Highway Toll Pricing *

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Abstract

For a tolled highway where consecutive sections allow vehicles enter and exit unrestrictedly, we propose a simple toll pricing method. We show that the method is the unique method that satisfies the classical axioms of Additivity and Dummy in the cost sharing literature, and the axioms of Toll Upper Bound for Local Traffic and Routing-proofness. We also show that the toll pricing method is the only method satisfying Routing-proofness and Cost Recovery. The main axiom in the characterizations is Routing-proofness which says that no vehicle can reduce its toll charges by exiting and re-entering intermediately. In the special case when there is only one unit of traffic (vehicle) for each (feasible) pair of entrance and exit, we show that our toll pricing method is the Shapley value (Shapley, 1953) of an associated game to the problem. In the case when there is one unit of traffic entering at each entrance but they all exit at the last exit, our toll pricing method coincides with the well-known airport landing fee solution-the Sequential Equal Contribution rule of Littlechild and Owen (1973).

Keywords: Highway toll, Cost sharing, Routing-proofness.

1 Introduction

This paper considers a highway toll pricing problem. Suppose that there is a highway divided into consecutive road sections by a number of entrances and exits. At each exit, a toll is charged for vehicles leaving the highway at that exit. There are costs associated with road sections and these costs depend on the length of the section and the traffic volume on that road. We seek for toll pricing methods that assign a toll charge for each vehicle such that the total toll collected from all vehicles at all exits cover exactly the total costs of the highway.

Traditionally, a flat or uniform toll is often used and its drawback is that it often induces cross-subsidization both amongst users and road sections. Other toll pricing methods have been proposed. For example, the Average Cost Pricing assigns the same price for all users, which treats all users homogeneously. Users have to pay the same share to all road sections independent of whether or not they use those roads. Thus, the Average Cost Pricing method may charge too much for vehicles so that they pay more than the costs of the roads that they actually use.

We propose in this paper a simple toll pricing method to impute the costs to each user that we think is fair and incentive-compatible. The pricing method simply charges each vehicle the sum of average cost it incurs in each section of the highway it travels through.

We provide two axiomatic characterizations for the toll pricing method. Theorem 1 shows that the toll pricing method is the unique method satisfying Additivity, Dummy, Toll Upper Bound for Local Traffic and Routing-proofness. Theorem 2 shows that the toll pricing method is the unique method satisfying Routing-proofness and Cost Recovery. Additivity and Dummy axioms are two classical axioms in the cost sharing literature (Moulin, 2002). Toll Upper Bound for Local Traffic says that local traffic pays no more than the average cost of all traffic passing through a given road section. Thus, it will not discriminate the local traffic. Routing-proofness is a new axiom recently proposed by Moulin (2008). The Cost Recovery axiom can be regarded as a core property.

The Routing-proof axiom says that no vehicle can reduce its toll charges by exiting and re-entering at intermediate exits and entrances. Without this property, a pricing method would induce strategic maneuvers by road users (vehicles) and generates unpredictable traffic patterns. A similar property (called No Transit) is used in a network cost allocation problem by Henriet and Moulin (1996), where the fixed cost of a communication network should be shared by users. No Transit axiom says that it is not profitable for any three users i, j, and k to make some of the traffic between i and j transit though k. Recently, Moulin (2008) considers a network connection problem, in which each user of the network needs to connect a pair of targeted nodes. Each pair of nodes has a connection cost that is independent of the traffic. Users must share the total cost of the efficient network. He considers cost sharing methods that satisfy Routing-proofness and Stand alone core stability. Our model is much simpler than his but we allow connection costs vary with the traffic.

The Cost Recovery axiom assumes that for a given road section between two consecutive exits, the total toll collected from all vehicles that use that road section should cover (be greater than or equal to) the cost of that section. Without this property, a pricing method would have to charge more to some vehicles on other road sections in order to make overall toll charges equal to the total costs of all road sections. In other words, some road sections would implicitly subsidize other road section(s). In practice, different exits (road sections) may belong to different jurisdictions. It is reasonable to require that there is no cross-subsidizations across road sections.[1]

The toll pricing method can be considered as a natural extension of the well-known Airport Landing Fee solution of Littlechild and Owen (1973), namely the Sequential Equal Contribution rule (see Thomson (2005) for a survey on Airport Landing Fee problem and its solutions). First, we show that the toll pricing method is the Shapley value of a associated game to the problem in which we assume that there is only one unit (vehicle) of traffic entering and exiting at each entrance and exit. Then, the toll pricing method is the Shapley value of the (cost) game that assigns to each coalition of agents (vehicles) the total costs of those road sections that are used by the vehicles in the coalition. Second, we assume that there is only one unit of traffic (one vehicle) entering at each entrance but all exit at the last exit. In this case,

[1]This does not means that the total toll charged at each exit must cover exactly the cost of that associated road section. In fact, the users of that road section may pay toll at any other late exits. This can be balanced by considering two-way traffic which is more realistic. It can be assumed that all vehicles returns to their origins.

the method coincides with the Sequential Equal Contribution rule for the Airport Landing Fee problem.

The rest of the paper is organized as follows. Section 2 introduces the model, method and an example. Section 3 gives two characterizations of the toll pricing method. Section 4 concludes.

2 The Model

Consider a linear highway divided into n sections by a set of entrances $E = \{1, ..., n\}$ and a set of exits $N = \{1, 2, ..., n\}$. Assume that exit $i, i = 1, ..., n-1$, is located in between the entrance i and entrance $i+1$, but exit n is located after the entrance n. An example with three entrances and exits is shown in Figure 1.

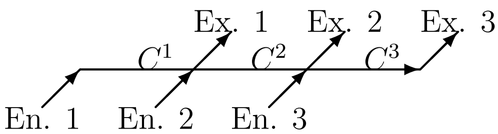

Figure 1

An agent is a pair $(i, j), i \leq j$ representing vehicles entering at entrance i and exiting at exit j. Let $V = \{(i, j)|i, j = 1, ..., n, i \leq j\}$ be the set of all agents. We assume that traffic is one way, i.e., all vehicles move in one direction (e.g., from left to right in Figure 1 above). Let $x_{ij}, i \leq j \in N$ be the number of vehicles that enter at entrance i and exit at exit j. We say that agent (i, j)'s demand is x_{ij}. A demand vector x is then

$$x = (x_{11}, ..., x_{1n}, x_{22}, ..., x_{2n}, ..., x_{(n-1)(n-1)}, x_{(n-1)n}, x_{nn}),$$

Let $C^i(x), i \in N$ be the cost of road section i (from entrance i to exit i) which depends on both the length of the road and the volume of the vehicles,

x, on that part of the road for certain period of time (e.g., a year). There is a large literature on how to determine or estimate the cost of a highway. For example, Johansson and Mattson (1994) argue that the maintenance cost can be decomposed into a fixed part and a variable part, of which the variable part is influence by traffic volume. Then, each section's cost consists of a fixed cost C_0^i and a variable cost $C_1^i(x)$. An example is to assume that $C_1^i(x) = (\max\{0, x-T\})^2, i = 1, ..., n,$ [2] where T is a constant (some threshold on the volume of vehicles above which there is an increasing cost incurred to the road). Therefore, $C^i(x) = C_0^i + (\max\{0, x-T\})^2$. For the purpose of this paper, we ignore how the cost functions are determined. Assume that road sections are ordered sequentially from 1 to n. Denote the total cost $C = \sum_i C^i$, where C^i is the cost of section $i (i \in N)$. Denote $\mathcal{C}$ the set of all such cost functions.

At each exit, a toll is charged to each vehicle. Let y_{ij} be the total toll charged to agent (i, j), $i, j = 1, ..., n, i \leq j$. For each agent $(i, j) \in V$, denote $K(i, j) = \{i, i + 1, ..., j\}$ the set of road sections used by agent (i, j). Our problem is to determine the toll (or equivalently the unit price y_{ij}/x_{ij} for each vehicle) at each exit so that the total cost $\sum_{i \in N} C^i(x)$ is equal to the total toll $\sum y_{ij}$ collected at all exits.

Formally, a *highway toll problem* is a triple (V, x, C). A solution to a problem (V, x, C) is a vector

$$y = (y_{11}, ..., y_{1n}, y_{22}, ..., y_{2n}, ..., y_{(n-1)(n-1)}, y_{(n-1)n}, y_{nn}),$$

such that

$$\sum_{i=1,...,n} \sum_{j=i,...,n} y_{ij} = \sum_{i=1,...,n} C^i(x)$$

[2]This cost function exhibits decreasing returns to scale, which embodies negative externalities of traffic usage. It is the externalities in usage among different users of the network that makes this cost sharing problem nontrivial. This kind of cost function is common in transportation literature, when construction cost and maintenance cost are included. See also Alberto Castaño-Pardo et.al. (1995).

A method is a mapping y that assigns to each problem (V, x, C) a solution $y(V, x, C)$.

In this paper, we propose and study the following *toll pricing method*:

$$y_{ij}(V,x,C) = \left[\sum_{l=i}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}}\right] x_{ij}, \text{ where } \ i \leq j = 1, ..., n. \quad (1)$$

In words, the pricing formula simply charges each vehicle the sum of average cost it incurs in each section of the highway it travels through.

Example 1. Consider the example in Figure 1. Let $x_{ij}, i \leq j = 1, 2, 3$ be the number of vehicles entering at entrance i and exiting at exit j. Let $C^k(x)$ be the costs related to the section from Entrance k to exit k, where $k = 1, 2, 3$ and x is the number of vehicles on the road.

Denote $y_{ij}, i \leq j = 1, 2, 3$ the total toll charges to the vehicles $x_{ij}, i \leq j = 1, 2, 3$, respectively. The Toll Sharing method defined in Equation (1) gives the following solution:

$$y_{11}(V,x,C) = \frac{C^1(x_{11}+x_{12}+x_{13})}{x_{11}+x_{12}+x_{13}} x_{11},$$

$$y_{12}(V,x,C) = \left[\frac{C^1(x_{11}+x_{12}+x_{13})}{x_{11}+x_{12}+x_{13}} + \frac{C^2(x_{12}+x_{13}+x_{22}+x_{23})}{x_{12}+x_{13}+x_{22}+x_{23}}\right] x_{12},$$

$$\begin{aligned} y_{13}(V,x,C) \quad &= \quad [\frac{C^1(x_{11}+x_{12}+x_{13})}{x_{11}+x_{12}+x_{13}} + \frac{C^2(x_{12}+x_{13}+x_{22}+x_{23})}{x_{12}+x_{13}+x_{22}+x_{23}} \\ &\quad + \frac{C^3(x_{13}+x_{23}+x_{33})}{x_{13}+x_{23}+x_{33}}] x_{13}, \end{aligned}$$

$$y_{22}(V,x,C) = \frac{C^2(x_{12}+x_{13}+x_{22}+x_{23})}{x_{12}+x_{13}+x_{22}+x_{23}} x_{22},$$

$$y_{23}(V,x,C) = \left[\frac{C^2(x_{12}+x_{13}+x_{22}+x_{23})}{x_{12}+x_{13}+x_{22}+x_{23}} + \frac{C^3(x_{13}+x_{23}+x_{33})}{x_{13}+x_{23}+x_{33}}\right] x_{23},$$

$$y_{33}(V,x,C) = \frac{C^3(x_{13}+x_{23}+x_{33})}{x_{13}+x_{23}+x_{33}} x_{33}.$$

We now introduce the following three properties for our pricing method.
Constraint (A)-Stand Alone Test for Vehicles: For all $(i, j) \in V$,

$$y_{ij} \leq \sum_{k \in K(i,j)} C^k(x)$$

This condition says that the cost borne by the vehicles travelling from i throughout to j should not exceed the total cost of the road sections used by them.
Constraint (B)-Cost Recovery (for Road Sections): For all $k \in N$,

$$\begin{aligned} & C^k(x) \\ \leq \; & \sum_{i<k}(\frac{y_{ik}}{x_{ik}} - \sum_{i \leq j < k} \frac{y_{jj}}{x_{jj}}) \cdot x_{ik} + y_{kk} + \sum_{i>k}(\frac{y_{ki}}{x_{ki}} - \sum_{k<j \leq i} \frac{y_{jj}}{x_{jj}}) \cdot x_{ki} + \\ & \sum_{i<k<j}(\frac{y_{ij}}{x_{ij}} - \sum_{i \leq s \neq k \leq j} \frac{y_{ss}}{x_{ss}}) \cdot x_{ij} \end{aligned}$$

This condition says that the total toll collected (may be at several different exits) that is related to a road section should not be lower than its cost. For a given road section k, the total toll is broken down into four parts. Part 1 is the toll collected from vehicles leaving from exit k but entering before entrance k; Part 2 is the toll paid by those from entrance k to exit k; Part 3 is the case for entering at k but exiting after k; and Part 4 is entering before k and exiting after k.
Constraint (C)-No Transit Constraint or Routing-proofness: For any $i \leq j$, and for any $j^{'} \in [i, j-1]$,

$$\frac{y_{ij}}{x_{ij}} \leq \frac{y_{ij'}}{x_{ij'}} + \frac{y_{j'+1,j}}{x_{j'+1,j}}$$

This condition ensures that no vehicle can reduce its toll charges by exiting and reentering intermediately. This axiom was first introduced in Henriet & Moulin (1996) (called Transit-proofness). It is called "Routing-proofness" in Moulin (2008).

It is helpful to compare our toll pricing method with other methods. In the following, we consider three methods. The first is the *Average Cost Pricing* method that treats all traffic homogeneously. The second is a *Weighted Average Cost Pricing* method. The last is our toll pricing method proposed in this paper. The examples show that the Average Cost Pricing violates both Constraint (A) the Stand Alone Test and the Constraint (B) Cost Recovery property while the Weighted Average Cost Pricing violates Constraint (B) Cost Recovery. Only our toll pricing method (1) satisfies all three properties.

Suppose $n = 3$, $V = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$. Suppose that cost functions $C^k(x) = C_0^k + (\max\{0, x - T\})^2, k = 1,2,3$, where $C_0^1 = 10, C_0^2 = 20, C_0^3 = 10$ and $T = 35$. Suppose that the traffic is as follows:

$$x_{11} = 10, x_{12} = 5, x_{13} = 5,$$

$$x_{22} = 5, x_{23} = 25, x_{33} = 10.$$

Since $x_{11} + x_{12} + x_{13} = 20 < 35$, then $C^1(x) = C_0^1 + C_1^1(x) = 10 + 0 = 10$.

Similarly, $x_{12} + x_{13} + x_{22} + x_{23} = 40 > 35 \Rightarrow C^2(x) = C_0^2 + C_1^2(x) = 20+25 = 45$, $x_{13}+x_{23}+x_{33} = 40 > 35 \Rightarrow C^3(x) = C_0^3+C_1^3(x) = 10+25 = 35$.

Example 2: Consider the Average Cost Sharing Method (i.e. proportional approach). It can be easily calculated that each vehicle should pay:

$$\frac{C^1(x) + C^2(x) + C^3(x)}{\sum_{i=1}^3 \sum_{j=i}^3 x_{ij}} = \frac{10 + 45 + 35}{10 + 5 + 5 + 5 + 25 + 10} = 1.5$$

That is, no matter how many road sections a vehicle has used, it is equally charged. Therefore, $y_{ij}/x_{ij} = 1.5$ for all $(i, j) \in V$, $y_{11} = 15, y_{12} = 7.5, y_{13} = 7.5, y_{22} = 7.5, y_{23} = 37.5, y_{33} = 15$. It can checked that $\sum_{i=1}^3 \sum_{j=1}^3 y_{ij} = \sum_{k=1}^3 C^k(x) = 90$.

Since $\frac{y_{ij'}}{x_{ij'}} = \frac{y_{j'+1,j}}{x_{j'+1,j}} = 1.5$, thus $\frac{y_{ij}}{x_{ij}} = 1.5 \le \frac{y_{ij'}}{x_{ij'}} + \frac{y_{j'+1,j}}{x_{j'+1,j}} = 3$, hence constraint (C) No Transit holds.

However, Constraint (B) for road section 2 now reduces to $C^2(x) \le y_{22} - y_{13} = 0$, which is a contradiction.

Now consider vehicles that use only road section 1. They have to pay a total of \$15 for the total cost of \$10 road section 1. Therefore, constraint (A) is also violated.

Example 3: Another toll sharing method often used in practice is the mile-Weighted Average Cost Pricing method:

$$\frac{C^1(x)+C^2(x)+C^3(x)}{\sum_{i=1}^{3}\sum_{j=i}^{3} x_{ij}\cdot(j-i+1)} = \frac{10+45+35}{10+5\times 2+5\times 3+5+25\times 2+10} = 0.9$$

Therefore, vehicles that travel though one, two, three road sections should pay \$ 0.9, \$ 1.8, \$ 2.7, respectively, and $y_{11}=9$, $y_{22}=4.5$, $y_{33}=9$, $y_{12}=9$, $y_{23}=45$, $y_{13}=13.5$. It can checked that $\sum_{i=1}^{3}\sum_{j=1}^{3} y_{ij} = \sum_{k=1}^{3} C^k(x) = 90$.

Because vehicles that use two (or three) road sections pay exactly twice (triple) of the toll paid by those use only one road section, constraint (C) holds. Constraint (A) also holds by simple calculations.

However, constraint (B) is violated by this method. Consider road section 2, whose condition can be explicitly expressed as follows:

$$C^2(x) \le y_{22} + (\frac{y_{12}}{x_{12}} - \frac{y_{11}}{x_{11}})\cdot x_{12} + (\frac{y_{23}}{x_{23}} - \frac{y_{33}}{x_{33}})\cdot x_{23} + (\frac{y_{13}}{x_{13}} - \frac{y_{11}}{x_{11}} - \frac{y_{33}}{x_{33}})\cdot x_{13}$$

It is easy to check that the RHS of this inequality is 36 while the LHS is $C^2(x)=45$, thus violating Constraint (B).

Example 4: Now we check that the toll pricing method (1) satisfies all three properties (A), (B), and (C). Indeed,

$$y_{11} = \frac{10}{20}\times 10 = 5,$$

$$y_{12} = [\frac{10}{20}+\frac{45}{40}]\times 5 = 8.125,$$

$$y_{13} = [\frac{10}{20}+\frac{45}{40}+\frac{35}{40}]\times 5 = 12.5,$$

$$y_{22} = \frac{45}{40}\times 5 = 5.625,$$

$$y_{23} = [\frac{45}{40}+\frac{35}{40}]\times 25 = 50,$$

$$y_{33} = \frac{35}{40} \times 10 = 8.75.$$

Thus, agent $(1,1)$ pays \$ 0.5 per vehicle, agent $(1,2)$ \$ 1.625 per vehicle, agent $(1,3)$ \$ 2.5 per vehicle, agent $(2,2)$ \$ 1.125 per vehicle, agent $(2,3)$ \$ 2 per vehicle, agent $(3,3)$ \$ 0.875 per vehicle.

It is easy to check that all three properties are satisfied. For example,

$$\begin{aligned}
C^1(x) &= 10 \\
&= y_{11} + (\frac{y_{12}}{x_{12}} - \frac{y_{22}}{x_{22}})x_{12} + (\frac{y_{13}}{x_{13}} - \frac{y_{22}}{x_{22}} - \frac{y_{33}}{x_{33}})x_{12} \\
&= 5 + (1.625 - 1.125) \times 5 + (2.5 - 1.125 - 0.875) \times 5 \\
&= 10
\end{aligned}$$

Note that Constraint (B) in fact is satisfied in equality. It will be shown in the next section that this is true in general. Also note that the toll pricing method satisfies a stronger property than Routing-proofness:

$$\frac{y_{ij}}{x_{ij}} = \frac{y_{ij'}}{x_{ij'}} + \frac{y_{j'+1,j}}{x_{j'+1,j}}, \text{ for all } i \le j' \le j-1. \tag{2}$$

3 Characterizations of the Method

Because traffic $x_{ij}, i \le j$ are integer numbers, it is convenient to consider the toll pricing problem from the perspective of the *discrete cost sharing model* (Moulin, 1995).

Thus, without loss of generality, we assume that $C(0) = 0$ and the fixed costs exist when traffic are not all zeros. We use the following two classical axioms in the cost sharing literature, Additivity and Dummy, together with Toll Upper Bound for Local Traffic and the Routing-proofness.

Additivity

$$y(V, x, C_1 + C_2) = y(V, x, C_1) + y(V, x, C_2), \ C_1, C_2 \in \mathcal{C}.$$

Dummy

If (i,j) is dummy, i.e., $\partial_{(i,j)}C(x)=0, \forall x$, then

$$y_{(i,j)}(V,x,C)=0.$$

where

$$\partial_{(i,j)}C(x)=C(x_{-ij},x_{ij}+1)-C(x).$$

Toll Upper Bound for Local Traffic

For any $k=1,...,n$,

$$\frac{y_{kk}}{x_{kk}}(V,x,C^k)\leq\frac{C^k(\sum_{(m,l)\in K^{-1}(k)}x_{ml})}{\sum_{(m,l)\in K^{-1}(k)}x_{ml}}. \tag{3}$$

This axiom says that no local traffic will pay a toll that is higher than the average cost of the total traffic on that road section. Note that this axiom is not implied by the Stand Alone Test for Vehicles and neither implies the latter.

Now, we show first that the highway toll pricing method (1) is Routing-proof.

Lemma 1 *The highway toll sharing method (1) satisfies Routing-proofness.*

Proof: We prove this lemma by mathematical induction.

Define $z_{ij}\equiv\frac{y_{ij}}{x_{ij}}=\sum_{l=i}^{j}\frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk}}$, then we need to prove $z_{ij}=z_{ij'}+z_{j'+1,j}$ for any $j^{'}\in[i,j-1]$.

From the definition of z_{ij},

$$\begin{aligned}
z_{ij} &= \sum_{l=i}^{j}\frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk}} \\
&= \frac{C^i(\sum_{m=1}^{i}\sum_{k=i}^{n}x_{mk})}{\sum_{m=1}^{i}\sum_{k=i}^{n}x_{mk}}+\sum_{l=i+1}^{j}\frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk}} \\
&= z_{ii}+z_{i+1,j}=z_{ii}+\frac{C^{i+1}(\sum_{m=1}^{i+1}\sum_{k=i+1}^{n}x_{mk})}{\sum_{m=1}^{i+1}\sum_{k=i+1}^{n}x_{mk}}+\sum_{l=i+2}^{j}\frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n}x_{mk}} \\
&= z_{ii}+z_{i+1,i+1}+z_{i+2,j} \\
&= z_{ii+1}+z_{i+2,j}
\end{aligned}$$

Then suppose $z_{ij} = z_{is} + z_{s+1,j}$ holds, then we only need to prove $z_{ij} = z_{is+1} + z_{s+2,j}$.

$$\begin{aligned}
z_{ij} &= z_{is} + z_{s+1,j} \\
&= \sum_{l=i}^{s} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} + \sum_{l=s+1}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} \\
&= \sum_{l=i}^{s} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} + \frac{C^{s+1}(\sum_{m=1}^{s+1}\sum_{k=s+1}^{n} x_{mk})}{\sum_{m=1}^{s+1}\sum_{k=s+1}^{n} x_{mk}} + \sum_{l=s+2}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} \\
&= \sum_{l=i}^{s+1} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} + \sum_{l=s+2}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} \\
&= z_{is+1} + z_{s+2,j}
\end{aligned}$$

Q.E.D.

We now prove the following characterization theorem.

Theorem 1 *The Toll Pricing method (1) is the unique method satisfying Additivity, Dummy, Toll Upper Bound for Local Traffic and Routing-proofness.*

Proof: It is easy to check that the toll pricing method (1) satisfies Additivity, Dummy, and Routing-proofness.

Now suppose that a method $y(V, x, C)$ satisfies the above three axioms. We show that it is uniquely determined.

By Routing-proofness, we have

$$\frac{y_{ij}}{x_{ij}} \leq \sum_{k=i}^{j} \frac{y_{kk}}{x_{kk}}. \tag{4}$$

Now we calculate y_{kk}/x_{kk}. By Additivity, we have

$$\frac{y_{kk}}{x_{kk}}(V, x, C) = \frac{y_{kk}}{x_{kk}}(V, x, C^1) + \cdots + \frac{y_{kk}}{x_{kk}}(V, x, C^k) + \cdots + \frac{y_{kk}}{x_{kk}}(V, x, C^n). \tag{5}$$

Note that (k, k) is dummy for all $C^l, l \neq k$. Thus, by Dummy axiom we have

$$\frac{y_{kk}}{x_{kk}}(V, x, C) = \frac{y_{kk}}{x_{kk}}(V, x, C^k). \tag{6}$$

Because y_{kk}/x_{kk} is the unit price for local traffic x_{kk} on the road section $k, k = 1, ..., n$, by Toll Upper Bound for Local Traffic we have

$$\frac{y_{kk}}{x_{kk}}(V, x, C^k) \leq \frac{C^k(\sum_{(m,l)\in K^{-1}(k)} x_{ml})}{\sum_{(m,l)\in K^{-1}(k)} x_{ml}}. \tag{7}$$

Therefore,

$$y_{ij}/x_{ij} \leq \sum_{k=i}^{j} \frac{C^k(\sum_{(m,l)\in K^{-1}(k)} x_{ml})}{\sum_{(m,l)\in K^{-1}(k)} x_{ml}},$$

and

$$y_{ij} \leq \sum_{k=i}^{j} \frac{C^k(\sum_{(m,l)\in K^{-1}(k)} x_{ml})}{\sum_{(m,l)\in K^{-1}(k)} x_{ml}} x_{ij}.$$

Summing over all $(i, j) \in V$,

$$\begin{aligned} \sum_{(i,j)\in V} y_{ij} &\leq \sum_{(i,j)\in V} \sum_{k=i}^{j} \frac{C^k(\sum_{(m,l)\in K^{-1}(k)} x_{ml})}{\sum_{(m,l)\in K^{-1}(k)} x_{ml}} x_{ij} \\ &= \sum_{k=1}^{n} C^k(x), \end{aligned}$$

By the budget balance condition, we thus have

$$y_{ij} = \sum_{k=i}^{j} \frac{C^k(\sum_{(m,l)\in K^{-1}(k)} x_{ml})}{\sum_{(m,l)\in K^{-1}(k)} x_{ml}} x_{ij}.$$

The theorem is proved. Q.E.D.

We can easily define a toll pricing method that satisfies Additivity, Dummy and Routing-proofness but violates the Toll Upper Bound for Local Traffic. This can be done by applying the well-known serial cost sharing method (Moulin and Shenker, 1992) as follows.

For any given problem, (V, x, C), consider cost function C^k, $k = 1, ..., n$. Consider $\{x_{ml}\}_{(m,l)\in K^{-1}(k)}$. Order them from the smallest to the largest, say, $x_{1k} \leq x_{2k} \leq \cdots \leq x_{kn}$. Apply the serial cost sharing method to problem

$(K^{-1}(k), x^{K^{-1}(k)}, C^k)$ (where $x^{K^{-1}(k)}$ is the restriction of x on the set $K^{-1}(k)$) to obtain

$$y_{ml}(k), (m,l) \in K^{-1}(k).$$

Let

$$y_{ml}(k) = 0, \text{ if } (m,l) \notin K^{-1}(k).$$

Define

$$y_{ml} = \sum_{k=1}^{n} y_{ml}(k). \tag{8}$$

Then it is easy to check that this defines a toll pricing method that satisfies Additivity, Dummy, and Routing-proofness. However, there is no guarantee that, e.g.,

$$\frac{y_{11}}{x_{11}} \leq \frac{C^1(x_{11} + x_{12} + \cdots x_{1n})}{x_{11} + x_{12} + \cdots x_{1n}}$$

would hold.

Consider the following example:

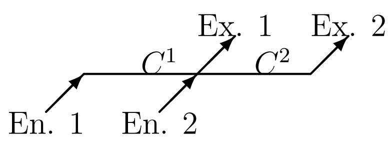

Figure 2

Let $x_{11} = 10, x_{12} = 5, x_{22} = 15$. According to (8) we have

$$y_{12} = \frac{C^1(2x_{12})}{2} + \frac{C^2(2x_{12})}{2} = \frac{1}{2}[C^1(10) + C^2(10)].$$

Thus, per unit price is

$$\frac{y_{12}}{x_{12}} = \frac{1}{10}[C^1(10) + C^2(10)].$$

If one unit of traffic x_{12} reroutes and becomes one unit of traffic $(1,1)$ and one unit of $(2,2)$, then the new traffic is $x'_{11} = 11, x'_{12} = 4, x'_{22} = 16$. This unit of traffic will pay the sum of the following

$$\begin{aligned}\frac{y'_{11}}{x'_{11}} &= \frac{1}{11}[\frac{C^1(2\times 4)}{2} + C^1(4+11) - C^1(2\times 4)] \\ &= \frac{1}{11}[\frac{C^1(8)}{2} + C^1(15) - C^1(8)] \\ &= \frac{1}{11}[C^1(15) - \frac{C^1(8)}{2}]\end{aligned}$$

and

$$\begin{aligned}\frac{y'_{22}}{x'_{22}} &= \frac{1}{16}[\frac{C^2(2\times 4)}{2} + C^2(4+16) - C^2(2\times 4)] \\ &= \frac{1}{16}[C^2(20) - \frac{C^1(8)}{2}]\end{aligned}$$

That is,

$$\frac{y'_{11}}{x'_{11}} + \frac{y'_{22}}{x'_{22}} = \frac{1}{11}[C^1(15) - \frac{C^1(8)}{2}] + \frac{1}{16}[C^2(20) - \frac{C^1(8)}{2}].$$

Without rerouting, that one unit of traffic of $(1,2)$ pays

$$\frac{y_{12}}{x_{12}} = \frac{1}{10}[C^1(10) + C^2(10)].$$

If we choose the following cost function

$$C^1(x) = C^2(x) = \begin{cases} 10 & \text{if} \quad x \le 10 \\ 10 + (x-10)^2 & \text{if} \quad x > 10 \end{cases}$$

Then

$$\frac{y_{12}}{x_{12}} = \frac{1}{10}[C^1(10) + C^2(10)] = 2.$$

But

$$\frac{y'_{11}}{x'_{11}} + \frac{y'_{22}}{x'_{22}} = \frac{1}{11}(35-5) + \frac{1}{16}(110-5) = \frac{30}{11} + \frac{105}{16} > 2.$$

There, it is Routing-proof.

However, it violates the Toll Upper Bound for Local Traffic because

$$\begin{aligned}\frac{y_{11}}{x_{11}} &= \frac{1}{10}[\frac{C^1(10)}{2} + C^1(15) - C^1(10)] \\ &= 3 \\ &\geq \frac{C^1(x_{11}+x_{12})}{x_{11}+x_{12}} \\ &= \frac{C^1(15)}{15} \\ &= \frac{35}{15}\end{aligned}$$

In the following, we further show that the toll pricing method is the unique method satisfying the properties of Routing-proofness and Cost Recovery. It is easy to observe that when there are 2 road sections, Cost Recovery and Routing-proofness give rise to the highway toll method (1) right away. This is shown as follows. By Cost Recovery and Routing-proofness, we have

$$\begin{cases} \frac{y_{12}}{x_{12}} \leq \frac{y_{11}}{x_{11}} + \frac{y_{22}}{x_{22}} \\ C^1 \leq y_{11} + (\frac{y_{12}}{x_{12}} - \frac{y_{22}}{x_{22}})x_{12} \\ C^2 \leq y_{22} + (\frac{y_{12}}{x_{12}} - \frac{y_{11}}{x_{11}})x_{12} \end{cases} \tag{9}$$

By definition, a pricing method satisfies the budget balance condition:

$$C^1 + C^2 = y_{11} + y_{12} + y_{22},$$

which implies that the above two inequalities of Cost Recovery must be equalities.

Solving the equation system, we have

$$\begin{cases} y_{11} = \frac{C^1}{x_{11}+x_{12}} \cdot x_{11} \\ y_{22} = \frac{C^2}{x_{12}+x_{22}} \cdot x_{22} \\ y_{12} = (\frac{C^1}{x_{11}+x_{12}} + \frac{C^2}{x_{12}+x_{22}}) \cdot x_{12} \end{cases} \tag{10}$$

Similarly, if there are 3 road sections, the equation system defined by Cost Recovery and Routing-proofness together with the budget balance condition is the following:

$$\begin{cases} \frac{y_{11}}{x_{11}} + \frac{y_{22}}{x_{22}} \geq \frac{y_{12}}{x_{12}} \\ \frac{y_{11}}{x_{11}} + \frac{y_{23}}{x_{23}} \geq \frac{y_{13}}{x_{13}} \\ \frac{y_{22}}{x_{22}} + \frac{y_{33}}{x_{33}} \geq \frac{y_{23}}{x_{23}} \\ \frac{y_{12}}{x_{12}} + \frac{y_{33}}{x_{33}} \geq \frac{y_{13}}{x_{13}} \\ C^1 \leq y_{11} + (\frac{y_{12}}{x_{12}} - \frac{y_{22}}{x_{22}})x_{12} + (\frac{y_{13}}{x_{13}} - \frac{y_{22}}{x_{22}} - \frac{y_{33}}{x_{33}})x_{13} \\ C^2 \leq y_{22} + (\frac{y_{12}}{x_{12}} - \frac{y_{11}}{x_{11}})x_{12} + (\frac{y_{13}}{x_{13}} - \frac{y_{11}}{x_{11}} - \frac{y_{33}}{x_{33}})x_{13} + (\frac{y_{23}}{x_{23}} - \frac{y_{33}}{x_{33}})x_{23} \\ C^3 \leq y_{33} + (\frac{y_{13}}{x_{13}} - \frac{y_{11}}{x_{11}} - \frac{y_{22}}{x_{22}})x_{13} + (\frac{y_{23}}{x_{23}} - \frac{y_{22}}{x_{22}})x_{23} \end{cases} \tag{11}$$

It is easy to check that the solution to this system is exactly the pricing method (1).

The following theorem summarizes this observation and provides an alternative characterization of the toll pricing method (1).

Theorem 2 *The Toll Pricing method (1) is the only method satisfying the properties of Routing-proofness and Cost Recovery.*

Proof: It is easy to check that the method satisfies Cost Recovery. Lemma 1 shows that it satisfies Routing-proofness. Now we show that it is the only method that satisfies these two properties.

First note that the method (1) can be rewritten as

$$\frac{y_{ij}}{x_{ij}} = \sum_{k\in K(i,j)} \frac{C^k(\sum_{(m,l)\in K^{-1}(k)} x_{ml})}{\sum_{(m,l)\in K^{-1}(k)} x_{ml}}$$

Now let us prove the following three inequalities:

$$\sum_{i<k}(\frac{y_{ik}}{x_{ik}} - \sum_{i\leq j<k} \frac{y_{jj}}{x_{jj}}) \leq \frac{y_{kk}}{x_{kk}}, \tag{12}$$

$$\sum_{i>k}(\frac{y_{ki}}{x_{ki}} - \sum_{k<j\leq i} \frac{y_{jj}}{x_{jj}}) \leq \frac{y_{kk}}{x_{kk}}, \tag{13}$$

$$\sum_{i<k<j}(\frac{y_{ij}}{x_{ij}} - \sum_{i\leq s\neq k\leq j} \frac{y_{ss}}{x_{ss}}) \leq \frac{y_{kk}}{x_{kk}}. \tag{14}$$

By Routing-proofness, we have

$$\begin{aligned}
\frac{y_{ik}}{x_{ik}} - \sum_{i \le j < k} \frac{y_{jj}}{x_{jj}} &= \frac{y_{ik}}{x_{ik}} - \frac{y_{ii}}{x_{ii}} - \frac{y_{i+1i+1}}{x_{i+1i+1}} - \cdots - \frac{y_{k-1k-1}}{x_{k-1k-1}} \\
&\le \frac{y_{i+1k}}{x_{i+1k}} - \frac{y_{i+1i+1}}{x_{i+1i+1}} - \cdots - \frac{y_{k-1k-1}}{x_{k-1k-1}} \\
&\le \frac{y_{i+2k}}{x_{i+2k}} - \frac{y_{i+1i+1}}{x_{i+1i+1}} - \cdots - \frac{y_{k-1k-1}}{x_{k-1k-1}} \\
&\le \frac{y_{k-1k}}{x_{k-1k}} - \frac{y_{k-1k-1}}{x_{k-1k-1}} \\
&\le \frac{y_{kk}}{x_{kk}}
\end{aligned}$$

Inequalities (13) and (14) can be similarly proved.

By Cost Recovery, we then have

$$\begin{aligned}
C^k &\le y_{kk} + \frac{y_{kk}}{x_{kk}} \Big(\sum_{i<k} x_{ik} + \sum_{i>k} x_{ki} + \sum_{i<k<j} x_{ij} \Big) \\
&= \frac{y_{kk}}{x_{kk}} \sum_{(i,j) \in K^{-1}(k)} x_{ij} \\
&= \frac{y_{kk}}{x_{kk}} \sum_{(m,l) \in K^{-1}(k)} x_{ml}
\end{aligned}$$

Thus

$$y_{kk} \ge \frac{C^k}{\sum_{(m,l) \in K^{-1}(k)} x_{ml}} x_{kk}$$

But note that y_{kk}/x_{kk} is the unit price (per vehicle) for vehicles on road section k only. It must be equal to

$$\frac{y_{kk}}{x_{kk}} = \frac{C^k}{\sum_{(m,l) \in K^{-1}(k)} x_{ml}}.$$

Now we show that for any $i \le j = 1, ..., n$,

$$y_{ij} = \left[\sum_{l=i}^{j} \frac{C^l \left(\sum_{m=1}^{l} \sum_{k=l}^{n} x_{mk} \right)}{\sum_{m=1}^{l} \sum_{k=l}^{n} x_{mk}} \right] x_{ij}. \tag{15}$$

By Routing-proofness,

$$\begin{aligned}
\frac{y_{ij}}{x_{ij}} &\leq \frac{y_{ii}}{x_{ii}} + \frac{y_{i+1,i+1}}{x_{i+1,i+1}} + \cdots + \frac{y_{jj}}{x_{jj}} \\
&= \frac{C^i}{\sum_{(m,l)\in K^{-1}(i)} x_{ml}} + \frac{C^{i+1}}{\sum_{(m,l)\in K^{-1}(i+1)} x_{ml}} + \cdots + \frac{C^j}{\sum_{(m,l)\in K^{-1}(j)} x_{ml}} \\
&= \sum_{l=i}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}}.
\end{aligned}$$

Thus,

$$y_{ij} \leq \sum_{l=i}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} x_{ij}. \tag{16}$$

By the budget balance condition, we have

$$\sum_{(i,j)\in V} y_{ij} \leq \sum_{(i,j)\in V} \sum_{l=i}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}} = \sum_{k=1}^{n} C^k = \sum_{(i,j)\in V} y_{ij},$$

we must have

$$y_{ij} = \left[\sum_{l=i}^{j} \frac{C^l(\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk})}{\sum_{m=1}^{l}\sum_{k=l}^{n} x_{mk}}\right] x_{ij}. \tag{17}$$

The theorem is proved. Q.E.D.

Now we relate our toll pricing method to the Shapley value in cooperative game theory and the Sequential Equal Contribution rule of the well-known airport landing fee problem (Littlechild and Owen (1973), see Thomson (2005) for a survey).

For any given coalition of agent, $S \subseteq V$, denote $K(S) = \cup_{(i,j)\in S} K(i,j)$ the set of road sections that are used by agents in S. Define the following game:

$$c(S) = \sum_{k\in K(S)} C^k, S \subseteq V. \tag{18}$$

The Shapley value of the game $c(\cdot)$ is

$$\phi_{ij}(c) = \sum_{S\subseteq V:(i,j)\in S} \frac{(|S|-1)!(|V|-|S|)!}{|V|!}[c(S) - c(S\setminus\{i,j\})], (i,j)\in V \tag{19}$$

On the other hand, in a highway toll problem assume that there is one unit of traffic for each pair of entrance and exit, i.e., $x_{mk} = 1, m \leq k = 1, ..., n$. We can easily see that the toll pricing method (1) gives rise to the following:

$$y_{ij}(V, x, C) = \sum_{l=i}^{j} \frac{C^l}{l(n-l+1)}, (i, j) \in V, \tag{20}$$

The following proposition shows that $\phi_{ij}(c) = y_{ij}(V, x, C), i \leq j = 1, ..., n$.

Proposition 1 *The Shapley value of the cost game $c(\cdot)$ defined in (18) coincides with (20), that is, the highway toll pricing method (1) when $x_{mk} = 1$, for all $m \leq k = 1, ..., n$.*

Proof: For any $k \in N$, consider the unit vector in R^n

$$\tilde{C}^k = (0, \ldots, 0, 1, 0, \ldots, 0),$$

where 1 is the k-th component of the vector $\tilde{C}^k$. For any $S \subseteq V$, the corresponding cost game defined in (18) is given by

$$\tilde{C}^k(S) = \begin{cases} 1 & \text{if } k \in K(S) \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

Clearly, all agents in $V \setminus K^{-1}(k) \subseteq V$ are dummy agents in the game $\tilde{C}^k$ and all agents in $K^{-1}(k)$ are symmetric, where $K^{-1}(k)$ is the set of agents that use the road section $k \in N$. Thus the Shapley value of the game $\tilde{C}^k$ is given by

$$\phi_{ij}(\tilde{C}^k) = \begin{cases} \frac{1}{|K^{-1}(k)|}, (i, j) \in K^{-1}(k) \\ 0, \text{ otherwise} \end{cases} \tag{22}$$

for all $(i, j) \in V$.

Since the cost vectors, $\{\tilde{C}^k\}(k \in \{1, \ldots, n\})$, form a basis of R^n, for any $C = (C^1, ..., C^n) \in R^n_+$, it can be uniquely written as $C = \sum_{k \in N} C^k \cdot \tilde{C}^k$. By the definition of the cost game, for $\emptyset \neq S \subseteq V$, we have

$$\begin{aligned} c(S) &= \sum_{i\in K(S)} C^i \\ &= \sum_{i\in K(S)} (\sum_{k\in N} [C^k \cdot \tilde{C}^k]_i) \\ &= \sum_{k\in N} C^k \cdot (\sum_{i\in K(S)} [\tilde{C}^k]_i) \\ &= \sum_{k\in N} C^k \cdot \tilde{C}^k(S) \end{aligned}$$

where $[\tilde{C}]_i$ is the i-th component of the vector $\tilde{C}$.

Because the Shapley value satisfies Additivity (and thus, *linearity* [3]), we have

$$\begin{aligned} \phi_{ij}(C) &= \sum_{k\in N} C^k \cdot \phi_{ij}(\tilde{C}^k) \\ &= \sum_{k\in N\setminus K(i,j)} 0 + \sum_{k\in K(i,j)} \frac{C^k}{|K^{-1}(k)|} \\ &= \sum_{k\in C(i,j)} \frac{C^k}{|K^{-1}(k)|} \\ &= \sum_{k=i}^{j} \frac{C^k}{|K^{-1}(k)|} \end{aligned}$$

for all $(i,j) \in V$.

What remains now is to calculate $|K^{-1}(k)|$. Recall that $|K^{-1}(k)|$ is the number of agents that have used road section k, which can be calculated as $C_k^1 \cdot C_{n-k+1}^1$, where C_k^1 means the number of entrance before exit k, and C_{n-k+1}^1 is the number of exits after entrance k. Since as long as the vehicle exits after entrance k, no matter from which entrance (before exit k, of course) it has entered, it has to use road section k. Thus the total number of agents that

[3] It is a well-known fact that Additivity plus Positivity (non-negative cost shares) implies Linearity, i.e., ϕ is linear with respect to C.

has used road section k is $C_k^1 \cdot C_{n-k+1}^1 = k(n-k+1)$. Therefore, the Shapley value is

$$\begin{aligned}\phi_{ij}(c) &= \sum_{k=i}^{j} \frac{C^k}{k(n-k+1)} \\ &= y_{ij}(V, x, C)\end{aligned}$$

for all $(i, j) \in V$. Q.E.D.

Proposition 2 *When there is only 1 unit of traffic entering at each entrance and all exit at the last exit n, the highway toll game coincides with the airport game and the toll pricing method coincides with the Sequential Equal Contribution rule of the airport game.*

Proof: Now $x_{11} = \cdots = x_{1(n-1)} = 0, x_{1n} = 1$; $x_{22} = \cdots = x_{2(n-1)} = 0, x_{2n} = 1$; ..., $x_{(n-1)(n-1)} = 0, x_{(n-1)n} = 1$; and $x_{nn} = 1$. Thus by (1), we have

$$\begin{gathered} y_{11} = \cdots = y_{1(n-1)} = 0, \\ y_{1n} = C^1 + \frac{C^2}{2} + \cdots + \frac{C^n}{n}, \\ y_{22} = \cdots = y_{2(n-1)} = 0, \\ y_{2n} = \frac{C^2}{2} + \cdots + \frac{C^n}{n}, \\ \vdots \\ y_{nn} = \frac{C^n}{n}. \end{gathered}$$

which is exactly the Sequential Equal Contribution rule of Littlechild and Owen (1973). Q.E.D.

4 Concluding Remarks

This paper proposes a highway toll pricing method that meets two tests for a toll pricing method-the stand alone test for users (vehicles) and the stand alone test (Cost Recovery) for suppliers (highway) and meanwhile satisfies a strategic property called Routing-proofness. We have shown that the Average Cost Pricing method violates the stand alone test for vehicles (users) and thus leads to cross subsidization among vehicles. On the other hand, the mile-Weighted Average Cost Pricing method fails the stand alone test (Cost Recovery) for road sections (suppliers) , thus generates subsidization across road sections.

We provide two characterizations for the toll pricing method. The first characterization uses the classical axioms of Additivity and Dummy in the cost sharing literature together with the Toll Upper Bound for Local Traffic and Routing-proofness (Theorem 1). The second characterization uses the Cost Recovery property and the Routing-proofness (Theorem 2).

Routing-proofness rules out any vehicle's (profitable) strategic maneuvers by exiting and re-entering the highway intermediately. For example, a player (vehicle), say $(1, 2)$, has no incentive to behave as two different players $(1, 1)$ and $(2, 2)$ travelling on road sections 1 and 2 consecutively by exiting at exit 1 and re-entering at entrance 2.

Cost Recovery implies that users (vehicles) of a road section do not need to subsidy users of other road section in terms of the cost of a road section. For example, if the cost of a road section is not covered by the total toll collected from the users of that section, then the budget balance condition forces some other road section(s) charges more on their users. Thus the users of the former road section are subsidized by the users (some of them may be the same users) of the latter road section.

In the case that all traffic flow is one unit, we show that the toll pricing method is the Shapley value of the associated game to the problem (Proposition 1). In an another special case where there is one unit traffic entering at each entrance but all exit at the last exit, we show that the pricing method coincides with the well-known Sequential Equal Contribution rule of the airport landing fee problem (Proposition 2). Thus, our toll pricing method can be regarded as a natural extension of the Sequential Equal Contribution rule.

Routing-proofness is an important axiom in managing network traffic, without it strategic routing may have efficient cost (Moulin, 2008). Combining with other axioms (e.g., stand-alone core stability), Routing-proofness is a powerful axiom in pricing traffic in networks.

Finally, we point out that the linear highway model can be easily generalized to deal with highway network of a tree structure. The toll pricing method can be defined accordingly. We omit the detail.

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