of the price competition game G in the limiting case of the prior beliefs z_θ. I defer their proofs to Appendix.

Proposition 17. *There exists no $\theta = \langle z_\theta, \alpha_\theta, \delta\rangle$ that can be supported as an equilibrium of the pricing game G such that the vector of prices α_θ is an equilibrium outcome of G as $\underline{z}$ converges to $\bar{z}$.*

Proposition 18. *There exists no $\theta = \langle z_\theta, \alpha_\theta, \delta\rangle$ that can be supported as an equilibrium of the pricing game G such that the vector of prices α_θ is an equilibrium outcome of G as $\underline{z}$ and $\bar{z}$ converge to zero at the same rate.*

Expected Payoff to the Sellers

I would like to remark that the results in this section imply that if a vector of primitives θ is supported as an equilibrium of the price competition game G, then we must have $\bar{z} > (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$. That is, each seller believes that the buyer is the strong player in his store, independent of the price he posts in stage 1. Therefore, the expected payoff to the sellers is uniquely determined and is equal to α_b, although a large set of prices can be supported in equilibrium when we perturb the higher order believes. The following corollary summarizes this point more formally.

Corollary 2. *Take any θ that can be supported as an equilibrium of the price competition game G. Then, each seller believes that his expected payoff (evaluated at time 0) in the game G is equal to α_b.*

4. Price Search Game

Consider the market set up (where there are two sellers and a buyer who freely moves back and forth between the stores and negotiates with the sellers one at a time) and the commitment types that I consider in Section 2. Here, I aim to answer the same question I asked in previous sections with one major difference: What prices (behavioral demands) the sellers post in equilibrium given that the buyer is allowed to haggle with them, but the sellers cannot announce their behavioral demands before the buyer visits their stores. Therefore, with a slight difference in the game that I analyze in Sections 2 and 3, in this section I suppose that the buyer cannot learn the sellers behavioral demands before visiting the stores. Thus, the game I consider next has the same spirit as the Diamond's search model.

The Price Search Game G_s**:** Before the game starts, nature makes its move and determines whether a player is a commitment type or not, and then the players privately learn their types. For simplicity, I assume that the demand of the commitment type buyer is some $\alpha_b \in (0,1)$ that is commonly known.

The price search game G_s consists of two stages. In the first stage, the buyer decides which store to visit first. Sellers do not have to make any decision regarding their behavioral demands in the first stage. Upon the buyer's arrival at store $i \in \{1,2\}$ at time 0 (stage 2 of the game), seller i chooses and announces his behavioral demand $\alpha_i \in [0,1)$, and the buyer and seller i immediately begin to play the concession game.[18] The buyer can leave the store he is currently visiting at anytime he wants. Upon the buyer's arrival at the second store, $j \in \{1,2\}$, $j \neq i$, the second seller decides and announces his behavioral demand, $\alpha_j \in [0,1)$ and they immediately begin to play the concession game.

The buyer can move back and forth between the sellers as much as he wants. If at any time the buyer decides to visit a seller that he visited before, for example seller 1, the buyer and seller 1 begin to play the concession game upon the buyer's arrival at store 1: Seller 1 either accepts the buyer's behavioral demand α_b or wait that the buyer accepts

[18]Without loss of generality we can restrict the seller i's strategy to announcing $\alpha_i \geq \alpha_b$ because in equilibrium seller i never announces a demand (strictly) lower than α_b since he has the option of ending the game by accepting the buyer's behavioral demand upon the buyer's arrival at his store. So, I assume that choosing the behavioral demand $\alpha_i \leq \alpha_b$ marks the completion of the game at time 0.

the behavioral demand that seller 1 has announced when the buyer visited store 1 for the first time. That is, I restrict the sellers' strategies so that they are not allowed to change their behavioral demands when the buyer revisits their stores.[19]

Concession of the buyer or seller $i \in \{1, 2\}$, while the buyer is in store i, marks the completion of the game. In case of simultaneous concession, surplus is split equally.[20] Throughout the game, each seller can perfectly observe the buyer's moves and the other seller's announcements. Thus, the players' actual types are the only source of uncertainty in the game. After leaving store i and traveling part way to store j, the buyer could, if he wished, turn back and enter store i again.

***Strategies in The Price Search Game*:** The strategies of the players in the game G_s are similar to the one that I define in Section 2 with slight modifications. The buyer's strategy has two parts. The first part σ_b determines the buyer's location as a function of history, but the buyer cannot condition his strategy σ_b on a seller's behavioral demand until the buyer visits the seller's store. The second part of the buyer's strategy is a pair of increasing and continuous distribution functions (concession game strategies) as given in Section 2.

Similarly, the sellers' concession game strategies are increasing and continuous distribution functions as presented in Section 2. To make the subsequent analysis more tractable, I assume that each seller's strategy of choosing a behavioral demand (upon the buyer's first visit to his store) is independent of the time that the buyer arrives at his store. However, this strategy depends on whether the buyer visits his store before or after visiting the other seller. That is, when the buyer visits, for example, store 1 first, seller 1 is allowed to ask a behavioral demand different than the one he would ask when the buyer visits his store after visiting seller 2. But, if the buyer visits store 2 first, then

[19]This restriction directly follows from the convergence result of the discrete-time bargaining games to the continuous-time war of attrition game when these simple behavioral (commitment) types exist. If a seller asks a price different than his initial announcement, the buyer will conclude that the buyer is not a commitment type. Hence, if the buyer insists on staying at the seller's store and asking his behavioral demand for long enough, then in any sequential equilibrium following this subgame, the seller gets a payoff arbitrarily close to α_b as the frequency between the offers are sufficiently close. Hence, in equilibrium of the continuous-time bargaining problem, the sellers prefer to accept the buyer's behavioral demand instead of revealing their rationality by offering a different price. For more detailed discussion of this convergence result, please refer to Section 4 of Ozyurt (2009).

[20]This assumption is not crucial for the results since this event will occur with zero probability in equilibrium.

seller 1's behavioral demand must be the same independent of the time that the buyer arrives at store 1.

Given that in equilibrium the buyer's timing and location decisions are summarized in σ_b, and the sellers choose α_1, α_2 upon the buyer's arrival at their store, Facts A and B characterize the equilibrium strategies of the players in the second stage concession games (of the price search game G_s.) Therefore in equilibrium, the buyer does not visit a store more than once if both sellers ask the same demand, or else the buyer may visit only the low demand store twice.[21] Likewise, the players' expected payoffs are no different than the ones derived in the previous sections.

Let $\theta = \langle z_1, z_2, z_b, \alpha_1, \alpha_2, \alpha_b, \delta \rangle$, in short, denote the vector of primitives of the price search game G_s, where $\alpha_i \geq \alpha_b$ for $i = 1, 2$, $\alpha_b \in (0, 1)$ and without loss of generality $\alpha_1 \geq \alpha_2$. For any θ given, let $\alpha_\theta = \langle \alpha_1, \alpha_2, \alpha_b \rangle$ and $z_\theta = \langle z_1, z_2, z_b \rangle$ denote the vector of behavioral demands and initial priors, respectively. I restrict our analysis on θ where z_θ is sufficiently small, more specifically I assume that θ satisfies the modified version of Assumption 1

Assumption 4: For each $i, j \in \{1, 2\}$ where $j \neq i$; *(a)* $z_j < A_i$ (whenever $A_i = \frac{1-\alpha_b - \frac{1-\alpha_i}{\delta}}{\alpha_j - \alpha_b} > 0$), and *(b)* $z_j < \alpha_b$.

Therefore, for any given θ and seller i, if $A_i > 0$, then we have $0 < X_i < 1$.

The vector of primitives $\theta = \langle z_\theta, \alpha_1, \alpha_2, \alpha_b, \delta \rangle$ can be supported as an equilibrium of the price search game G_s means that there exists a sequential equilibrium of the game G_s in which the sellers choose their posted prices α_1 and α_2 upon the buyer's arrival at their stores given z_θ, α_b and δ.

Suppose that θ can be supported as an equilibrium of the price search game. Then, according to the unique equilibrium strategies of the second stage concession games, at most one player (either the buyer or the seller) can make an initial probabilistic concession. A player is strong if he receives this probabilistic gift from his opponent and weak if he does not. Thus, in light of Facts A and B, we retain the definitions of being weak, locally weak and distance-corrected strong as we defined in Section 3 with

[21] For more elaborate discussion of this claim, see Ozyurt (2009).

only modification that $z_i^j = z_i$ for all $i \in \{1, 2, b\}$ and $j \in \{1, 2\}$ since in this section we do not assume perturbation in higher order beliefs as we did in Section 3.

Equilibrium prices of the Price Search Game

I start with an important result which is a re-statement of the Proposition 3 in Section 3 under the appropriate notation. Since a similar proof can be provided by slight modifications on the notation, I do not provide a formal proof.

Proposition 19. *Suppose that the vector of primitives θ can be supported as an equilibrium of the price search game G_s and seller $i \in \{1, 2\}$ is weak in his store at θ, or his expected payoff in stage 2 if the buyer visits his store first is α_b. Then it must be true that for all $\alpha_i \geq \alpha_b$, seller i is weak in his store at θ', which is equivalent to θ except α_i. That is, $z_b \geq z_i^{\frac{\alpha_b}{1-\alpha_b}}$.*

I start by analyzing the case where the behavioral demands are farther apart relative to the stores. That is, suppose that θ can be supported as an equilibrium of the price search game G_s where $\alpha_1 > \alpha_2$ and $1 - \alpha_1 < \delta(1 - \alpha_2)$.

Consider the subgame where the buyer visits seller 1 first. According to the equilibrium strategies (Fact B), seller 1 immediately accepts the buyer's behavioral demand and finishes the game if he is rational. Otherwise, the buyer leaves store 1 (immediately) knowing that seller 1 is the commitment type and he never returns to this store. The buyer leaves store 1 upon his arrival regardless of being (locally) weak or strong in store 2 at θ. If seller 2 is locally strong in store 2 at θ, i.e. $z_b > z_2^{\alpha_b/1-\alpha_2}$, then seller 2 will (profitably) deviate to some price $\alpha_2' > \alpha_2$ as long as the inequality $\delta(1 - \alpha_1) \geq 1 - \alpha_2'$ holds.[22] Thus, seller 2 must be weak in his store at θ in order to ensure that θ can be supported as an equilibrium. More formally:

Proposition 20. *Suppose that the vector of primitives $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$, satisfying $1 - \alpha_1 < \delta(1 - \alpha_2)$, can be supported as an equilibrium of the price search game G_s. Then for sufficiently small values of z_θ, seller 2 must be locally weak in his store at θ, i.e. $z_b \geq z_2^{\frac{\alpha_b}{1-\alpha_2}}$.*

[22] Otherwise, the rational buyer prefers to leave store 2 and go to store 1 to accept seller 1's behavioral demand α_1.

Note that this is not a characterization result. It just shows that θ can be supported as an equilibrium for small values of z_θ if seller 2 is weak in his own store at θ. Clearly, this condition is not sufficient to ensure that θ can be supported as an equilibrium since there might be a price $\alpha_2' < \alpha_2$ such that seller 2 is strong in his store at θ', which is the same with θ except α_2'. However, along with Proposition 20, we attain the characterization result as stated next.

Proposition 21. *The vector of primitives $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$, satisfying $1 - \alpha_1 < \delta(1 - \alpha_2)$, can be supported as an equilibrium of the price search game G_s for sufficiently small values of z_θ if and only if $z_b \geq z_i^{\frac{\alpha_b}{1-\alpha_b}}$ for $i = 1, 2$.*

Proposition 21 shows that θ, in which the behavioral demands are far apart relative to the location of the sellers, can be supported as an equilibrium of the price search game if no seller has an option of asking a behavioral demand (above α_b) to become locally strong in his own store.

Next result considers the vector of primitives θ such that the behavioral demands are closer relative to the location of the stores. Recall that according to Fact A, the buyer will stay a positive amount of time in either store (whenever the buyer is not distance-corrected strong relative to either seller).

Proposition 22. *Suppose that the vector of primitives $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$, satisfying $1 - \alpha_1 \geq \delta(1 - \alpha_2)$, can be supported as an equilibrium of the price search game G_s. Then, for sufficiently small values of z_θ, and for any seller $i, j \in \{1, 2\}$ where $j \neq i$, (a) the buyer must be distance-corrected strong relative to seller j at θ if $A_i > 0$, and (b) if $A_i \leq 0$, then seller i must be locally weak in his store at θ.*

Note that Proposition 22 includes the cases where both sellers ask the same demand. If θ can be supported as an equilibrium, then the buyer must be distance corrected strong relative to each seller, i.e. $z_b \geq X_i$ for $i = 1, 2$, given that $A_i > 0$. If $A_i \leq 0$, then seller i must be locally weak in his store, i.e. $z_b \geq z_i^{\frac{\alpha_b}{1-\alpha_i}}$. Hence, each seller's expected payoff following the subgame where the buyer visits his store first is exactly α_b. Moreover, according to Fact A, if both sellers are rational (not commitment type), then the price search game finishes at time 0 upon the buyer's arrival at the first store by concession of the seller.

Proposition 23. *The vector of primitives $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$, satisfying $1 - \alpha_1 \geq \delta(1 - \alpha_2)$, can be supported as an equilibrium of the price search game G_s for sufficiently small values of z_θ if and only if (a) the buyer is distance-corrected strong relative to both sellers at θ if applicable, i.e. $z_b \geq X_i$ for $i = 1, 2$ given that $0 < A_i$, and (b) $z_b \geq (z_i)^{\frac{\alpha_b}{1-\alpha_b}}$ for $i = 1, 2$.*

In light of Propositions 21 and 23, the next result ensures that for any $\alpha_b \in (0, 1)$ given, all pairs of behavioral demands (α_1, α_2) where $\alpha_i \geq \alpha_b$ can be supported as an equilibrium of the price search game G_s with small values of initial priors.

Corollary 3. *For any δ, α_b and pair of behavioral demands (α_1, α_2) where $\alpha_i \geq \alpha_b$ for $i = 1, 2$, there exists z_1, z_2 and z_b small enough such that $\theta = \langle \alpha_b, \alpha_1, \alpha_2, z_b, z_1, z_2, \delta \rangle$ can be supported as an equilibrium of the price search game G_s whenever $z_b \geq X_i$ (given that $0 < A_i$) and $z_b \geq (z_i)^{\frac{\alpha_b}{1-\alpha_b}}$ for $i = 1, 2$.*

Take any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the price search game G_s. By the statement "the vector of prices α_θ is an equilibrium outcome of G_s as z_θ converges to $\mathbf{0} = (0, 0, 0)$ at the same rate", I mean the following. Take any sequence of prior beliefs $z_\theta^n = (z_1^n, z_2^n, z_b^n)$ (where $z_\theta^0 = z_\theta$) converging to $\mathbf{0}$, i.e. $\forall \epsilon > 0$, $\exists M > 0$ such that $|z_\theta^m - \mathbf{0}| < \epsilon$, $\forall m > M$, such that for all $i \in \{1, 2\}$ and $n \geq 0$, $z_b^n = K_i z_i^n$ for some $K_i > 0$. Then, the vector of primitives $\theta_m = \langle z_\theta^m, \alpha_\theta, \delta \rangle$ can be supported as an equilibrium of the price search game G_s for all $m > M$. The following result characterizes the set of equilibrium prices that can be supported as an equilibrium of the price search game G_s when the initial prior beliefs about the commitment types vanish.

Corollary 4. *Take any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$ that can be supported as an equilibrium of the price search game G_s. The vector of prices α_θ is an equilibrium outcome of G_s as z_θ converges to $\mathbf{0}$ at the same rate if and only if $\alpha_i \geq \alpha_b \geq \frac{1}{2}$ for $i = 1, 2$.*

Corollary 5. *Take any θ that can be supported as an equilibrium of the price competition game G_s where z_θ is sufficiently small. Then, the expected payoff to each seller when the buyer visits his store first is α_b, and the expected payoff to the buyer is less than $1 - \alpha_b$, but converging to this as initial priors converge to 0 at the same rate.*[23]

Proof. Directly follows from Propositions 21 and 23 □

[23]Expected payoffs are evaluated at time 0

For any θ that can be supported as an equilibrium of the price competition game G_s where z_θ is sufficiently small, the expected payoff to the buyer (evaluated at time 0) is strictly less than $1-\alpha_b$ whenever $\alpha_i > \alpha_b$ for each $i \in \{1,2\}$. It is exactly equal to $1-\alpha_b$ only when $\alpha_i = \alpha_b$ for some seller i.

5. Competition with Posted or Bargain Prices

This section aims to understand whether each seller wants to announce his price (behavioral demand) to the buyer before he makes his decision regarding which store to visit first, or if he prefers to reveal it after the buyer's arrival at his own store. In an environment where there is a flow of consumers entering the market, posting a price and making it public naturally reduces the seller's ability to price discriminate among the buyers with different demands. For this reason, it is rather plausible to ask this question in a "dynamic" context where the sellers are facing more than one potential buyer. Given this situation, I nevertheless would like to answer this question within this "static" model that I analyze in this paper because it is unclear which strategy dominates the other even in this "*simpler*" context.

In this section, I consider the *pricing game* G_p which is a combination of the price search game G_s and the price competition game G. The game G_p consists of two stages. In the first stage, each seller simultaneously decides whether to reveal his behavioral demand (by advertising the posted price) to the buyer before he visits any store or not. The seller who chooses to announce his behavioral demand picks his price and then, before moving to the second stage, the buyer observes the posted prices (if any seller chooses to reveal) and decides which store to go to first.

At time 0, the beginning of the second stage, the buyer arrives at the store that he chooses to visit first; for example, store 1. If seller 1 announces his posted price in stage 1, the buyer and seller 1 immediately begin to play the concession game upon the buyer's arrival at store 1, where the buyer either accepts the seller's posted price α_1 or waits to be conceded by the seller. However, if seller 1 does not reveal his demand in stage 1, then upon the buyer's arrival at his store, seller 1 immediately announces his demand and begins to play the concession game with the buyer. If seller 1 chooses to announce his demand such that $\alpha_1 \leq \alpha_b$, this ends the game by the buyer's acceptance of the seller

1's demand. If seller 1 picks his demand so that $\alpha_1 > \alpha_b$, then the buyer and seller 1 play the concession game until the time that the buyer leaves the store or either one of the players concedes.

Upon the buyer's arrival at store 2 after leaving store 1, the buyer and seller 2 immediately begin to play the concession game given that seller 2 posts his price in stage 1. Otherwise, seller 2 immediately reveals his demand and begins to play the concession game with the buyer. The buyer can leave the stores at any time, and visit them as much as he wants.

Therefore, if both sellers choose to announce their prices in stage 1, the rest of the game G_p is exactly the same as the price competition game G. If no seller chooses to post his price in stage 1, then it is identical to the price search game G_s. A seller's strategy in the game G_p is the same as the price competition game G if the seller announces his demand in stage 1, and the same as the price search game G_s if he does not. On the other hand, the buyer can condition his timing and location decisions on a seller's posted price (behavioral demand) before visiting this seller's store only if the seller announces his demand in stage 1. Therefore, all the players' equilibrium strategies in the concession games as well as the buyer's optimal departure time are as summarized in Facts A and B.

Suppose that the vector of primitives θ where $\alpha_i \in [0, 1)$ for $i = 1, 2$ satisfies Assumption 4, and can be supported as an equilibrium of the price posting game G_p. First note that there is no equilibrium supporting θ (for sufficiently small values of z_θ) in which both sellers choose not to reveal their demands in stage 1. Suppose for a contradiction that there exists one such equilibrium. So, according to Corollary 5, the expected payoff (evaluated at time 0) to each seller when the buyer visits his store first is α_b, and to the buyer is less than $1 - \alpha_b$.

Therefore, the seller (that is not visited by the buyer with probability one at time 0) will deviate and post his behavioral demand as, for example, α_b in stage 1, and he will ensure that the buyer visits his store first at time 0 with certainty.[24] We know that this deviation is profitable because in equilibrium for small values of the initial prior beliefs, each seller prefers to have the buyer visiting his store first at time 0 with certainty.

[24] If the buyer's expected payoff of visiting other store is exactly equal to $1 - \alpha_b$, then the seller can post a price slightly below α_b to make the buyer visit his store first.

Similarly, there is no equilibrium in which one seller announces his demand in stage 1 and the other does not. Suppose for a contradiction that there is an equilibrium of this sort (suppose, without loss of generality, that seller 1 announces his demand in stage 1, but seller 2 does not). In an equilibrium that supports θ, if the buyer does not visit seller 1 first at time 0 with certainty, then seller 1 would deviate and announce his demand as α_b (or slightly less than α_b) in stage 1 so that the buyer visits his store first in stage 2.[25] Thus, by such unilateral deviation, seller 1 guarantees himself the payoff (if not exactly equal to, arbitrarily close to) α_b in stage 2, and this deviation is profitable because seller 1 prefers to be the first store the buyer visits in stage 2 since θ satisfies Assumption 4, i.e. $z_2 < \alpha_b$.

On the other hand, if the buyer does not visit seller 2 first in stage 2 with certainty, then seller 2 would deviate and announce his demand as α_b in stage 1 (given that seller 1's posted price in stage 1 is strictly higher than α_b), and thus guarantees himself a higher payoff. This unilateral deviation of seller 2 is profitable since θ satisfies assumption 4. If however, seller 1's posted price is positive but less than or equal to α_b, then seller 2 would prefer to price undercut and get a positive payoff by announcing a demand in stage 1, that is slightly below α_1.[26] Hence, arguments in the last two paragraphs show that there is no equilibrium in which only one seller announces his demand (and ask $\alpha_i > 0$) in stage 1.

Hence, there can be equilibrium only when (i) both sellers announce their demands in stage 1, or (ii) both sellers choose 0 as their demands and at least one seller announces it in stage 1. However, we know that if each seller announces his behavioral demand in stage 1, then the pricing game becomes identical to the price competition game G, and according to the Proposition 1, the equilibrium of this game is unique (for small values of the prior beliefs on players' commitment types) in which both sellers choose to post the price of 0.[27] The next result summarizes the discussion we make about the equilibrium of the pricing game G_p.

[25]Note that in an equilibrium supporting θ, the buyer cannot have payoff strictly more than $1 - \alpha_b$ in stage 2 if he first visits the store whose seller does not announce his price in stage 1. This is true because this seller always has an option of announcing his demand as α_b upon the buyer's arrival in his store and finishing the game with the payoff of α_b.

[26]Note that, in equilibrium if the buyer visits store 1 first and his posted price in stage 1 is less than or equal to α_b, then the expected payoff of seller 2 in the game is zero.

[27]Recall that we can still have Proposition 1 when we assume that each player's initial prior is different.

Proposition 24. *For sufficiently small values of the initial priors about the players' commitment types, the equilibrium of the pricing game G_p is that both sellers choose their demands as* 0 *and at least one seller announces his behavioral demand in stage 1.*

When the sellers have the opportunity to announce their prices in stage 1, each seller can guarantee to be the first store the buyer visits at time 0 by undercutting his opponent's price and announcing his demand (for example below α_b) in stage 1. Therefore, in equilibrium of the pricing game G_p where both sellers demand strictly above zero, we must have that each seller believes that the buyer visits his store first in stage 2 with certainty (Proposition 2). Therefore, the uniqueness result Proposition 24 suggests is not robust to perturbations in higher order beliefs as we investigate in Section 3.

The arguments in this section are correct under the assumption that both sellers agree on the content of the knowledge that they believe is common. However, if their beliefs do not coincide as in the analysis we pursue in Section 3, we can support a continuum of prices $\alpha_i \geq 1/2$ in equilibrium whereas each seller believes that the buyer will visit his store in stage 2 with certainty. In this case, the sellers do not have incentive to announce their demands in stage 1.

As a result, if the sellers' priors are not common, which could be the case when the sellers do not have enough experience about their opponents, then we may expect them not to announce their prices to the buyer before he makes his visits to their stores. However, if the sellers agree on the prior probabilities of all the players' commitment types (which is very possible especially when the two sellers operate in the market for a long time and have the chance to learn about one another through their experiences) we expect them to announce their prices before the buyer makes his visits to the stores and post their marginal costs (zero). In the latter case, if the sellers can commit themselves to not announce their demands before the buyer visits their stores, then they can (weakly) improve their payoffs. As a result, if the sellers, who may know each other quite well, for example, through repeating interactions, do not announce their prices before the buyer's arrival at their stores, then the model identifies this act as a collusive behavior.

Costly Announcement

Proposition 24 relies on the assumption that the sellers can announce their prices in stage 1 with no additional cost. However, in many circumstances, advertising posted prices and making the buyers aware of the sellers' demand do not necessarily have negligible cost. If each seller i is subject to an additional cost of announcing his demand in stage 1, $c_i > 0$, then $(0, 0)$ pricing is not the equilibrium outcome of the pricing game G_p.

As long as $c_i < \alpha_b$ for each seller i, then in equilibrium both sellers are willing to announce their demands in stage 1 and pay that cost. Therefore, the equilibrium outcome of the pricing game is identical to the equilibrium outcome of the Bertrand competition where each seller i's marginal cost is c_i.

However, if $c_i = \alpha_b$ for each seller i, then no seller is willing to post his price below α_b. So, there is an equilibrium where both sellers post their prices in stage 1 as α_b.[28] There is another equilibrium where neither seller announces his price in stage 1 and both sellers demand α_b upon the buyer's arrival at their stores. However, there cannot be an equilibrium where no seller announces his price in stage 1 and both demand strictly more than α_b when the buyer arrives at their stores.

6. Optimal Demand of the Buyer

Consider the price competition game G and price search game G_s. Now, I modify these two games by adding another stage (stage 0) where the buyer chooses his behavioral demand $\alpha_b \in (0, 1)$ and reveals his choice to the sellers. Therefore, regardless of the game the players play following stage 0, both sellers know the buyer's demand α_b in stage 1. Then, the obvious question is what the buyer would choose in equilibrium in stage 0. Clearly, the answer depends on which game (G_p or G) the players play following the buyer's decision in stage 0.

Suppose first that the players play the price search game G_s following stage 0. Recall that according to Corollary 4, for any $\theta = \langle z_\theta, \alpha_\theta, \delta \rangle$, the vector of prices α_θ can be supported as an equilibrium of the price search game G_s as z_θ converges to $\mathbf{0}$ at the same

[28]Moreover, the buyer believes that if one seller deviates and does not post his price, this seller asks a demand no less than α_b when the buyer visits his store.

rate if and only if $\alpha_i \geq \alpha_b \geq \frac{1}{2}$ for $i = 1, 2$. Therefore, if the buyer chooses $\alpha_b < 1/2$ in stage 0, then for sufficiently small values of the initial priors no couple of demands (α_1, α_2) can be supported as an equilibrium of the price search game G_s.

This is true because when α_b takes values strictly less than 1/2, the seller (that the buyer visits first in stage 2) can find a price that is sufficiently close to α_b so that by asking this price, the seller can make himself locally strong in his store and increase his payoff through such a unilateral deviation. Hence in equilibrium, it must be the case that the buyer chooses his demand α_b such that $\alpha_b \geq 1/2$.

Therefore, suppose that the buyer chooses his demand strictly above 1/2 in equilibrium. According to Corollary 3, if the buyer visits store i first, then the buyer's expected payoff v_b^i evaluated at time 0 is either

$$\left\{(1 - z_i) + \delta z_i \left(1 - \frac{z_j}{z_b^{(1-\alpha_j)/\alpha_b}}\right)\right\}(1 - \alpha_b) + \frac{\delta z_1 z_2}{z_b^{(1-\alpha_j)/\alpha_b}}(1 - \alpha_j) \tag{12}$$

whenever $A_i > 0$, or else

$$(1 - \frac{z_i}{z_b^{(1-\alpha_i)/\alpha_b}})(1 - \alpha_b) + \frac{z_i}{z_b^{(1-\alpha_i)/\alpha_b}}(1 - \alpha_i) \tag{13}$$

Remark that whether it is evaluated by (12) or (13), v_b^i is a decreasing function of α_b for sufficiently small values of the initial priors: If the buyer's expected payoff is as given by (12), then

$$\frac{\partial v_b^i}{\partial \alpha_b} = -(1 - z_i(1 - \delta)) + \frac{\delta z_1 z_2}{z_b^{(1-\alpha_j)/\alpha_b}} - \frac{\delta(\alpha_j - \alpha_b)(1 - \alpha_j)\ln(z_b) z_1 z_2}{\alpha_b^2 z_b^{(1-\alpha_j)/\alpha_b}} \tag{14}$$

Similarly, if v_b^i is as given by (13), then

$$\frac{\partial v_b^i}{\partial \alpha_b} = -1 + \frac{z_i}{z_b^{(1-\alpha_i)/\alpha_b}} - \frac{(\alpha_i - \alpha_b) z_i (1 - \alpha_i)\ln(z_b)}{\alpha_b^2 z_b^{(1-\alpha_j)/\alpha_b}} \tag{15}$$

In either case, $\frac{\partial v_b^i}{\partial \alpha_b}$ is negative for sufficiently small values of the initial priors. To see this point suppose that for each seller i, $z_i = K_i z_b$ for some $K_i > 0$. Then, as z_1, z_2 and z_b converge to zero at the same rate (i.e. any sequence of $\{(z_1^n, z_2^n, z_b^n)\}$ converging to $\mathbf{0}$ such that for each n and i, $z_i^n = K_i z_b^n$), the second and the third terms of $\frac{\partial v_b^i}{\partial \alpha_b}$ in (14) and (15) converge to zero because $\alpha_b \geq 1/2$, and thus $\frac{1-\alpha_j}{\alpha_b} < 1$.

As a result, in equilibrium the buyer uniquely determines his demand α_b in stage 0 by 1/2 (for sufficiently small values of the initial priors). Hence, in equilibrium as the prior beliefs converge to zero (at the same rate) the share of the buyer and the share of the seller (that the buyer visits first in stage 2) are equal to 1/2. Recall that 1/2 is the relative ratio of the players' time discounts, when their interest rates are normalized to one and is equal to the buyer's share in the "standard" bargaining game (Rubinstein's alternating offer bargaining protocol with no commitment types) between the buyer and a single seller in the limit where the players can make increasingly frequent offers.

This result suggests that the buyer is not better off with the existence of an additional seller who is selling the same item in the market place if the sellers do not announce their demands before the buyer decides which store to visit first. This result is independent of the search cost (δ) arising due to the time required to travel from one store into the other. Also, the sellers are not worse off with the existence of the additional seller as long as they believe that in equilibrium the buyer will visit their stores first.

However, if the players play the price competition game G following stage 0, then independent of the buyer's choice in this stage, the vector of posted prices $(0, 0)$ can be supported as an equilibrium outcome in stage 1. Furthermore, it is the unique equilibrium outcome of the price competition game when the sellers agree on the knowledge that they believe is common (Section 2). Yet, as I show in Section 3, a slightest perturbation in higher order beliefs ensures that a large set of prices (above 1/2) can be supported as an equilibrium outcome of the price competition game in stage 1.

Therefore, if the buyer is sure that the sellers' beliefs coincide (as in Section 2), then he will be indifferent among all the possible demands he could choose in stage 0. However, if the buyer believes that the sellers may hold different opinions about the content of the knowledge that they believe is common, then he would prefer to choose his demand strictly less than 1/2 and thus by doing so, he ensures that the sellers will uniquely determine their prices as zero in equilibrium.

7. Conclusion

In this paper, I investigate the properties of the equilibrium outcomes observed in markets where the sellers announce their demands, but do not necessarily commit, while the buyers can move back and forth between the sellers to negotiate for a better deal.

Furthermore, I characterize the optimal strategies of the market participants and identify the conditions when it is optimal for the sellers to compete with posted or bargain prices.

If the initial priors about the players' commitment types are sufficiently small and the sellers must announce their prices before the buyer's arrival at their stores, the unique equilibrium outcome is in the spirit of Bertrand, i.e. each seller posts his marginal cost of zero. However, this uniqueness result is not robust to perturbation on higher order beliefs about the initial priors.

On the other hand, if the buyer can learn the sellers' prices upon his arrival to their stores, then a continuum of prices can be supported in equilibrium. However, independent of the sellers' announcements, the payoff to the seller (visited by the buyer first) is always the same, and the buyer's payoff approaches a unique limit as the initial priors vanish.

If the sellers agree on the prior probabilities of the players' commitment types, then we expect them to announce their prices before the buyer makes his visits to the stores, and thus each seller posts his marginal cost in equilibrium. However, if the sellers' priors are not common, then they do not have incentive to announce their prices.

Therefore, in the former case where the sellers know each other quite well, for example, through repeating interactions, if the sellers can commit themselves not to announce their demands before the buyer visits their stores, then they can (weakly) improve their payoffs. Accordingly, if these sellers do not announce their prices before the buyer's arrival, then the model identifies this as collusive behavior.

Appendix

Proof of Proposition 1. First, note that posting the price of 0 is an equilibrium strategy for both sellers in stage 1 for all values of z. This is true because the buyer can end the game by accepting the seller's posted price upon his arrival at the store whenever the seller's posted price is no more than α_b. Therefore, within the set of prices $\alpha_i \leq \alpha_b$ for any $i \in \{1, 2\}$, undercutting the opponent's price is a dominant strategy for the sellers, and thus $(0, 0)$ is the unique equilibrium outcome in stage 1.

Regarding the uniqueness of $(0, 0)$, suppose for a contradiction that there exists some $\alpha_i > \alpha_b$ for $i = 1, 2$ that can be supported as an equilibrium of the game G. Suppose without loss of generality that in this equilibrium the buyer chooses to visit store 1 first. Also suppose that the buyer visits store 1 with certainty at time 0. Then according to this strategy, the second seller's expected payoff (in the game G) must be less than or equal to $\delta z \alpha_2$. However, by posting the price of α_b in stage 1, seller 2 would guarantee the buyers visit to his store first at time 0 and a payoff of $\alpha_b > \delta z \alpha_2$. For sufficiently low values of z, this is a profitable deviation, contradicting the assumption that (α_1, α_2) can be supported as an equilibrium. So, if there is an equilibrium (α_1, α_2) where $\alpha_i > \alpha_b$ for $i = 1, 2$, then the buyer should visit each store at time 0 with some positive probability.

Now suppose that the buyer visits each store i at time 0 with a positive probability ($p_i \in (0, 1)$) according to the equilibrium strategy. Also notice that in equilibrium, both sellers' expected payoffs in stage 2 must be strictly higher than α_b (evaluated at the beginning of the second stage). Suppose for a contradiction that this is not the case and without loss of generality seller 1's continuation payoff is less than or equal to α_b. Then, since in equilibrium the buyer visits store 1 with probability p_1, seller 1's expected payoff in the game G is no more than $p_i \alpha_b$. However, by posting α_b in stage 1, seller 1 guarantees the payoff of α_b (total payoff in the game), which yields the desired contradiction.[29]

Thus, in equilibrium both sellers's continuation values at time 0 are strictly higher than α_b. That means that $F_i(0) = 0$ for each seller i and thus the buyer's expected payoff of visiting store 2 at time 0 (evaluated at the beginning of stage 2) is $1 - \alpha_i$. Hence, there cannot be an equilibrium where $\alpha_1 > \alpha_2 > \alpha_b$ because in this case the buyer will select

[29]With some slight manipulation of the notation, I do not indicate the cost of delay occurring due to the buyer's travel time from his location to the stores. Introducing this cost does not alter the result.

seller 2 whose posted price is lower, contradicting that in equilibrium we have $p_i > 0$ for $i = 1, 2$.

Therefore, if there is an equilibrium outcome in stage 1 other than $(0, 0)$, we must have that $\alpha_1 = \alpha_2 = \alpha > \alpha_b$. I now claim that there cannot be an equilibrium as such. Suppose for a contradiction that there exists one.If α is very close to α_b so that $A \leq 0$ or $X = \left(\frac{z}{A}\right)^{\lambda_b/\lambda} > 1$, then we have $F_i(0) = 0$ whenever $z < z^{\lambda_b/\lambda}$. Then, seller 1 for example, can get the buyer by reducing his price slightly below α with certainty since the buyer is weak in both stores and he strictly prefers to visit the store whose posted price is lower. We can always find a profitable deviation of this sort for the sellers where each seller can increase his expected payoff (slightly) by price undercutting. However, this contradicts that α can be supported as an equilibrium. So assume that α is sufficiently far from α_b. Then, $F_i(0) = 0$ whenever $z \leq \left(\frac{z}{A}\right)^{\lambda_b/\lambda} z^{\lambda_b/\lambda}$ holds.[30]

Consider the case where seller 1, for example, decreases his posted price to $\alpha_1 = \alpha - \epsilon$ for sufficiently small ϵ in stage 1 so that we still have that $z < X_2 z^{\lambda_b^1/\lambda_1}$. Since, $\frac{\lambda_b}{\lambda} > \frac{\lambda_b}{\lambda_1}$, for sufficiently small z's $\left(\frac{z}{A_1}\right)^{\lambda_b/\lambda} z^{\lambda_b^1/\lambda_1}$ is larger than $X_2 z^{\frac{\lambda_b}{\lambda}}$, implying that the buyer is (still) weak in both stores and he attains the payoff of $1 - \alpha$ in store 2, $1 - \alpha_1$ in store 1. Since $\alpha_1 < \alpha$, the buyer will choose seller 1 (the lower posted price) to visit first at time 0. With this deviation, seller 1 increases his expected payoff slightly higher than his initial expected payoff because this time the buyer is visiting his store at time 0 with certainty.[31] This shows that there is a profitable deviation for the sellers in this case, implying that there is no equilibrium in which $\alpha_i = \alpha > \alpha_b$ for $i = 1, 2$. Hence, this contradiction yields the desired result.

Proof of Proposition 3. Suppose that θ can be supported as an equilibrium (with $\alpha_1 \geq \alpha_2$), and without loss of generality consider the subgame where the buyer visits store 1 first at time 0. Therefore, the rest of the arguments are from the point of view of seller 1. Also suppose that seller 1 believes that he is weak in his store at θ, or his expected payoff in stage 2 is α_b.[32] If seller 1's expected payoff at these prices is α_b, then the only profitable deviation would occur if there exists a price $\alpha_1' > \alpha_b$ such that when seller 1 chooses α_1' in stage 1, the buyer continues to visit his store first at time 0 and

[30] Note that $\frac{\lambda_b}{\lambda} = \frac{\alpha_b}{1-\alpha}$

[31] For the given values of p_1, α and α_b, we can find sufficiently small ϵ and z guarantees that $z \leq X_2 z^{\lambda_b^1/\lambda_1}$ and that seller 1's expected payoff increases with this deviation.

[32] The former argument is equivalent to the latter one whenever $(1 - \alpha_1) \geq \delta(1 - \alpha_2)$.

seller 1 is strong at θ' that is the same with θ except α_1'.

If, α_1' is high enough so that $1-\alpha_1' < \delta(1-\alpha_2)$, then in equilibrium the buyer leaves store 1 immediately even if he visits store 1 at time 0. Thus, seller 1 cannot improve his expected payoff with such a deviation.

On the other hand, if α_1' is closer to α_2 relative to the distance between the stores (or just the same as α_2), then seller 1 may become strong and hence have a payoff higher than α_b only if $z_b^1 < (z_1^1)^{\frac{\alpha_b}{1-\alpha_1'}} \hat{X}_1^1$ where $\hat{X}_1^1 = (\frac{z_2^1}{\hat{A}_1})^{\frac{\alpha_b}{1-\alpha_2}}$ and $\hat{A}_1 = \frac{1-\alpha_b-\frac{1-\alpha_1'}{\delta}}{\alpha_2-\alpha_b}$ given that $\hat{X}_1^1 \leq 1$ ($\hat{X}_1^1$ is the reputation that the buyer must build in order to be able to go to store 2 if his initial reputation is lower than this.)

Otherwise, that is when α_1' is very close to α_b so that $\hat{A}_1 \leq 0$ or $\hat{X}_1^1$, then the buyer will never leave this store (in equilibrium) and hence seller 1 may become strong only if $z_b^1 < (z_1^1)^{\frac{\alpha_b}{1-\alpha_1'}}$. Note that for all values of (z_1^1), $(z_1^1)^{\frac{\alpha_b}{1-\alpha_1'}}$ is bigger than $(z_1^1)^{\frac{\alpha_b}{1-\alpha_1'}} \hat{X}_1^1$ as long as $0 < \hat{X}_1^1 \leq 1$. So, if seller 1 becomes strong at α_1' for all values of z_b^1 less than the latter ($(z_1^1)^{\frac{\alpha_b}{1-\alpha_1'}} \hat{X}_1^1$), he prevails to be strong for the values of z_b^1 less than the former one. However, such a profitable deviation contradicts that θ is an equilibrium. Hence, for all $\alpha_1' > \alpha_b$ we must have that $z_b^1 \geq (z_1^1)^{\frac{\alpha_b}{1-\alpha_1'}}$. So, this implies that we must have $z_b^1 \geq (z_1^1)^{\frac{\alpha_b}{1-\alpha_b}}$ (or equivalently $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$) otherwise, we can find a price very close to α_b so that seller 1 becomes strong and hence gets a payoff higher than α_b.

Proof of Proposition 4. Since $\bar{z} > \underline{z}$, each seller believes that he is the weakest seller so, each believes that in equilibrium the buyer will choose his store to visit first with probability 1 whenever the buyer is distance-corrected strong relative to himself, (which is direct implication of Proposition 2.4 in Ozyurt (2009)), implying the condition (1).

On the other hand, since condition (1) holds, then each seller believes that in equilibrium the buyer will visit his store first and his expected payoff will be exactly α_b. Therefore, according to Proposition 3 the second condition must hold as well.

Note that the second inequality must be satisfied in equilibrium where a seller believes that his expected payoff will be α_b when the buyer visits his store first. If it does not hold, then that seller decreases his posted price in stage 1 to a price that is slightly higher than α_b, so the buyer prefers to visit his store first even if the seller is strong relative to the buyer (locally) with this price.

Without loss of generality suppose that seller 1's expected payoff is α_b in the subgame where the buyer visits his store first. So, if $\bar{z} < (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$ holds, then we can always find a price α_1' such that in equilibrium the buyer prefers to visit sore 1 first; the buyer's expected payoff in store 2 is strictly less than $1-\alpha_b$ (in stage 2), say it is $1-\alpha_b-v$. So, if seller 1 posts the price of $\alpha_1' = \alpha_b + \frac{v}{2}$ in stage 1, the buyer will choose store 1 to visit first regardless of his relative weakness in store 1. Moreover, by posting the price α_1' in stage 1, seller 1 can get a payoff slightly higher than α_b in stage 2 (this is true only if seller 1 can be locally strong in his store with this price.) These arguments show that this condition is both necessary and sufficient.

Notice that these conditions are necessary and sufficient if α is sufficiently far from α_b so that A is positive. When α is close to α_b, so $A < 0$, i.e the buyer finds it not worth to leave the store he visits first, then the only change in our conditions will be the modification in part (1). What we should have is that $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha}}$. However, condition (2) implies this inequality.

Proof of Proposition 5. Condition 1 in Proposition 4 holds for all α where $\alpha+\alpha_b \geq 1$ and sufficiently small values of $\underline{z}$. For the values of α where $\alpha+\alpha_b < 1$, condition 1 in Proposition 4 implies that $\ln \bar{z} \geq \frac{\alpha_b}{1-\alpha}(\ln \underline{z} - \ln A)$ and condition 2 yields that $\ln \bar{z} \geq \frac{\alpha_b}{1-\alpha_b} \ln \underline{z}$. These inequalities indicates that there always exists some $\bar{z}$ and $\underline{z}$ such that Proposition 4 holds for any $\alpha, \alpha_b \in (0,1)$ such that $\alpha \geq \alpha_b$.

Proof of Proposition 6. Condition 2 in Proposition 4 implies that $\alpha_b \geq \frac{|\ln \bar{z}|}{|\ln \underline{z}|+|\ln \bar{z}|}$. Taking the limit of the right hand side as $\underline{z}$ converges to $\bar{z}$ yields the inequality $\alpha_b \geq 1/2$ in part 1. It also ensures that in equilibrium, we can support α's that are close to α_b (whenever $A \leq 0$), since in the limit $\bar{z} \geq (\bar{z})^{\frac{\alpha_b}{1-\alpha}}$ holds for such values of α.[33] These arguments imply the first condition. If $A > 0$, the first inequality of Proposition 4 implies that as $\underline{z}$ converges to $\bar{z}$, we have $\left(\frac{\alpha_b}{1-\alpha} - 1\right) \geq \frac{\ln A}{\ln \bar{z}} := \bar{\epsilon}$ where $0 < \bar{\epsilon} < 1$, implying that $\alpha \geq 1 - \frac{1}{1+\bar{\epsilon}}\alpha_b$.

Proof of Proposition 7. The requirement that $\alpha_b \geq 1/2$ follows from the inequality I derived in the proof of Proposition 6. From the first inequality in Proposition 4, we have $(\frac{\alpha_b}{1-\alpha} - 1) \geq \frac{\ln K + \frac{\alpha_b}{1-\alpha}\ln A}{\ln \underline{z}_n}$ where $K = \bar{z}_n/\underline{z}_n$ for all n. The limit of the right hand side of this inequality as $\underline{z}_n \to 0$ is 0. So, in the limit we must have $\alpha_b + \alpha \geq 1$, which yields the

[33]Note that this inequality is the only requirement to make the claim of Proposition 4 true whenever A is non-positive.

desired result.

Proof of Lemma 1. Both sellers believe that the buyer's expected payoff is as given in Equation (5) in store 1 and in Equation (6) in store 2, where u_b is as given in (7).

Therefore, for seller 1, $v_b^1(1) > v_b^2(1)$ if and only if

$$\bar{z} > \left[\frac{(1-\alpha_b)\left(1 - \frac{1-\underline{z}}{1-\delta\underline{z}}\right)}{(\alpha_2 - \alpha_b)} \right]^{\frac{\alpha_b}{\alpha_2+\alpha_b-1}} \tag{16}$$

and for seller 2, $v_b^2(2) > v_b^1(2)$ if and only if

$$\underline{z} < \frac{(1-\alpha_b)\left[1 - \frac{1-\bar{z}}{1-\delta\bar{z}}\right]}{(\alpha_2 - \alpha_b)} (\bar{z})^{\frac{1-\alpha_2}{\alpha_b}} \tag{17}$$

Along with Equation(16) and (17) we must have $\bar{z} > (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$ so that θ can be supported as an equilibrium in this case. The reason why we need the first inequality can be summarized as follows. Either seller may decrease his price to something lower than α, that is very close to α_b, and by doing this he becomes locally strong in his store, and make the buyer visit his store first with certainty. By doing this the seller guarantees a payoff slightly higher than α_b. However, the first inequality ensures that such deviations are not profitable for either sellers because they will never be locally strong in their store even if they post a price slightly above α_b. This yields the desired result.

Proof of Lemma 2. In this case seller 1 believes that the buyer compares Equation (9) with (10) where u_b is as given in (7), which implies the inequality in 2. When this inequality holds, then seller 1 believes that the buyer will visit his store first at time 0.

On the other hand, seller 2 still compares Equations (5) and (6) where u_b is as given in (7). So, seller 2 believes that the buyer will visit his store if Equation (17) is satisfied.

Proof of Lemma 3. In this case seller 2 also believes that the buyer compares Equation (9) with (10) (again u_b is as given in (7), which implies that

$$\bar{z} < \left(\frac{\delta\underline{z}\bar{z}(\alpha_2 - \alpha_b)}{\delta\underline{z}\bar{z}(1-\alpha_b-\delta(1-\alpha_2)) - (\bar{z}-\underline{z})(1-\delta)(1-\alpha_b)} \right)^{\frac{\alpha_b}{1-\alpha_2}} \tag{18}$$

The important question is whether it is possible that the inequality in part 2 of Lemma 2 and (18) hold at the same time because $\alpha_b + \alpha_2 \geq 1$. The answer is yes if and only if the terms inside the parentheses of the Equation (18) is no less than 1, or else its denominator is non-positive. That is, Equation (18) holds if either

$$\bar{z} \geq \frac{\underline{z}}{1 - \delta \underline{z}(\frac{1-\alpha_2}{1-\alpha_b})} \tag{19}$$

or

$$\underline{z} \leq \frac{(\bar{z} - \underline{z})(1-\delta)(1-\alpha_b)}{\delta \bar{z}(1 - \alpha_b - \delta(1-\alpha_2))} \tag{20}$$

holds.

Proof of Lemma 4. Seller 1 believes that the buyer compares Equations (5) and (6) where $u_b = 1 - \alpha_2$. Then $v_b^1(1) > v_b^2(1)$ if and only if

$$\underline{z} < \frac{\alpha_2 - \alpha_b}{1 - \alpha_b - \delta(1-\alpha_2)} \tag{21}$$

Note that since $\delta < 1$ we can have $\underline{z}$ that satisfies both $z_1^* < \underline{z}$ and Equation (21). Seller 2 believes that the buyer is locally strong in his store. So, from our previous analysis, it mush be true that $\underline{z}$ satisfies Equation (17) so that seller 2 believes that the buyer will visit his store at time 0 with certainty.

The inequality (1) in Lemma 1 is also required in this case because seller 2 believes that he is locally weak in his own store. Therefore, his expected payoff is α_b in equilibrium. As long as this inequality holds, there will be no manipulation by seller 2.

Proof of Lemma 5. Seller 1 compares Equations (11) and (10). So, seller 1 believes that the buyer will visit his store with probability 1 at time 0 if and only if the inequality (2) of Lemma 5 holds. Then, the rest follows.

Proof of Lemma 6. Seller 2 compares the equations as he was doing so in the case of Lemma 3. On the other hand, seller 1 is in the same situation he is in the case of Lemma 5. Therefore, the results directly follows.

Proof of Lemma 7. Suppose that $\bar{z} \leq (\underline{z})^{\frac{\alpha_b}{1-\alpha_2}}$. In this case, seller 1 has an opportunity for a profitable deviation, so there is no equilibrium of the form $\alpha_b < \alpha_2 < \alpha_1$ where $\delta > \frac{1-\alpha_1}{1-\alpha_2}$ as the following discussions show. We know that in equilibrium (under these assumptions) seller 1's expected payoff is α_b (at most) in case buyer visits his store first at time 0. However, I claim that seller 1 would instead prefer to post a price $\alpha_1' < \alpha_1$ which is very close to α_b so that according to his equilibrium strategies, the buyer prefers to visit his store first at time 0, and seller 1's payoff is slightly higher than α_b (since with this deviation he becomes locally strong relative to the buyer).

The main question is whether the buyer really wants to visit store 1 in the case of (α_1', α_2). If α_1' is very close to α_b so that $\frac{1-\alpha_1'}{1-\alpha_b} > \delta$, i.e., the buyer prefers to accept seller 1's posted price instead of bearing the travel cost to visit seller 2, then the buyer's expected payoff in store 1 is $1-\alpha_1'$ (since the buyer will finish the game in store 1.)

However, if the buyer ever visits store 2 (high price store now) his payoff will depend on the relationship between $\frac{1-\alpha_2}{1-\alpha_1'}$ and δ. If the later is larger than the former, then according to seller 1

$$v_b^2 = (1 - z_2^1)(1 - \alpha_b) + \delta z_1^1 (1 - \alpha_1') \tag{22}$$

Otherwise,

$$v_b^2 = \left[1 - \bar{z}\left(\frac{X_2^1}{\bar{z}}\right)^{\frac{1-\alpha_2}{\alpha_b}}\right](1-\alpha_b) + \bar{z}\left(\frac{X_2^1}{\bar{z}}\right)^{\frac{1-\alpha_2}{\alpha_b}}(1-\alpha_2) \tag{23}$$

In either case we can find α_1' such that $v_b^1 > v_b^2$: Equation (22)) implies

$$\alpha_b < \alpha_1' < 1 - \frac{1-\bar{z}}{1-\delta\bar{z}} + \alpha_b \frac{1-\bar{z}}{1-\delta\bar{z}}$$

which is always more than α_b, whereas Equation (23) implies

$$\alpha_1' < \alpha_b + (\alpha_2 - \alpha_b)\bar{z}\left(\frac{X_2^1}{\bar{z}}\right)^{\frac{1-\alpha_2}{\alpha_b}}$$

Hence, the result follows.

Proof of Proposition 8. First, Lemma 1-7 imply that in equilibrium we must have , $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$. Then in the limit where $\underline{z}$ converges to $\bar{z}$, we have $\alpha_b \geq 1/2$. Hence, a vector of primitives θ can be supported as an equilibrium (in the limit) if and only if $\alpha_b + \alpha_2 \geq 1$.

However, there cannot be prices falling into this category where $\underline{z} > z_1^*$ holds as well (Lemma 1). This is because inequality (2) in Lemma 1 implies that in the limit, where $\underline{z}_n \to \bar{z}$, $W < \bar{z}$ must be true whereas inequality (3) implies $W > \bar{z}$ where $W = \left[\frac{(1-\alpha_b)\left[1-\frac{1-\bar{z}}{1-\delta\bar{z}}\right]}{\alpha_2-\alpha_b}\right]^{\frac{\alpha_b}{\alpha_b+\alpha_2-1}}$.

Similarly, there cannot be prices falling into the category where $z_1^* < \bar{z}$ holds either (Lemma 3). This is because in the limit, Lemma 3 implies that we must have that $W' < \bar{z} < W'$ where $W' = \left[\frac{\alpha_2-\alpha_b}{1-\alpha_b-\delta(1-\alpha_2)}\right]^{\frac{\alpha_b}{1-\alpha_2}}$.

Proof of Proposition 9. First, Lemma 1-7 imply that in equilibrium we must have $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$. Then in the limit where both $\underline{z}$ and $\bar{z}$ converge to zero at the same rate, we have $\alpha_b \geq 1/2$. Hence, θ can be supported as an equilibrium (in the limit) if and only if $\alpha_b + \alpha_2 \geq 1$.

So, we need to consider three cases analyzed in Lemma 1-3. However, Lemma 2 becomes redundant in the limit because z_1^* cannot be both negative and positive. Given that $\bar{z}_n = K(\underline{z}_n)$ for some $K > 1$ and all $n \geq 0$, inequality (2) of Lemma 1 implies that $\frac{\alpha_2-1}{\alpha_b} < \frac{W}{\ln \underline{z}_n}$ where $W = \ln(1-\alpha_b) + \ln(1-\delta) - \ln(1-\delta\underline{z}_n) - \ln(\alpha_2-\alpha_b) - \left(\frac{\alpha_2+\alpha_b-1}{\alpha_b}\right)\ln K$. As $\underline{z}_n \to 0$ we have $\frac{\alpha_2-1}{\alpha_b} < 0$ if and only if $\alpha_2 < 1$.

However, inequality (3) of Lemma 1 implies that $\frac{\alpha_2-1}{\alpha_b} > \frac{W'}{\ln \underline{z}_n}$ where

$$W' = \ln\left[\frac{K^{\frac{\alpha_2-1}{\alpha_b}}(1-\alpha_b)\frac{(1-\delta)K}{1-\delta\underline{z}_n K}}{\alpha_2-\alpha_b}\right]$$

implying that in the limit we have $\alpha_2 > 1$. Hence, in equilibrium there cannot be prices α_2 and $\alpha_b \geq 1/2$ such that $z_1^* < 0$.

On the other hand, the first inequality of condition (2) in Lemma 3 will be satisfied for sufficiently small $\underline{z}_n$'s. Inequality (2) of Lemma 2 (which has to be satisfied for Lemma 3 to be true) implies that $1 - \frac{\alpha_b}{1-\alpha_2} < \frac{\frac{\alpha_b}{1-\alpha_2}W''}{\ln(K\underline{z}_n)}$ where

$$W'' = \ln\left[\frac{\delta(\alpha_2-\alpha_b)}{(K-1)(1-\delta)(1-\alpha_b)+\delta K\underline{z}_n(1-\alpha_b-\delta(1-\alpha_2))}\right]$$

implying that in the limit we have $\alpha_b + \alpha_2 > 1$. Hence, the desired result follows.

Proof of Proposition 10. Note that $w_1 > \gamma_1$ since $\bar{z} > \underline{z}$. So, the only region where both sellers believe that the buyer is weak is for the values of $\bar{z}$ satisfying $\bar{z} \leq \gamma_1$. However, for this values of $\bar{z}$, seller 1 believes that the buyer will visit store 2 first (since the buyer is weak, i.e. his expected payoff of visiting store i is $1-\alpha_i$ and the posted price in store 2 is lower). Hence, we cannot have an equilibrium in this region. However, for all other values of $\bar{z}$, the buyer is strong in store 1. That means that if there is any equilibrium for the values of $\bar{z} > \gamma_1$, at least seller 1's expected payoff in stage 2 is at most α_b even if the buyer visits store 1 first (the same could be possible for seller 2). Hence, according to Proposition 3, in equilibrium we must have $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$. Otherwise, the seller whose payoff is α_b (or lower) may deviate to a price slightly above α_b and get the buyer for sure with a payoff of slightly higher than α_b (since, the seller may be locally strong)

Proof of Proposition 11. First note that for sufficiently small values of $\bar{z}$, by this we need that $\bar{z} < (A_j)^{\frac{\alpha_b+\alpha_i-1}{1-\alpha_i}}$ for $i, j \in \{1, 2\}$ with $j \neq i$, we have that $\bar{z}$ is strictly greater than $\max\{w_2, w_4, \gamma_2, \gamma_4\}$. So, both sellers believe that the buyer's expected payoff is calculated according to 3.

Then the buyer selects store 2 first if and only if either $\bar{z} \leq \left(\frac{\alpha_2-\alpha_b}{\alpha_1-\alpha_b}\right)^{\frac{\alpha_b}{\alpha_1-\alpha_2}}$ or else $\bar{z} < \frac{(1-\delta)(1-\alpha_b)(\bar{z}-\underline{z})}{\delta \underline{z} R}$ holds, where $R = \left((\alpha_1 - \alpha_b)/(\bar{z})^{\frac{1-\alpha_1}{\alpha_b}}\right) - \left((\alpha_2 - \alpha_b)/(\bar{z})^{\frac{1-\alpha_2}{\alpha_b}}\right)$.

On the other hand, seller 1 believes that the buyer visits his store first if and only if either $\bar{z} > \left(\frac{\alpha_2-\alpha_b}{\alpha_1-\alpha_b}\right)^{\frac{\alpha_b}{\alpha_1-\alpha_2}}$, or else $\underline{z} < \frac{(\bar{z}-\underline{z})(1-\delta)(1-\alpha_b)}{\delta \bar{z} \bar{R}}$ holds, where $\bar{R} = \left((\alpha_2 - \alpha_b)/\bar{z}^{\frac{1-\alpha_2}{\alpha_b}}\right) - \left((\alpha_1 - \alpha_b)/\bar{z}^{\frac{1-\alpha_1}{\alpha_b}}\right)$.

Furthermore, each seller believes that in equilibrium their expected payoffs in stage 2 is α_b. Therefore, we need to have $\bar{z} \geq (\underline{z})^{\frac{\alpha_b}{1-\alpha_b}}$. This completes the proof.

Proof of Proposition 13. First note that in the limit we must have $\alpha_b \geq 1/2$ by Corollary 1. Then according to Proposition 11 condition 1 (the second inequality) we have $\left[1 - K\left(\frac{1-\alpha_2}{\alpha_b}\right)\right] > \frac{\ln W - W_n'}{\ln \underline{z}_n}$ where $W = \frac{(K-1)(1-\delta)(1-\alpha_b)}{K\delta}$ and

$$W_n' = \ln\left[(\alpha_2 - \alpha_b) - (\underline{z})^{\frac{\alpha_1-\alpha_2}{\alpha_b}}(\alpha_1 - \alpha_b)\right]$$

Hence, in the limit we have $1 - K\left(\frac{1-\alpha_2}{\alpha_b}\right) > 0$, implying the desired result.

Proof of Proposition 15. Suppose that $\alpha_b + \alpha_2 \geq 1$. Then, $\bar{z} \geq (\bar{z})^{\frac{\alpha_b}{1-\alpha_2}}$, i.e. both sellers believe that the buyer is locally strong in both stores. So, for seller 1, the buyer strictly prefers store 1 over store 2 (i.e., $u_b^1(1) > u_b^2(1)$) if and only if $(1 - \underline{z}A)(1 - \alpha_b) + \underline{z}A(1 - \alpha_1) > (1 - \bar{z}B)(1 - \alpha_b) + \bar{z}B(1 - \alpha_2)$ where $A = (1/\bar{z})^{\frac{1-\alpha_1}{\alpha_b}}$ and $B = (1/\bar{z})^{\frac{1-\alpha_2}{\alpha_b}}$. The last inequality implies the condition 2. On the other hand, seller 2 believes that the buyer will select his store if and only if $\underline{z}B(\alpha_2 - \alpha_b) < \bar{z}A(\alpha_1 - \alpha_b)$, implying the third condition.

Proof of Proposition 16. In this case the threshold X_1^1 is the highest and for sufficiently small values of $\bar{z}$ we have $\bar{z} > X_1^1$ when $\alpha_b + \alpha_2 > 1$. That is, in equilibrium both sellers believe that the buyer is distance-corrected strong relative to seller 2.

So, the buyer's expected payoff of visiting store 2 first is

$$v_b^2 = \left(1 - z_2 z_b^{-\lambda_2/\lambda_b^2}\right)(1 - \alpha_b) + z_2 z_b^{-\lambda_2/\lambda_b^2}(1 - \alpha_2)$$

whereas his payoff of visiting store 1 first is

$$\left[(1-z_1)+\delta z_1\left(1-\frac{z_2}{z_b^{\lambda_2/\lambda_b^2}}\right)\right](1-\alpha_b)+\frac{\delta z_1 z_2}{z_b^{\lambda_2/\lambda_b^2}}(1-\alpha_2)$$

Therefore, seller 2 believes that the buyer will visit his store first if and only if condition 2 holds and seller 1 believes that the buyer visits his store first when condition 3 holds.

Proof of Proposition 17. According to Proposition 15, in the limit we have $(\bar{z})^{\frac{\alpha_1-\alpha_2}{\alpha_b}} < \frac{\alpha_2-\alpha_b}{\alpha_1-\alpha_b}$ by condition 2 and $(\bar{z})^{\frac{\alpha_1-\alpha_2}{\alpha_b}} > \frac{\alpha_2-\alpha_b}{\alpha_1-\alpha_b}$ by condition 3. These inequalities yields the required results. On the other hand, by Proposition 16, in the limit we have $\bar{z}^{\frac{1-\alpha_2}{\alpha_b}} > W$ by condition 2 and $\bar{z}^{\frac{1-\alpha_2}{\alpha_b}} < W^{\frac{\alpha_b}{\alpha_b+\alpha_2-1}}$ where $W = \frac{(1-\delta\bar{z})(\alpha_2-\alpha_b)}{(1-\delta)(1-\alpha_b)}$. The first inequality implies that $W < 1$. However, $\alpha_b/(\alpha_b+\alpha_2-1) > 1$, which implies that $W^{\frac{\alpha_b}{\alpha_b+\alpha_2-1}} < W$, which yields the required result.

Proof of Proposition 18. According to condition 3 in Proposition 15, we have $\frac{\alpha_2-\alpha_1}{\alpha_b} > \frac{W}{\ln \underline{z}_n}$ where $W = \ln\left[K^{\frac{\alpha_b+\alpha_1-\alpha_2}{\alpha_b}}\left(\frac{\alpha_1-\alpha_b}{\alpha_2-\alpha_b}\right)\right]$, implying that in the limit it is $\alpha_2 > \alpha_1$, which contradicts our assumption. On the other hand, according to condition 2 of Proposition 16 we have $\frac{\alpha_2-1}{\alpha_b} > \frac{W'}{\ln \underline{z}_n}$ where $W' = \ln\left[\frac{K^{\frac{1+\alpha_b-\alpha_2}{\alpha_b}}(1-\delta)(1-\alpha_b)}{(1-\delta K\underline{z}_n)(\alpha_2-\alpha_b)}\right]$, implying that in the limit we have $\alpha_2 > 1$, which contradicts our assumption.

Proof of Proposition 20. Suppose that θ can be supported as an equilibrium. Since z_θ is sufficiently small, more specifically $z_i < \alpha_b$ for all $i \in \{1,2,b\}$, in equilibrium the buyer will never leave store 2 until the buyer and seller 2 reaches an agreement (otherwise, both will get the payoff of zero), whether the buyer visits store 2 first at time 0, or after visiting seller 1. Therefore, suppose for a contradiction that seller 2 is locally strong in his store at θ, so his expected payoff is

$$v_2 = \left(1-\frac{z_b}{z_2^{\alpha_b/1-\alpha_2}}\right)\alpha_2+\frac{z_b}{z_2^{\alpha_b/1-\alpha_2}}\alpha_b$$

which is equivalent to $v_2 = \alpha_2-(\alpha_2-\alpha_b)\frac{z_b}{z_2^{\alpha_b/1-\alpha_2}}$.

As long as seller 2 remains strong in his store, the impact of increasing α_2 on the second seller's expected payoff is positive for sufficiently small values of z_θ. To see this, consider the sign of $\partial v_2/\partial\alpha_2$. Note that

$$\frac{\partial v_2}{\partial \alpha_2} = 1 - z_b z_2^{-\frac{\alpha_b}{1-\alpha_2}} + (\alpha_2-\alpha_b)\alpha_b z_2^{\frac{\alpha_b}{1-\alpha_2}}\ln z_2$$

Note that as z_2 converges to zero, the value of the last part of this equation converges to zero. Hence, for small values of z_2's $\partial v_2/\partial\alpha_2$ gets positive values since $z_b < z_2^{\alpha_b/1-\alpha_2}$ and thus $z_b z_2^{-\frac{\alpha_b}{1-\alpha_2}} < 1$.

Therefore, as long as $z_b < z_2^{\alpha_b/1-\alpha_2}$, we can find α_2' that is very close to but higher than α_2, so that it increases seller 2's expected payoff.

Proof of Proposition 21. In equilibrium, whether the buyer visits store 2 first at time 0 or not, Proposition 20 shows that θ can be supported as an equilibrium of the price search game G_s only if seller 2 is locally weak in his store at θ. Therefore, along with Proposition 20, we get the desired result. Similarly, seller 1's expected payoff in equilibrium is α_b since the buyer will never play the concession game with seller 1 (Fact B). Hence, Proposition 20 also yields the desired result for this case.

Since neither seller has an opportunity to make himself locally strong by deviating to a lower price once the buyer arrives at his store, θ can be supported as an equilibrium of the price search game G_s.

Proof of Proposition 22. Suppose that θ can be supported as an equilibrium where $1-\alpha_1 \geq \delta(1-\alpha_2)$ and for a contradiction that the buyer is not distance-corrected strong relative to seller 2 while $A_2 > 0$. That is, $z_b < X_1$. So, in equilibrium following the subgame where the buyer visits store 1 first at time 0, the buyer has to play the concession game until he builds up his reputation up to X_1. So, at the time of departure, T_1^d, the buyer's reputation will be $z_b(T_1^d) = X_1$.

Since θ is an equilibrium, the buyer leaves store 1 at time T_1^d believing that in store 2, the price will be α_2 and he will be locally strong in store 2 with his reputation X_1. That is, $z_b(T_1^d) = X_1 > (z_2)^{\frac{\alpha_b}{1-\alpha_2}}$. So, seller 2's continuation payoff in the subgame where the buyer enters store 2 after visiting store 1 is α_b. Since θ is an equilibrium, there should be no price α_2' such that seller 2 becomes strong relative to the buyer with reputation X_1 in his store (Proposition 20). That means, we must have $X_1 > (z_2)^{\frac{\alpha_b}{1-\alpha_b}}$, implying that

$$(z_2)^{\frac{\alpha_b}{1-\alpha_2}} > A_1^{\frac{\alpha_b}{1-\alpha_2}}(z_2)^{\frac{\alpha_b}{1-\alpha_b}}$$

So, we must have that

$$(z_2)^{\frac{\alpha_2-\alpha_b}{1-\alpha_b}} > A_1$$

The last inequality holds true only for α_2's that are very close to α_b. However, for sufficiently small values of z_2, this inequality will not be satisfied, contradicting that θ is an equilibrium.

On the other hand, if $A_i \leq 0$ for some seller i, then the seller must be locally weak in his store because otherwise seller i would increase his demand upon buyer's arrival at his store at time 0 and increase his expected payoff (for sufficiently small values of z_θ). Therefore, similar arguments in the proof of Proposition 20 yields the desired result.

Proof of Proposition 23. Proposition 22 shows that if θ can be supported as an equilibrium, then (a) must hold whenever possible. If for some seller i we have $A_i \leq 0$, then according to 22, seller i must be locally weak in his store at θ which is implied by the condition (b). Therefore in equilibrium, each seller's expected payoff in the game is at most α_b, and so Proposition 19 implies the condition (b). Moreover, both (a) and (b) imply that θ can be supported as an equilibrium of the price search game G_s since no seller has an incentive to deviate from his behavioral demand.

Proof of Corollary 3. According to Propositions 21 and 23, in equilibrium each seller's expected payoff when the buyer visits his store must be α_b. Moreover, as the parameters satisfy the two conditions given, no seller has chance for a profitable deviation by increasing or decreasing his behavioral demand upon the buyer's arrival at his store. On the other hand, since no seller can make the buyer visit his store first by a unilateral deviation (the buyer chooses which store to visit first before observing the behavioral demands), whichever store the buyer visits first in equilibrium, the seller who will be visited the second does not alter his behavioral demand, which yields the desired result.

Proof of Corollary 4. For any θ that can be supported as an equilibrium, condition $z_b \geq (z_i)^{\frac{\alpha_b}{1-\alpha_b}}$ implies that we must have $\alpha_b \geq 1/2$ in the limit. In addition, the condition $z_b \geq X_i$ whenever it is applicable, implies that in the limit we must have $\alpha_b + \alpha_i \geq 1$. However, the former condition implies the latter one.

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