

# Liquidity Effects and Cost Channels in Monetary Transmission <sup>\*</sup>

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## Abstract

We study liquidity effects and cost channels within a model of nominal rigidities and imperfect competition that gives explicit role for money-credit markets and investment decisions. We find that cost channels matter for monetary transmission, amplifying the impact of supply shocks and dampening the effects of demand shocks. Liquidity effects only obtain when the policy is specified by an interest rate policy rule and money-credit conditions are determined endogenously. We also find that determinacy issues are particularly relevant when models include the cost channel and explicit money-credit markets.

JEL Codes: E52, E51, E41, E44.

Keyword: Liquidity effect, cost channel, investment finance, Taylor rule, indeterminacy

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# 1 Introduction

The impact of monetary policy on real variables is the central concern in actual monetary policymaking. To obtain monetary non-neutralities dynamic stochastic general equilibrium (DSGE) models suggest various forms of demand (e.g. sticky wages and prices) or supply side frictions (e.g. adjustment costs, credit frictions). Nevertheless, these models' main focus remains on interest rate channels of monetary transmission that works via intertemporal consumption decisions. In this paper we investigate in detail the supply side of monetary transmission. We aim to address three associated issues that require the presence of money-credit markets within a New Keynesian analysis. First, we provide a rich framework for monetary transmission that incorporates explicitly an interaction between credit markets and investment decisions. Second, we analyze money credit expansions and contractions (i.e. liquidity effects) associated with monetary policymaking. Third, we are concerned about the stability of this environment with well specified credit and money markets.

The New Keynesian literature mainly considers a cashless economy (Woodford (1999, 2003)), where money-credit markets are absent. A satisfactory analysis of liquidity considerations requires an explicit link between demand and supply of credit in the economy. We examine the role of liquidity restrictions on the demand side and factor finance considerations on the production side within a stylized model of sticky prices. In our model, supply of loans is determined by the cash-in-advance constraint and monetary injections (see Lucas (1990), Dow (1995) and Christiano and Eichenbaum (1992, 2005)) and the demand for loans is determined by working capital assumptions. As firms have to borrow to pay for the capital input and salaries, they are subject to cost channels of monetary transmission.

We show that (i) the cost channel impacts the transmission of monetary policy, dampening the output response to an exogenous monetary shock and, when an active monetary policy is assumed, supply shock effects are amplified while demand shock effects are dampened; (ii) liquidity effects of monetary policy are obtained when one considers a model with active monetary policy rule; and finally (iii) augmenting a monetary model with cost channels and a well defined money-credit market exposes the framework to indeterminacy problems, forcing the central bank to be more aggressive towards inflation and to adopt interest rate smoothing.

Our particular emphasis is on the investment finance-cost channel next to the standard labour finance-cost channel of the monetary transmission mechanism. Financial conditions affect firms' ability to produce which is accomplished by investing in capital and by how investment is financed. Together with a cash-in-advance constraint, the investment and labour cost financing allow us to establish a credit market and to generate intertemporal linkages for consumption, labour supply and investment. We show in detail how consumption and labour supply are affected by the evolution of real money balances and how investment and employment are affected by the short-term interest rates. We argue that these effects are important to understand observed comovements of key macro-

economic variables generated by a series of exogenous shocks. To the best of our knowledge we are the first to focus explicitly on the investment finance.

Ravenna and Walsh (2006) study the cost channel of monetary transmission within a simple cash-in-advance economy without capital. They show that variations in the nominal rates will affect real marginal costs; thereby directly influence the New Keynesian Phillips Curve. Their key assumption is that firms must borrow from financial intermediaries to pay labour costs at the beginning of the period and they have to pay back these loans by the end of the period. Therefore, cost of borrowing is directly influenced by the nominal rates<sup>1</sup>. Our model captures this supply side perspective with an extension to capture investment finance considerations.

Are supply side effects important? There is compelling empirical evidence that cost channels matter. Barth and Ramey (2001), for instance, show that at the manufacturing industries level strong supply-side channels in the monetary transmission are present in the short to medium run. They show that following an unanticipated monetary contraction prices rise and output falls in key US manufacturing industries after controlling for both the price puzzle and the cost effects of oil shocks. They suggest that the monetary policy may well be acting as a supply side shock to important industries in the U.S. economy. Similarly Ravenna and Walsh (2006) present corroborating econometric evidence for the direct (costly) influence of monetary policy on the U.S. inflation adjustment equation.

In the case of standard New Keynesian monetary analysis with demand channels only, a policymaker's job is straightforward. The ex-ante real interest rate stabilizes inflation via its impact on the output gap. As the policy change stabilizes both business cycle fluctuations and inflation, policymakers will not face intricate policy questions such as output inflation trade off. (e.g. Clarida et al. (1999) or Woodford (1999)). In the case of supply channels, however, a policy dilemma arises. When a contractionary monetary policy triggers a change in the marginal cost of production, associated production falls can be accompanied by a rise in prices due to supply shortages. The presence of supply channels implies that, given an exogenous demand shock, a monetary policy response dampens the impact on output, the opposite occurs with supply shocks. This asymmetric result is linked to Ravenna and Walsh's (2006) contribution, which indicate that the cost channel creates an inflation output trade-off; thereby generating a tension between aggregate demand and supply channels. We show that investment dynamics are much more significant in producing these effects in such a model setting.

Although there is ample empirical evidence on the liquidity effect of monetary policy, that is a negative comovement between money balances and nominal interest rates (Christiano et al. (1999)), this effect is difficult to obtain within standard monetary models. In these models, an exogenous expansion in money supply typically increases nominal interest rates. In practice, however,

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<sup>1</sup>For details of this version of the working capital assumption (financing wage bills via borrowing from financial intermediaries) see Christiano and Eichenbaum (1992, 2005).

the monetary policy is conducted by interest rate announcements and open market operations such that money liquidity is determined endogenously in money markets. We suggest that to obtain more realistic liquidity effects, we need to study movements in endogenous money balances in response to an implementation of an interest rate rule. We show that movements in money balances, that are movements in the demand for loans, are conditional on the type of exogenous shocks and on the central bank's stabilization response to these shocks. Our model shows that an inflation shock generates a contemporaneous liquidity effect, whereas aggregate demand shocks generate delayed liquidity effects.

Altering the standard New Keynesian model by incorporating supply side considerations and money-credit markets seems to have important local determinacy implications.<sup>2</sup> We first find that determinacy regions are much more narrow as compared to the literature and second the Taylor-Woodford principle is often violated. Our analysis suggests that when the monetary transmission is characterized by supply side as well as demand side channels, inflation conservatism may be paramount to obtain locally unique equilibrium, reinforcing the demand channel effect of a policy response. Furthermore, in such settings strong interest rate smoothing becomes critical.

In sum, we claim that when both supply and demand channels of monetary transmission are present, monetary policymaker's stabilization attempts may be subject to further challenges and trade-offs. We, therefore, argue that monitoring money/credit conditions can potentially offer useful information for the implementation of monetary policy.

The rest of the paper is organized as follows. Section 2 presents the outline of the cash-in-advance model with labour and investment cost channel considerations. Section 3 presents the steady state of the economy. Section 4 sets out equilibrium conditions for the cash-in-advance model and the standard benchmark New Keynesian macro-model with cost channels. Section 5 presents the quantitative evaluation of the model where we discuss monetary policy transmission, liquidity effects and indeterminacy conditions. Finally, Section 6 concludes.

## 2 A Cash-in-advance Model

The economy consists of a representative household, a firm and a financial intermediary.

### 2.1 Households

Formally, the household is maximizing its discounted lifetime utility given by:

$$\max_{C_t, M_{t+1}, D_t, A_t, H_t} E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right), \quad \beta \in (0, 1) \quad \sigma, \eta > 0 \quad (1)$$

<sup>2</sup>See also Surico (2008) on a related concern.

where  $C_t$  denotes the household's total consumption,  $H_t$  denotes hours worked. The curvature parameters  $\sigma, \eta$  are strictly positive.  $\beta$  is the discount factor. The family faces the following budget and cash in advance constraints:

$$C_t + \frac{D_t}{P_t} + \frac{M_{t+1}^d}{P_t} + \frac{Q_{t,t+1}A_t}{P_t} \leq \frac{W_t}{P_t}H_t + \frac{A_{t-1}}{P_t} + \frac{R_t D_{t-1}}{P_t} + \frac{M_t}{P_t} \quad (2)$$

$$+ \int_0^1 \Pi_{i,t} di + \Pi_t^{FI} - T_t$$

$$C_t + \frac{D_t}{P_t} \leq \frac{M_t}{P_t} \quad (3)$$

where  $R_t$  is the rate of return of a one period deposit  $D_{t-1}$ ,  $M_{t+1}^d$  are money holdings carried over to period  $t+1$ ,  $A_t$  represents alternative physical assets valued at the stochastic discount factor  $Q_{t,t+1}$ ,  $\int_0^1 \Pi_{i,t} di$  represents dividends accrued from the intermediate producers to households,  $\Pi_t^{FI}$  represents profits of the financial intermediary accrued to the household, and finally  $T_t$  stands for the lump-sum taxes households has to pay. Similar to our paper, Wang and Wen (2006) consider a case with a cash-in-advance in consumption and investment, but they do not introduce the cost channel.

The cash-in-advance constraint imposes the condition that the household needs to allocate money balances for consumption purposes net of deposits she has decided to allocate to the financial intermediary. The literature also considers the cash-in-advance model without a labour supply effect or a constraint given by

$$C_t + \frac{D_t}{P_t} \leq \frac{M_t}{P_t} + \frac{W_t}{P_t}H_t,$$

where the wage payments are made at the beginning of period  $t$ . This specification implies that the labour supply is not affected by real balances (see Christiano and Eichenbaum (1992)). Another alternative specification regards the assumption on the timing of deposits, which is important for the evolution of consumption. In our benchmark model we assumed deposits are made in the current period and paid back in the next. In the appendix we present in more detail the implications of assuming deposits are paid back in the same period (intra-period deposits) and assuming the cash-in-advance constraint includes wage payments.

## 2.2 Firms

The final goods representative firm produces goods combining a continuum of intermediate goods  $i \in [0, 1]$  with the following production function:

$$Y_t = \left[ \int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (4)$$

As standard this implies a demand function given by:

$$y_{it} = \left( \frac{p_{it}}{P_t} \right)^{-\varepsilon} Y_t \quad (5)$$

where the aggregate price level is:

$$P_t = \left[ \int_0^1 p_{i,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (6)$$

The intermediate sector is constituted of a continuum of firms  $i \in [0, 1]$  producing differentiated goods with the following constant returns to scale production function:

$$y_i = K_i^\alpha H_i^{1-\alpha} \quad (7)$$

where  $K$  is the capital stock and  $H$  is the labour used in the production. The firm hires labour and buys capital (goods) from the final good producer. It is assumed that the firm must borrow money to pay for these expenses.

To characterize the problem of intermediate firms, as standard, we split their decision into a pricing decision given the real marginal cost and then solve for their cost minimization problem.

Following the standard Calvo pricing scheme, firm  $i$ , when allowed, sets prices  $P_{i,t}$  according to:

$$\max_{P_{i,t}} E_t \left\{ \sum_{s=0}^{\infty} Q_{t,t+s} \omega^s y_{i,t+s} \left[ \frac{P_{t,i}}{P_{t+s}} - \Lambda_{t+s} \right] \right\} \quad (8)$$

subject to the demand function (5), where  $\Lambda_t$  is the real marginal cost of the firm. To obtain the real marginal cost, we need to solve firm's intertemporal cost minimization problem. That is:

$$\min_{K_{i,t+1}, H_{i,t}} E_t \left\{ \sum_{t=0}^{\infty} Q_{t,t+1} (R_t^{\gamma_1} W_t H_{i,t} + R_t^{\gamma_2} P_t I_{i,t}) \right\} \quad (9)$$

subject to the production function (7) and investment equation  $I_{i,t} = K_{i,t+1} - (1 - \delta)K_{i,t}$ ; where  $W_t$  is the nominal wage, and  $R_t$  the rate the bank charge for the loan made in period  $t$ , to be paid in  $t + 1$  and  $\Lambda_t$  is the multiplier of the constraint (7).

Expression  $R_t^{\gamma_1} W_t H_{i,t} + R_t^{\gamma_2} P_t I_{i,t}$  in the cost minimization exercise characterizes the costs of firms given that they need to borrow from the financial intermediary to finance wage and investment payments. Parameters  $\gamma_1 \in [0, 1]$ ,  $\gamma_2 \in [0, 1]$  specify the importance of the cost channel of labour and investment, respectively. Full cost channel is represented by  $\gamma_1 = \gamma_2 = 1$ . Later on, we will study different versions of cost channel by varying these parameters. Finally, to clarify notation, the stochastic discount factor in period  $t$  for period  $t$  is given by  $Q_{t,t} = 1$ . Firms use the stochastic discount factor obtained from the household consumption condition in their production decisions.

### 2.3 Financial Intermediary

We assume that the financial intermediary acts in the interest of the household. That means she will optimize the discounted cash flow of the consumer. The competitive market financial intermediary (*FI*) gets deposits from the household and lends money to the firms in the form of loans (*L*). In addition to that the central bank might inject money (*V*) into the economy by giving it directly to the financial intermediary, who will then add to the other funds used for lending. At the end of each period the money injected then forms part of the household assets (as money holdings). Formally the financial intermediation profits, which are part of the household budget constraint (given  $V_t$ ), are given by:

$$\begin{aligned} & \underset{D_t}{Max} R_{1,t}^L D_t - R_{1,t} D_t \\ & R_{1,t}^L = R_{1,t}. \end{aligned}$$

Loan market equilibrium requires:

$$L^s(V_t, R_{1,t}^L) = L^D(R_{1,t}^L).$$

In equilibrium the demand for credit to pay the production input must be equal to the supply of credit made by the banking system. The credit supply is determined by deposits and the monetary injection. Therefore, the credit market condition is given by:

$$\gamma_1 W_t H_t + \gamma_2 P_t I_t = D_t + V_t. \quad (10)$$

Once again,  $\gamma_1 > 0$  implies firms must borrow to pay the wage bill and  $\gamma_2 > 0$  implies firms must borrow to invest.

The money supply is determined by the government. Then, the government budget constraint is given by:

$$T_t + V_t = 0, \quad (11)$$

where  $T$  stands for taxes received. Note that any profit accruing to the FI due to monetary injections are transferred to the household by the end of the period in the form of financial intermediary dividends.

### 2.4 Equilibrium

The equilibrium of the economy is defined as the vector of Lagrange multipliers  $\{\lambda_t, \mu_t, \Lambda_t\}$ , the allocation set  $\{C_t, H_t, K_{t+1}, D_t, M_{t+1}, Y_t\}$ , and the vector of prices  $\{p_{i,t}, P_t, W_t, R_t\}$  such that the household, the final good firm and intermediate firms maximization problems, the market clearing conditions and the government budget constraint hold.

Consumer problem is represented by the following first order conditions:

$$\beta^2 E_t \left( \frac{R_{t+1} C_{t+2}^{-\sigma}}{\pi_{t+2} \pi_{t+1}} \right) = E_t (C_t^{-\sigma}) \quad (12)$$

$$\frac{\chi H_t^\eta}{\beta E_t (C_{t+1}^{-\sigma})} = \frac{W_t}{P_t E_t (\pi_{t+1})}. \quad (13)$$

In this case, the labour supply shows a *real money balance effect*, i.e. expected future inflation rates directly affect the labour-consumption decision and the labour supply depends on future consumption.

From the consumer problem we obtain the stochastic discount factor:

$$Q_{t,t+1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} = \beta E_t \left[ \frac{C_{t+2}^{-\sigma}}{C_{t+1}^{-\sigma} \pi_{t+2}} \right].$$

The goods market clearing condition is given by:

$$Y_t = C_t + I_t. \quad (14)$$

The capital and labour market clearing condition are given by:

$$K_t = \int_0^1 K_{i,t} di \text{ and } H_t = \int_0^1 H_{i,t} di. \quad (15)$$

Investment evolves according to:

$$I_{i,t} = K_{i,t+1} - (1 - \delta) K_{i,t}. \quad (16)$$

The price setting equation is given by solving (8):

$$P_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \{ \sum_{s=0}^{\infty} Q_{t,t+s} \omega^s \Lambda_{t+s} y_{i,t+s} \}}{E_t \left\{ \sum_{s=0}^{\infty} Q_{t,t+s} \omega^s \frac{y_{i,t+s}}{P_{t+s}} \right\}} \quad (17)$$

Finally, from the firm problem we obtain the demand for capital and labour and the optimal price. Next, we describe the three main cases of cost channels through which nominal rates affect real marginal cost of the firm.

**Full Cost Channel:** After rearranging first order conditions and using the stochastic discount factor  $Q_{t,t+1} = \beta E_t \frac{(C_{t+2}^{-\sigma})}{(C_{t+1}^{-\sigma}) \pi_{t+2}}$ , we obtain the following equilibrium conditions under the full cost channel:

$$\Lambda_t = \frac{R_t^{\gamma_1} W_t H_{i,t}}{P_t Y_{i,t} (1 - \alpha)} \quad (18)$$

$$R_t^{\gamma_2} = \beta E_t \left\{ \frac{(C_{t+2}^{-\sigma}) \pi_{t+1}}{(C_{t+1}^{-\sigma}) \pi_{t+2}} \left[ \Lambda_{t+1} \frac{\alpha Y_{i,t+1}}{K_{i,t+1}} + (1 - \delta) R_{t+1}^{\gamma_2} \right] \right\}. \quad (19)$$

As conditions (18) and (19) reveal, when both cost channels of labour and investment are present, the real marginal cost of the firm will be, among others, a function of both current and future expected short term rates. The investment cost channel also reveals the impact of the expected labour supply decisions on the real marginal cost.



**Cost Channel in Labour:** Ravenna and Walsh (2006) derive their aggregate supply equation based on the impact of policy changes on labour cost financing leaving aside the impact of the policy decisions on the investment finance costs. To get Ravenna and Walsh (2006), we need to remove the cost channel in investment, i.e.  $\gamma_2 = 0$ . Setting  $\gamma_2 = 0$  we obtain<sup>3</sup>:

$$\Lambda_t = \frac{R_t^{\gamma_1} W_t H_{i,t}}{P_t Y_{i,t} (1 - \alpha)} \quad (20)$$

$$1 = \beta E \left\{ \frac{(C_{t+2}^{-\sigma}) \pi_{t+1}}{(C_{t+1}^{-\sigma}) \pi_{t+2}} \left[ \Lambda_{t+1} \frac{\alpha Y_{t+1}}{K_{t+1}} + (1 - \delta) \right] \right\}. \quad (21)$$

As revealed in conditions (20) and (21), in the case of the labour cost channel only, expected labour supply and nominal rates in period  $t$  still affect the real marginal cost. Carlstrom and Fuerst (2005) present a model without cash-in-advance and with capital accumulation. Note that their equilibrium conditions are recursive, while in our model cash-in-advance constraint leads to an impact of the expected two period ahead inflation rate on real variables.

**Cost Channel in Investment:** By assuming  $\gamma_1 = 0$  we obtain the case for the cost channel only in investment (see Kurozami and Van Zandweghe (2008)):

$$\Lambda_t = \frac{W_t H_{i,t}}{P_t Y_{i,t} (1 - \alpha)} \quad (22)$$

$$R_t^{\gamma_2} = \beta E_t \left\{ \frac{(C_{t+2}^{-\sigma}) \pi_{t+1}}{(C_{t+1}^{-\sigma}) \pi_{t+2}} \left[ \Lambda_{t+1} \frac{\alpha Y_{t+1}}{K_{t+1}} + (1 - \delta) R_{t+1}^{\gamma_2} \right] \right\}. \quad (23)$$

In the case of the investment cost channel only, both current and expected rates influence real marginal costs. Note that Dow (1995) obtains a similar expression for investment using a slightly different discount factor, since firms have to pay the capital input in advance and an increase in nominal interest rates raises the capital cost.

To establish the money market condition we need an expression that links short term rates and monetary injections ( $V_t$ ). We obtain this by combining the cash-in-advance specification (3) and the credit market clearing condition (10). At the credit market equilibrium, the demand for credit to pay the production input must be equal to the supply of credit made by the banking system. The credit supply is determined by the deposits and the monetary injection. Therefore, the credit market condition is given by:

$$\gamma_1 W_t H_t + \gamma_2 P_t I_t = D_t + V_t. \quad (24)$$

Combining this result with the cash-in-advance constraint and money market flow equation gives the following money/credit condition:

<sup>3</sup>Note that Ravenna and Walsh's (2006) model has a cash-in-advance constraint with wage payments and deposits clear at the same period (see appendix).

$$\gamma_1 \frac{W_t H_t}{P_t} + \gamma_2 I_t + C_t = \frac{M_{t+1}}{P_t}. \quad (25)$$

### 3 Steady State with Flexible Prices and Zero Inflation

The New Keynesian literature considers that the government implements zero inflation at the steady state (see Ravenna and Walsh (2006)). While we retain this assumption, it is important to note that even in the steady-state with flexible prices, the monetary policy is non-neutral. The reason is that the cost channel implies a steady state relation between investment/employment and the nominal rate, which in turn is determined by the inflation rate. To show this we write from the consumption Euler condition:

$$R = \left( \frac{\pi}{\beta} \right)^2 \quad (26)$$

where  $R$  and  $\pi$  are directly associated. We are going to normalize the aggregate price level  $P = 1$ . When wage payments are made at beginning of the period we obtain the conventional labour supply condition:

$$\frac{\chi H^\eta}{C^{-\sigma}} = \omega$$

where  $\omega$  is now the real wage. In the case of a cash constrain without wage-payment, however, the labour supply is affected by the nominal interest rate, therefore, monetary policy is not neutral at the steady-state, even in the case of flexible prices:

$$\frac{\pi \chi H^\eta}{\beta C^{-\sigma}} = \omega$$

i.e. inflation affects the labour supply and the real wage.

Finally in the case of full cost channel we show the effect of inflation in the firm employment and investment decision. At the symmetric equilibrium, price over marginal cost is given by  $P = \frac{1}{\mu} MC$ , since  $P = P_i$ . Now, since  $\Lambda$  is the real marginal cost and  $\mu = \frac{MC}{P} = \Lambda$ , at the steady state marginal cost relations read as:

$$\mu = \omega \frac{H}{Y(1-\alpha)} \left( \frac{\pi}{\beta} \right)^2 \quad (27)$$

$$\left( \frac{\pi}{\beta} \right)^2 (1 - \beta(1 - \delta)) = \beta \left[ \mu \frac{\alpha Y}{K} \right] \quad (28)$$

Equations (27) and (28) indicate that at the steady state the policymaker can, in principle, inflate away mark-up distortions. However, in what follows we

ignore such price inflation effects on the real economy under flexible prices, i.e. we will work with log-linearised model around a zero inflation steady state (i.e.,  $\pi = 1$ ) as in Ravenna and Walsh (2006).

## 4 The Linearized Model

The linear model, based on equilibrium conditions (7), (12), (14) - (19) and (25), for the set of variables  $\{\hat{c}_t, \hat{r}_t, \hat{\Lambda}_t, \hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{k}_{t+1}, \hat{h}_t, \hat{m}_t\}$  is summarized as follows:

$$E_t(\hat{c}_t) = E_t(\hat{c}_{t+2}) - \frac{1}{\sigma} E_t[\hat{r}_{t+1} - \hat{\pi}_{t+1} - \hat{\pi}_{t+2}] \quad (29a)$$

$$\begin{aligned} \gamma_2 \hat{r}_t &= -\sigma(E_t(\hat{c}_{t+2}) - \hat{c}_{t+1}) - E_t(\hat{\pi}_{t+2}) + E_t(\hat{\pi}_{t+1}) + \\ &+ (1 - \beta(1 - \delta))E_t[\hat{y}_{t+1} + \hat{\Lambda}_{t+1} - \hat{k}_{t+1}] + \beta\gamma_2(1 - \delta)E_t(\hat{r}_{t+1}) \end{aligned} \quad (29b)$$

$$\hat{\Lambda}_t = \gamma_1 \hat{r}_t + (1 + \eta)\hat{h}_t + \sigma E_t \hat{c}_{t+1} + E_t \hat{\pi}_{t+1} - \hat{y}_t \quad (29c)$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa \hat{\Lambda}_t \quad (29d)$$

$$\hat{y}_t = s_c \hat{c}_t + s_I \hat{i}_t \quad (29e)$$

$$\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + \delta \hat{i}_t \quad (29f)$$

$$\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha)\hat{h}_t \quad (29g)$$

$$\hat{m}_{t+1} = \hat{m}_t - \hat{\pi}_t + v_t \quad (29h)$$

$$\left[ \begin{array}{l} \gamma_1(\hat{\Lambda}_t - \gamma_1 \hat{r}_t) = \frac{m/Y}{\varpi} \hat{m}_{t+1} - \frac{s_c}{\varpi} \hat{c}_t - \frac{\gamma_2 s_I}{\varpi} \hat{i}_t - y_t \quad \text{for } \gamma_1 \neq 0 \\ \hat{m}_{t+1} = \frac{s_c}{(s_c + \gamma_2 s_I)} \hat{c}_t + \frac{\gamma_2 s_I}{(s_c + \gamma_2 s_I)} \hat{i}_t \quad \text{for } \gamma_1 = 0 \end{array} \right] \quad (29i)$$

where  $\kappa = (1 - \omega)(1 - \omega\beta)/\omega$ ,  $s_c = C/Y$ ,  $s_I = I/Y$  and  $\varpi = m/Y - (s_c + \gamma_2 s_I)$ .

Equation (29h) shows the flow of real money balances. Under this framework monetary policy is exogenously set through monetary injections. In order to study active monetary policy making we also consider a cash-in-advance model where this equation is replaced by an interest rate rule of the form:

$$\hat{r}_t = \epsilon_y \hat{y}_t + \epsilon_\pi \hat{\pi}_t + \epsilon_r \hat{r}_{t-1} + v_t. \quad (30)$$

For comparison purposes we also present the New Keynesian model ( $NK\_i$ ) for a cashless economy with investment. This reference linear model is presented for the following set of variables  $\{\hat{c}_t, \hat{r}_t, \hat{\Lambda}_t, \hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{k}_t, \hat{h}_t\}$ :

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \frac{1}{\sigma} E_t [\widehat{r}_t - \widehat{\pi}_{t+1}] \quad (31a)$$

$$0 \approx -E_t [\widehat{r}_t - \widehat{\pi}_{t+1}] + \quad (31b)$$

$$+(1 - \beta(1 - \delta)) E_t [\widehat{y}_{t+1} + \widehat{\Lambda}_{t+1} - \widehat{k}_{t+1}]$$

$$\widehat{\Lambda}_t = (1 + \eta) \widehat{h}_t + \sigma \widehat{c}_t - \widehat{y}_t \quad (31c)$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa \widehat{\Lambda}_t \quad (31d)$$

$$\widehat{r}_t = \epsilon_y \widehat{y}_t + \epsilon_\pi \widehat{\pi}_t + \epsilon_r \widehat{r}_{t-1} \quad (31e)$$

$$\widehat{y}_t = s_c \widehat{c}_t + s_I \widehat{i}_t \quad (31f)$$

$$\widehat{k}_{t+1} = (1 - \delta) \widehat{k}_t + \delta \widehat{i}_t \quad (31g)$$

$$\widehat{y}_t = \alpha \widehat{k}_t + (1 - \alpha) \widehat{h}_t \quad (31h)$$

The  $NK_i$  model is useful benchmark to understand the transmission of the monetary policy in the cash-in-advance model. The monetary transmission in the NK model is guided by the evolution of real interest rates, which affect consumption and investment, while in the cash-in-advance model the monetary policy is transmitted through a set of additional channels that we summarize below<sup>4</sup>:

- Real balances effect on consumption Euler condition: in the cash-in-advance model, consumption is determined by consumption two period ahead  $E_t(\widehat{c}_{t+2})$ , and the future evolution of interest rates. In the New Keynesian model the consumption is determined by future consumption and real interest rates only.
- Real balances effect on labour supply: in the cash-in-advance model, the labour supply is affected by future consumption and future inflation. These effects are transmitted to inflation through the marginal costs. In the Appendix, we show that this effect is not present when the salaries are paid at the same time that consumers make decision about deposits.
- Cost channel: in the cash-in-advance model, the existence of the money-credit market imposes a direct interaction of cost channel and the demand for loans and the interest rate. This money market condition and channel cost effect are absent in the standard NK Model.

## 5 Quantitative Evaluation

In this section we assess quantitative properties of the three models we have (cash-in-advance model, cash-in-advance model with a Taylor Rule and the NK

<sup>4</sup> Assuming that deposits are paid intra-period and wage payments are included into the cash-in-advance constraint one can isolate the effect of the cost channel on the monetary transmission, offsetting the real balance effect on consumption and labour supply. Simulation results under these assumptions are presented in a technical appendix available upon request.

macro model). In Section 5.1. we discuss the assigned parameter values. In Section 5.2. we present the monetary transmission with the unlikely case of exogenous monetary policy. In Section 5.3.1 we discuss monetary transmission after the implementation of interest rate rules under various shock scenarios. In Section 5.3.2 we will pay particular attention to liquidity effects with Taylor rule specifications. Quantitative evaluation concludes with the discussion of (in)determinacy problems.

## 5.1 Parameter Values

The cash-in-advance model has 12 free parameters:  $\sigma, \gamma_1, \gamma_2, \delta, \eta, s_c, s_I, \alpha, \beta, \omega, \epsilon_v$  and  $\rho$ . We set the parameter of intertemporal elasticity of substitution  $\sigma = 1$  and the parameter of intertemporal elasticity of labour supply  $\eta = 1.03$ . The discount factor,  $\beta$ , is calibrated to be 0.99, which is equivalent to an annual steady state real interest rate of 4 percent. The depreciation rate,  $\delta$ , is set equal to 0.02 per quarter. We set  $\alpha = 0.33$  which roughly implies a steady state share of labour income in total output of 66%. The share of steady state consumption ( $s_c$ ) is set equal to 0.7, while the share of steady state investment ( $s_I$ ) is set equal to 0.3.  $\epsilon_v$  represents the parameter for the credit-money markets. We resort to Christiano and Eichenbaum (2005) in calculating the parameter who report that the steady state velocity of money  $\frac{m}{Y} = 0.44$ . This implies in our case a value for  $\epsilon_v = \frac{m}{m-Y} = 1.7857$ . Parameters  $\gamma_1, \gamma_2$  regulate the importance of labour and investment cost channels, where  $\gamma_{1,2} = 1$  implies full-cost channel,  $\gamma_{1,2} = 0$  implies no-cost channel. Throughout simulations shocks are persistent. The parameter  $\rho$  represents the persistence of shocks and we set this equal to 0.5 in all simulations. We set the value of the Calvo parameter  $\omega$  (fraction of firms which do not adjust their prices) as equal to 0.75 consistent with the findings reported in Gali and Gertler (1999). Finally, we consider the following parameters in the Taylor rule  $\epsilon_y = 0.5$ ,  $\epsilon_\pi = 1.5$ , and  $\epsilon_r = 1$ . Later on, we study the stability of the model under various Taylor rule parameters.

## 5.2 Transmission of Monetary Policy with an Exogenous Money Supply Rule

In this section we discuss the case of passive monetary policy where the policymaker follows an exogenous money supply rule, focusing particularly on the importance of cost channels on policy transmission by varying the values of  $\gamma_1$  and  $\gamma_2$ . A brief discussion on the existence of unique equilibrium is in order before presenting the impulse responses. Unique equilibrium obtains while holding  $\gamma_2 = 1$  fixed and varying  $\gamma_1$  from zero to one, however, indeterminacy obtains for values of  $\gamma_2$  smaller than 0.45 (holding  $\gamma_1 = 1$ ). When both  $\gamma_1$  and  $\gamma_2$  are reduced together, the lower possible value to ensure determinacy is  $\gamma_1 = \gamma_2 = 0.48$ . A more detail analysis of indeterminacy in our framework will be analyzed in section 5.3.3. In order to compare different degrees of cost

channel we analyze the response of key macroeconomic variables given a money shock for four cases: (1) full cost channel ( $\gamma_1 = \gamma_2 = 1$ ), small cost channel ( $\gamma_1 = \gamma_2 = 0.5$ ), labour bias cost channel ( $\gamma_1 = 1$  and  $\gamma_2 = 0.5$ ) and investment bias cost channel ( $\gamma_1 = 0.5$  and  $\gamma_2 = 1$ ).

Figure 1 shows the comparison between the first two, highlighting the effects of reducing  $\gamma_1$  and  $\gamma_2$  together. Firstly, while the inflation response is very similar, the output response to a monetary shock is considerably smaller in the full cost channel case. More strikingly, the components of output, consumption and investment, move in opposite directions, while in the full cost case consumption increases while investment initially decreases, in the small cost case investment increases while consumption initially decreases (albeit mildly). The lower  $\gamma_1$  and  $\gamma_2$  are, lower the fund constraint and interest rate cost of investment will be, allowing investment to respond more to a monetary stimulus, overcrowding the consumption increase. Furthermore, as output respond more, real rates are greater, further depressing the initial consumption response.

**[Insert Figure 1 here]**

Figure 2 presents the impulse responses when  $\gamma_1$  and  $\gamma_2$  are reduced separately. When the investment cost channel is stronger ( $\gamma_2 > \gamma_1$ ), or as  $\gamma_1$  decreases, consumption and inflation responses remain the same, but as the fund constraint on investment decreases, investment does not decrease as much, leading to an increase in output. On the other hand as  $\gamma_2$  decreases, both the fund constraint and cost of investment decreases, leading to an initial increase in investment. Consumption, however, decreases, leading to a smaller increase in output. Given the similarities between the full cost and investment cost channel case, we conclude that the investment dynamics and channel are very significant in determining the dynamic responses in our model.

**[Insert Figure 2 here]**

In all simulation results presented so far we observe an increase in the nominal interest rate after money supply expands, hence, we do not observe a liquidity effect in any of the specifications. In fact real rates also increase in all cases. Although the money injection increases the supply of funds, the amount of deposits decreases and the demand of funds increases such that the equilibrium in the credit market is such that real rates go up. The fact that deposits take a period to clear, which imply deposits tend to decrease after a monetary injection, and that firms must borrow to invest, which implies that holding real rates constant imply a sharp increase in the demand for loans for investment purposes, are the main drivers of this result<sup>5</sup>.

Nonetheless, the assumption that money supply follows an exogenous process is at odds with monetary policymaking. Given a set of macroeconomic conditions, monetary policy is conducted by interest rate announcements and open market operations such that money liquidity is determined endogenously in money markets. Hence, in section 5.3.3 we analyze liquidity effects as response

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<sup>5</sup>When only the labour cost is present, deposits clear in the same period, wages are included into the cash-in-advance constraint and money supply shock are *iid*, we are able to obtain a liquidity effect. The simulation results are available upon request.

to an active monetary policy in the presence of aggregate demand and supply shocks.

### 5.3 Monetary Policy with a Taylor Rule

It has been extensively argued that monetary policy rules, where monetary authority reacts to inflation and output gap, are remarkably successful for stabilization purposes. Therefore, as a corollary to our cash-in-advance model with money supply process we want to evaluate the Cash-in-advance model under interest rate policy rules. A typical simple Taylor rule with interest rate smoothing takes the following form:

$$\hat{r}_t = \epsilon_y \hat{y}_t + \epsilon_\pi \hat{\pi}_t + \epsilon_r \hat{r}_{t-1} + v_t \quad (32)$$

where  $\epsilon_y$  is the output gap coefficient and  $\epsilon_\pi$  is the inflation coefficient in the Taylor rule.

Closing the model with a Taylor rule, instead of a monetary condition as given by (29h), allow us *(i)* to study the economy's response to different demand and supply shocks under an active monetary policy, as observed in reality, inferring the relevance of cost channels to this process, *(ii)* to study liquidity effects, that is the relation between monetary aggregates and the nominal rate of interest through reverse engineering. In principle, the standard way to look at the liquidity effects of monetary policy changes is to first assume an innovation to money supply process and see how in equilibrium interest rates will be affected. Nevertheless, given that in our model we allow for explicit credit market condition, we can assess the implications of simple Taylor rule type of policies on equilibrium demand for credit, and therefore, for monetary conditions. And finally, it allow us *(iii)* to verify the conditions on monetary policy making, or more precisely the constraints on the Taylor rule's parameters to ensure determinacy of equilibrium.

#### 5.3.1 Transmission of Shocks

Before we study the statistical properties and impulse responses of key variables with respect to a range of shocks we briefly recall the cash-in-advance model implications for monetary transmission. Our model assumptions so far imply an important role for investment in the monetary transmission. As argued in the introduction, most NK models assume that monetary transmission actually occurs through an intertemporal substitution channel in consumption. If the policymaker follows a Taylor rule, this modelling approach implies that the policymaker faces no trade off between real output and inflation under aggregate demand shocks. In contrast, here we allow for supply side implications of policy changes thereby introducing a trade off between real output and inflation. Ultimately, the outcome of policy changes will depend on the relative importance of aggregate demand versus aggregate supply channels.

We consider four types of shocks: a taste shock directly associated with the consumption Euler equation, an investment shock, that reflects an unex-

pected boost in investment, an inflation (or supply) shock associated with the New Keynesian Phillips Curve and finally a policy shock to the Taylor Rule. The vector of shocks is defined as  $\xi_t = [\varepsilon_{c,t}, \varepsilon_{I,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t}]'$ . All shock processes are assumed to have an autocorrelation coefficient equal to 0.5; their standard deviations are set equal to 1%.

In Table 1 we present the correlation structure between real output and key macroeconomic variables and in Table 2 we present the ratio of standard deviations of these variables and of real output. Here we assume that the economy is independently hit by each of the four different types of shocks simultaneously (*all shocks*). We compare correlations and standard deviations statistics with the US data as reported by Stock and Watson (1999) in the final column. In reporting the data we refer to non-durables consumption as a proxy for consumption, total fixed investment as a proxy for investment, consumer price index as a proxy for inflation, federal funds rate for nominal short term rates and finally real gross domestic product as a proxy for real output.

We compare the results for three basic models, New Keynesian model with investment (*NK<sub>i</sub>*), our benchmark cash-in-advance model with a Taylor rule (*CIATR*), and the standard New Keynesian model (*NK*).

**Table 1: Contemporaneous Correlations with output (in %)**

	<i>c - shock</i>	<i>i - shock</i>	$\pi$ - shock	<i>r - shock</i>	<i>all shocks</i>	<i>Data</i>
<i>NK<sub>i</sub></i>						
<i>c</i>	-85.1	-81.4	21.3	20.4	-25.4	74
<i>i</i>	91.5	96.2	98.1	98.9	90.2	82
$\pi$	-96.0	-21.5	-99.3	99.4	-66.9	35
<i>r</i>	-96.3	83.6	-98.1	97.1	-0.8	38
<i>CIATR</i>						
<i>c</i>	45.0	-53.1	35.8	70.3	-1.21	74
<i>i</i>	45.3	95.2	94.7	98.9	94.4	82
$\pi$	43.7	46.9	-96.6	91.6	-60.5	35
<i>r</i>	12.2	58.4	-87.8	67.7	-24.2	38
<i>NK</i>						
$\pi$	72	-	-91	99	-20	35
<i>r</i>	58	-	-99	-98	-80	38



**Table 2 : Standard Deviations vis-a-vis output**  $\left(\frac{\sigma_x}{\sigma_y} \text{ in } \%\right)$

	<i>c - shock</i>	<i>i - shock</i>	$\pi - shock$	<i>r - shock</i>	<i>all shocks</i>	<i>Data</i>
<i>NK_i</i>						
<i>c</i>	417	117	27	21	81	67
<i>i</i>	1268	577	326	327	327	299
$\pi$	39	16	37	28	28	87
<i>r</i>	23	49	12	14	14	89
<i>CIATR</i>						
<i>c</i>	158	68	46	26	50	67
<i>i</i>	370	438	311	293	354	299
$\pi$	23	17	38	32	34	87
<i>r</i>	80	69	36	23	42	89
<i>NK</i>						
$\pi$	16	-	46	39	35	87
<i>r</i>	94	-	45	19	51	89

**Effects of All Shocks Together:**

The most notable difference in the results presented in Table 1 (column 5) is that in the case of the *NK\_i* model there is a significant negative association (-25.4%) between consumption and real output whilst in the *CIATR* model this correlation is significantly smaller (-1.2%). By definition *NK* benchmark model introduces 100% correlation between consumption and real output. Investment is very highly correlated with real output in both models with investment. This is because in the *CIATR* model any change in real interest rates leads to an instantaneous adjustment in credit demand for investment/associated production. In the *NK\_i* model as credit markets are undefined there is no constraint on the investment adjustment. Furthermore, models with investment exhibit significant negative relation with nominal rates, i.e. tight monetary policy is associated with production falls.

In Table 2 (column 5) we report relative standard deviations with all shocks. Both the *NK\_i* model and the *CIATR* model mimic the typical rankings of relative volatilities of key variables found in actual data. In other words, we find that investment is more volatile than real output and real output is more volatile than consumption in both models. The *CIATR* model seems to do better in matching the correlation between interest rate and output and consumption and output.

**Effects of Individual Shocks:**

Now we evaluate implications of individual shocks separately. Firstly, we observe that for all individuals shocks consumption and output are more positively correlated in the *CIATR* model than in the *NK\_i* model. Moreover, in the case of a taste shock, while consumption is countercyclical in the *NK\_i* model, it is procyclical in the *CIATR* model. These results indicate that the *CIATR* model, when shocks are appropriately adjusted by variances and covariances, should be more compatible with the actual data.

Secondly, due to the presence of the supply channels and the real balance effects due to the introduction of a money-credit market, both taste and investment shocks generate a positive comovement of inflation and output. This comovement is not observed in the  $NK\_i$  model.

Thirdly, in the case of  $NK\_i$  model standard deviations of consumption and investment are particularly high when the economy is subject to aggregate demand shocks as compared to the  $CIATR$  model. In general, an explicit incorporation of the money credit markets associated with cash-in-advance constraints seem to dampen investment and consumption volatilities. Furthermore, interest rates are more volatile in the  $CIATR$  model than in the  $NK\_i$  for all individual shocks, bringing the results closer to the ones observed in the data.

Finally we analyze the impulse responses to the key variables for each of the four shocks considered assuming a full cost channel ( $\gamma_1 = \gamma_2 = 1$ ) and a small cost channel ( $\gamma_1 = \gamma_2 = 0.3$ )<sup>6</sup>. Figure 3 shows the output responses while figure 4 shows the inflation responses to the each of the four shocks (consumption and investment responses are shown in the appendix). While under a full cost channel output is more responsive to supply and policy shocks, under a small cost channel output is more responsive to aggregate demand shocks. It turns out that a reduced cost channel dampens the cost of credit for investment and consumption under aggregate demand shocks, while a full cost channel has more pronounced implications on the cost of financing, i.e. the demand for credit to finance labour and investment. Following the same reasoning inflation responses are likewise stronger under small cost channel with aggregate demand shocks while aggregate supply shocks and policy shocks lead to stronger reactions in the case of full cost channel.

**[Insert Figures 3 and 4 here]**

### 5.3.2 Liquidity Effects

In this section we discuss liquidity effect results with an active policymaking as discussed in Section 5.3. In other words, liquidity conditions are determined endogenously where the policymaker implements Taylor Rules as given by Equation (32). In figure 5 we present the liquidity responses (money balances) of the cash-in-advance economy with one-period deposits together with the associated nominal interest rates under a series of shocks on consumption (taste), investment, inflation and interest rates. In this set of simulations we report results for full and small cost channels. The variable  $gnom$  stands for changes in the nominal money balances and  $R$  for nominal interest rates.

**[Insert Figure 5 here]**

We observe that, in the case of full cost channel, after an aggregate demand shock (a shock in consumption or investment) nominal rates rise. This is consistent with the observation that as consumption increases, credit demand (liquidity) and inflationary pressures on the side of the central bank increase.

<sup>6</sup>Note that small cost channel in the previous section was with ( $\gamma_1 = 0.5$  and  $\gamma_2 = 0.5$ ) since due to indeterminacy we could not decrease it further. Assuming an active monetary policy allow us to find unique equilibrium for lower values of  $\gamma_1$  and  $\gamma_2$ .

To curb this effect the central bank needs to raise interest rates. This amounts, together with an increase in real wages, to a decline in investment as current and expected future marginal costs increase (see (19)). The net effect on output is, however, an initial increase that sharply contracts thereafter as the consumption growth declines steadily. While liquidity demand increases in the first instance, it turns sharply negative due to cost channels of labour and investment. In other words, if liquidity effect represents the credit conditions in the economy, an aggregate demand shock causes an initial rise in the demand for money credit, followed by a contractionary monetary policy and a strong decline in the credit demand; equivalent to a liquidity contraction as one would expect with liquidity effects. A similar reasoning applies to investment shocks. The impact of a taste shock and an investment shock are more pronounced under the full cost channel than the small cost channel framework.

In the case of an inflation shock, i.e. a shock to the Phillips Curve, we observe an immediate increase in the nominal interest rates due to stabilization preferences of the central bank. As inflation goes up, consumers start to increase their current consumption due to intertemporal elasticity concerns (future value of consumption will be relatively eroded), while the investment demand goes down as both expected inflation and a rise in the policy interest rate cause the current and future expected marginal cost to increase. The sharp decline in investment credit demand is not met by an increase in consumption demand. In turn, the rise in the policy rates due to supply shocks is associated with a sharp decline in the demand for credit to finance investment and labour costs. Therefore, the liquidity effect obtains.

In the case of policy shocks, i.e. a shock to the Taylor Rule, we do not observe the liquidity effect. In fact, we observe that nominal rates initially respond negatively to the shock in the Taylor Rule. While, at first, this may seem puzzling, we argue that this is exactly what should happen in equilibrium. First, note that we are initially at the steady state where both inflation and output gap are set equal to zero. A positive shock in interest rates means that the current output and inflation both will decrease below their steady state values, or below what the central bank prefers these to be. From the expression of the marginal cost and associated investment equation we observe that investment should go down as the expected real marginal cost will increase in the economy while consumption should likewise decrease due to intertemporal substitution effects. The forward looking nature of both investment and consumption pins down the aggregate goods and credit demand in our model. The only way to avoid output and inflation to fall below their potential values is to cut the policy rates. That is exactly what happens in the model.

In order to compare our results with the benchmark model used in the literature, we once again adopt the New Keynesian model with investment. However, this is normally set up as a cashless model, hence, we introduce an ad-hoc money demand specification ( $m - y = -\phi r$ ), which would be observed if one had assume households derive utility from real money balances. Gali (2008, p. 52-53) performs the same exercise with a standard NK model. Results are presented in the appendix. The main change is that after the aggregate demand

shocks (taste and investment) money balances react negatively straight after the shock without the delay observed in the *CIATR* model. The existence of the credit market and the real balance effect on the Euler equation drive this delay observed in the *CIATR* model. Note that Gali (2008, p. 52-53) shows that a contractionary policy shock leads to a rise in nominal interest rates and a decline in the residual money demand. This result is undone when one incorporates investment into the NK model. After a policy shock liquidity effects are not observed in the *NK<sub>i</sub>* model similar to the case with the *CIATR* model.

### 5.3.3 Determinacy

There are important determinacy problems with cash-in-advance models even under Taylor Rule closures. Taylor (1993) provided theoretical arguments for why an inflation coefficient greater than one is crucial to macroeconomic stabilization. In this framework, a more than one-to-one increase in nominal rates in response to an increase in expected inflation effectively raises real interest rates; therefore a decline in aggregate demand alleviates inflationary tendencies. Woodford (2003) formally discusses conditions for determinacy of equilibrium within the setting of a cashless New Keynesian framework (Taylor-Woodford principle). He argues that when a monetary policymaker targets output gap next to inflation she effectively relaxes the conditions for equilibrium determinacy. He also shows that interest rate smoothing is useful in obtaining a locally unique equilibrium. We find that determinacy considerations appear to be more serious in the presence of cost channels and money credit markets. While we concur that interest rate smoothing is indeed important to achieve a unique local equilibrium, targeting output gap is in fact counter productive for determinacy purposes. In Figures 6 we present (in)determinacy regions of our standard cash-in-advance model with Taylor rules.

**[Insert Figure 6 here]**

Firstly, we present in Figure 6 (a) conditions of indeterminacy (grey shaded area) and determinacy (non-shaded area) for a range of parameter values for Taylor rule variables of inflation and lagged interest rates when the central bank does *not* target output and when there is full cost channel. It is evident that the area of determinacy is much narrower than prescribed by for instance Woodford (2003). Here, a strong interest rate smoothing ( $\epsilon_r = 1$ ) helps to obtain a unique local equilibrium, however the size of the inflation coefficient that is required for determinacy is larger the smaller the coefficient on lagged interest rates. We note that for determinacy we need  $\frac{\Delta\epsilon_\pi}{\Delta\epsilon_r} < 0$ . In the unlikely case of the central bank not concerned about inflation and output, very strong interest rate smoothing ( $\epsilon_r > 1$ ) is sufficient to ensure determinacy in our model.

In Figure 6 (b) we present (in)determinacy regions when the full cost channel model includes a modest output gap target in Taylor Rule ( $\epsilon_y = 0.5$ ). Remarkably, we observe here a very significant decline in the area of local determinacy. The model requires a high level of interest rate smoothing together with a very aggressive stand against inflation to ensure that a unique local equilibrium ex-

ists.

We motivate this as follows: as in our model a contractionary policy change leads to a contraction of the economy and two opposing implications for inflation, targeting output together with inflation requires the determinacy concerned policymaker to act in order to make sure aggregate demand channels dominate the cost channels.

In Figure 6 (c) we modify the model such that only investment cost channel is present and the central bank still has a modest output target ( $\epsilon_y = 0.5$ ). As we observe the determinacy region is somewhat larger; however central bank still needs to be highly aggressive against inflation if the coefficient of lagged interest rate is relatively small. In sum, a rather small modification in the New Keynesian model, i.e. inclusion of money-credit markets and cost channels of investment and labour imply much more narrow determinacy regions. Very aggressive stand against inflation and strong interest rate smoothing appear to be necessary requirements to ensure locally unique equilibrium.

Next, in Figures 6 (d) and 6 (e) we evaluate the significance of cost channels for determinacy. Firstly, in figure 6 (d) we assume that  $\epsilon_y = 0.5$ ,  $\epsilon_r = 1$  and  $\gamma_2 = 1$  and plot (in)determinacy regions for parameter values  $\epsilon_\pi$  and  $\gamma_1$ . We note that as the labour cost channel ( $\gamma_1$ ) loses its significance, the area of determinacy widens. In the extreme case of the absence of labour cost channel, the required inflation coefficient is somewhat lower than 1 violating the Taylor-Woodford principle. Finally, in Figure 6 (e) we present the determinacy case under no output targeting. Remarkably, irrespective of the size of labour cost channels, equilibrium is locally determinate even when  $\epsilon_\pi$  is close to 0. Requirements for determinacy are strong interest rate smoothing and absence of an output target.

For the sake of completeness, we also report in the Appendix determinacy regions for the  $NK\_i$  model (in Figures 10 (a) to 10 (c)). Again same qualitative results obtain as in the case of  $CIATR$  model. When cost channels are present, targeting output gap in the Taylor rule increases areas of indeterminacy. This comparison is particularly relevant to show that this result is not driven by the real balance effects but cost channels. Surico (2008) finds qualitatively similar results in a three equation NK model with a cost channel in wage payments.

We conclude that indeterminacy problems are more severe within these model settings. The Taylor-Woodford principle that prescribes simple conditions for ensuring macroeconomic stability is often violated. Therefore, if the macroeconomic environment includes supply side as well as demand side considerations together with a role for money credit markets, very aggressive stand against inflation may be paramount in achieving policy outcomes that yield model determinacy.

## 6 Conclusions

We study a model with an explicit role for money/credit markets. It allows us to analyze the transmission of monetary policy via the variation of real money balances and via various cost channels. Investment dynamics play a crucial role

in this model. Our results can be summarized as follows:

Firstly, cost channels matter in terms of the effectiveness of monetary policy and performance of the economy in a stochastic environment. While full cost channel output is more volatile in the case of supply and monetary policy shocks, small cost channel output tends to be more volatile in the case of aggregate demand shocks. Inflation responses are more pronounced under small cost channel case with aggregate demand shocks while aggregate supply and policy shocks lead to strong volatility in the case of full cost channel. In other words, working capital finance plays an important role in the monetary transmission.

Secondly, to understand the liquidity effect we study the more plausible case of interest rate being determined by a central bank and money balances are determined in the credit markets endogenously. We show that aggregate supply shocks lead to desired contemporaneous liquidity responses, whereas aggregate demand shocks, that are shocks to investment and consumption, lead to lagging liquidity effects. The reason underlying these result is that the loan demand are very different under different type of shocks. We also show that in this economic environment while a narrow range of Taylor rules are successful in achieving economic stabilization, there is potentially useful information contained in the evolution of the money-credit conditions for an appropriate design of the monetary policy.

Thirdly, our quantitative analysis shows that the presence of cost channels and real money balance effects have important implications for model stability. In particular, when the central bank follows a money growth rule, model stability is only ensured in the presence of a strong cost channel. In our model settings, when the central bank implements an interest rate policy rule stability obtains only under a narrow set of parameters in the rule specification, where interest rate smoothing becomes essential. In addition, the Taylor-Woodford principle is often violated.

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## 7 Appendix

### 7.1 Cash in-advance and the evolution of consumption and labour supply:

In order to incorporate all possible cases of cash-in-advance and type of deposits we will present the model with two indicator parameters,  $\theta_1$  and  $\theta_2$ . If  $\theta_1 = 1$  and  $\theta_2 = 0$  we are back to our benchmark model. If  $\theta_1 = 0$ , then deposits clear in the same period and if  $\theta_2 = 1$ , then wages are paid at the beginning of the

period. Formally, the household is maximising its discounted lifetime utility given by:

$$\max_{c_t, M_{t+1}, D_t, A_t, H_t,} E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right), \quad \beta \in (0, 1) \quad \sigma, \eta > 0 \quad (33)$$

where  $C_t$  denotes the household's total consumption,  $H_t$  denotes hours worked. The curvature parameters  $\sigma, \eta$  are strictly positive.  $\beta$  is the discount factor. The family faces the following budget and cash in advance constraints:

$$C_t + \frac{D_t}{P_t} + \frac{M_{t+1}^d}{P_t} + \frac{Q_{t,t+1}A_t}{P_t} \leq \frac{W_t}{P_t}H_t + \frac{A_{t-1}}{P_t} + \Pi_t^{FI} + \frac{R_t(\theta_1 D_{t-1} + (1-\theta_1)D_t)}{P_t} + \frac{M_t}{P_t} + \quad (34)$$

$$+ \int_0^1 \Pi_{i,t} di - T_t \quad (35)$$

$$C_t + \frac{D_t}{P_t} \leq \frac{M_t}{P_t} + \theta_2 \frac{W_t}{P_t} H_t \quad (36)$$

where  $R_t$  is the rate of return on deposits. As introduced above the parameter  $\theta_1 \in \{0, 1\}$  determines if the rate of return is paid at the end of the period deposits were done as in Ravenna and Walsh(2006) or if they are paid next period as it is more commonly done.  $M_{t+1}^d$  are money holdings carried over to period  $t+1$ ,  $A_t$  represents alternative physical assets valued at the stochastic discount factor  $Q_{t,t+1}$ ,  $\int_0^1 \Pi_{i,t} di$  represents dividends accrued from the intermediate producers to households,  $\Pi_t^{FI}$  represents profits of the financial intermediary accrued to the household, and finally  $T_t$  stands for the lump-sum taxes households has to pay.

Cash-in-advance constraint imposes the condition that the household needs to allocate money balances for consumption purposes net of deposits she has decided to allocate to the financial intermediary. The parameter  $\theta_2 \in \{0, 1\}$  determines whether the wage income can be used for consumption in the current period or not.

The consumption Euler equation and labour supply condition are given by:

$$\theta_1 = 1 \quad \beta^2 E_t \left( \frac{R_{t+1} C_{t+2}^{-\sigma}}{\pi_{t+2} \pi_{t+1}} \right) = E_t (C_t^{-\sigma}) \quad (37)$$

$$\theta_1 = 0 \quad \beta E_t \left( \frac{R_t C_{t+1}^{-\sigma}}{\pi_{t+1}} \right) = E_t (C_t^{-\sigma}) \quad (38)$$



$$\theta_2 = 0 \quad \frac{\chi H_t^\eta}{\beta E_t (C_{t+1}^{-\sigma})} = \frac{W_t}{P_t E_t (\pi_{t+1})} \quad (39)$$

$$\theta_2 = 1 \quad \frac{\chi H_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} \quad (40)$$

## 7.2 Solution to cash-in-advance Model:

Form the Lagrangian:

$$\max_{C_t, M_{t+1}, D_t, H_t, A_t} E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right) + \quad (41)$$

$$\sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \frac{W_t}{P_t} H_t + \frac{R_t D_{t-1}}{P_t} + \frac{A_{t-1}}{P_t} + \frac{M_t}{P_t} + \int_0^1 \Pi_{i,t} di + \Pi_t^{FI} - T_t - C_t - \frac{D_t}{P_t} - \frac{M_{t+1}^d}{P_t} - \frac{Q_{t,t+1} A_t}{P_t} \right] \quad (42)$$

$$+ \sum_{t=0}^{\infty} \beta^t \mu_t \left[ \frac{M_t}{P_t} - C_t - \frac{D_t}{P_t} \right] \quad (43)$$

First order conditions are:

$$\frac{\partial V}{\partial C_t} = E_t [C_t^{-\sigma} - \lambda_t - \mu_t] = C_t^{-\sigma} - \lambda_t - \mu_t = 0 \quad (44)$$

$$\frac{\partial V}{\partial D_t} = \beta E_t \left( \frac{R_{t+1} \lambda_{t+1}}{P_{t+1}} \right) - E_t \left[ \frac{1}{P_t} (\mu_t + \lambda_t) \right] = 0 \quad (45)$$

$$\frac{\partial V}{\partial M_{t+1}} = \beta E_t \left[ \frac{(\lambda_{t+1} + \mu_{t+1})}{P_{t+1}} \right] - E_t \left[ \frac{\lambda_t}{P_t} \right] = 0 \quad (46)$$

$$\frac{\partial V}{\partial H_t} = E_t \left[ \chi H_t^\eta - \frac{W_t}{P_t} \lambda_t \right] = 0 \quad (47)$$

$$\frac{\partial V}{\partial A_t} = E_t \left[ -\frac{Q_{t,t+1} \lambda_t}{P_t} + \beta \frac{\lambda_{t+1}}{P_{t+1}} \right] = 0 \quad (48)$$

Combining (44) into (46), and (44) into (45) gives respectively

$$\beta E_t \left( \frac{R_{t+1} \lambda_{t+1}}{P_{t+1}} \right) = E_t \left( \frac{C_t^{-\sigma}}{P_t} \right)$$

$$\beta E_t \left[ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] = \lambda_t$$

Combining these two results gives the Consumption Euler equation

$$\beta E_t \left( \frac{R_{t+1} \beta E_t \left[ \frac{C_{t+2}^{-\sigma}}{\pi_{t+2}} \right]}{P_{t+1}} \right) = E_t \left( \frac{C_t^{-\sigma}}{P_t} \right)$$

With the model without the effect on the labour supply, the cash in advance constraint is  $\sum_{t=0}^{\infty} \beta^t \mu_t \left[ \frac{M_t}{P_t} + \frac{W_t}{P_t} H_t - C_t - \frac{D_t}{P_t} \right]$ . The first order condition with respect to hours worked is then given by:

$$\frac{\partial V}{\partial H_t} = E_t \left[ \chi H_t^\eta - \frac{W_t}{P_t} (\lambda_t + \mu_t) \right] = 0 \quad (49)$$

### 7.3 New Keynesian Macro-model with Cost Channels

The New Keynesian model presented in the main text does not include cost channels. This can be introduced in an ad-hoc way. In fact the final system of linear equations, presented below, would be the same as a cash-in-advance model with intra-period deposits and cash-in-advance constraint including wages. That way only the cost channels are introduced but the real balance effects are eliminated.

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma} E_t [\hat{r}_t - \hat{\pi}_{t+1}] \quad (50a)$$

$$\gamma_2 \hat{r}_t \approx -E_t [\hat{r}_t - \hat{\pi}_{t+1}] + (1 - \beta(1 - \delta)) E_t [\hat{y}_{t+1} + \hat{\Lambda}_{t+1} - \hat{k}_{t+1}] + \beta \gamma_2 (1 - \delta) E_t \hat{r}_{t+1} \quad (50b)$$

$$\hat{\Lambda}_t = \gamma_1 \hat{r}_t + (1 + \eta) \hat{h}_t + \sigma \hat{c}_t - \hat{y}_t \quad (50c)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{\Lambda}_t \quad (50d)$$

$$\hat{r}_t = \epsilon_y \hat{y}_t + \epsilon_\pi \hat{\pi}_t + v_t \quad (50e)$$

$$\hat{y}_t = s_c \hat{c}_t + s_I \hat{i}_t \quad (50f)$$

$$\hat{k}_{t+1} = (1 - \delta) \hat{k}_t + \delta \hat{i}_t \quad (50g)$$

$$\hat{h}_t = \alpha \hat{k}_t + (1 - \alpha) \hat{h}_t \quad (50h)$$

### 7.4 Appendix 2: Log-linear Approximations:

Remember Campbell calculus. Suppose we have:

$$y_t = f(x_t, y_t)$$

Then, we should have that:

$$\bar{y}\hat{y}_t = f_x(\bar{x}, \bar{z})\bar{x}\hat{x}_t + f_z(\bar{x}, \bar{z})\bar{z}\hat{z}_t$$

which is often written as:

$$\begin{aligned} \hat{y}_t &= \frac{f_x(\bar{x}, \bar{z})\bar{x}}{\bar{y}}\hat{x}_t + \frac{f_z(\bar{x}, \bar{z})\bar{z}}{\bar{y}}\hat{z}_t \\ \text{or} \\ \hat{y}_t &= \frac{f_x(\bar{x}, \bar{z})\bar{x}}{f(\bar{x}, \bar{z})}\hat{x}_t + \frac{f_z(\bar{x}, \bar{z})\bar{z}}{f(\bar{x}, \bar{z})}\hat{z}_t \end{aligned}$$

1. Log-linearisation of the consumption Euler condition (12):

$$E_t(\hat{c}_t) = E_t(\hat{c}_{t+2}) - \frac{1}{\sigma}E_t[\hat{r}_{t+1} - \hat{\pi}_{t+1} - \hat{\pi}_{t+2}] \quad (51)$$

2. Labour supply equation (13):

$$\eta\hat{h}_t + \sigma E_t(\hat{c}_{t+1}) + E_t(\hat{\pi}_{t+1}) = \hat{w}_t - \hat{p}_t \quad (52)$$

3. Log-linearization of the investment equation (19). Note that  $\beta E_t \frac{(C_{t+2}^{-\sigma})}{(C_{t+1}^{-\sigma})^{\pi_{t+2}}} = Q_{t,t+1}$ . Log-linearization of this gives us  $\hat{q}_{t,t+1}$

$$\gamma_2\hat{r}_t \approx \hat{q}_{t,t+1} + E_t(\hat{\pi}_{t+1}) + \ln E_t \left[ \Lambda_{t+1} \frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)R_{t+1}^{\gamma_2} \right]$$

Define  $F_t = \ln E_t \left[ \Lambda_{t+1} \frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)R_{t+1}^{\gamma_2} | \Omega_{t,1} \right]$  and at the steady state  $F = \left[ \Lambda \frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2} \right]$

$$\hat{F}_t = \frac{f_y(\bar{x}, \bar{z})\bar{y}}{f(\bar{x}, \bar{z})}\hat{y}_{t+1} + \frac{f_\Lambda(\bar{x}, \bar{z})\bar{\Lambda}}{f(\bar{x}, \bar{z})}\hat{\Lambda}_{t+1} + \frac{f_r(\bar{x}, \bar{z})\bar{r}}{f(\bar{x}, \bar{z})}\hat{r}_{t+1} + \frac{f_k(\bar{x}, \bar{z})\bar{k}}{f(\bar{x}, \bar{z})}\hat{k}_{t+1} \quad (53)$$

$$\begin{aligned} \hat{F}_t &= \frac{\Lambda \frac{\alpha Y}{K}}{\left[ \Lambda \frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2} \right]} E_t \left[ \hat{y}_{t+1} - \hat{k}_{t+1} \right] + \frac{\frac{\alpha Y}{K} \Lambda}{\left[ \Lambda \frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2} \right]} E_t \left( \hat{\Lambda}_{t+1} \right) + \\ &+ \frac{\gamma_2 R^{\gamma_2} (1-\delta)}{\left[ \Lambda \frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2} \right]} E_t(\hat{r}_{t+1}) \end{aligned} \quad (54)$$

therefore

$$\gamma_2\hat{r}_t \approx \hat{q}_{t,t+1} + E_t(\hat{\pi}_{t+1}) + \epsilon_1 E_t \left[ \hat{y}_{t+1} - \hat{k}_{t+1} + \hat{\Lambda}_{t+1} \right] + \epsilon_2 E_t(\hat{r}_{t+1}) \quad (55)$$

$$\epsilon_1 = \frac{\Lambda \frac{\alpha Y}{K}}{\left[ \Lambda \frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2} \right]}, \epsilon_2 = \frac{\gamma_2 R^{\gamma_2} (1-\delta)}{\left[ \Lambda \frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2} \right]}$$

To simplify the elasticities in the above expression, we use that the steady state expression for capital:

$$R^{\gamma_2} = \beta \left[ \Lambda \frac{\alpha Y}{K} + (1 - \delta) R^{\gamma_2} \right] \quad (56)$$

$$\frac{R^{\gamma_2}}{\beta} = \left[ \Lambda \frac{\alpha Y}{K} + (1 - \delta) R^{\gamma_2} \right] \quad (57)$$

$$\Lambda \frac{\alpha Y}{K} = R^{\gamma_2} \frac{(1 - \beta(1 - \delta))}{\beta} =$$

To simplify  $\epsilon_2 = \beta \gamma_2 (1 - \delta)$ ,  
 $\epsilon_1 = (1 - \beta(1 - \delta))$

3b. Now, we compute the log-linearization of  $\beta E_t \frac{(C_{t+2}^{-\sigma})}{(C_{t+1}^{-\sigma})^{\pi_{t+2}}} = Q_{t,t+1}$

$$\hat{q}_{t,t+1} = -\sigma (E_t (\hat{c}_{t+2} - \hat{c}_{t+1})) - E_t (\hat{\pi}_{t+2})$$

$$\begin{aligned} \gamma_2 \hat{r}_t \approx & -\sigma (E_t (\hat{c}_{t+2} - \hat{c}_{t+1})) - E_t (\hat{\pi}_{t+2}) + E_t (\hat{\pi}_{t+1}) + \\ & + (1 - \beta(1 - \delta)) E_t [\hat{y}_{t+1} + \hat{\Lambda}_{t+1} - \hat{k}_{t+1}] + \beta \gamma_2 (1 - \delta) E_t (\hat{r}_{t+1}) \end{aligned}$$

To compare this equation with the conventional NK model,  $\gamma_2 = 0$ , and we

$$0 \approx \hat{q}_{t,t+1} + E_t (\hat{\pi}_{t+1}) + (1 - \beta(1 - \delta)) E_t [\hat{y}_{t+1} - \hat{k}_{t+1} + \hat{\Lambda}_{t+1}]$$

$\hat{q}_{t,t+1} = -\sigma (E_t (\hat{c}_{t+1} - \hat{c}_t)) - E_t (\hat{\pi}_{t+1}) = -\hat{r}_t + E_t (\hat{\pi}_{t+1})$  than the NK interest rate will reflect the monopoly distortions ( $\hat{\Lambda}_{t+1}$ ) and the marginal cost of capital without the cost channel. Hence,

$$(\hat{r}_t - E_t (\hat{\pi}_{t+1})) = (1 - \beta(1 - \delta)) E_t [\hat{y}_{t+1} - \hat{k}_{t+1} + \hat{\Lambda}_{t+1}]$$

4. Log-linearization of lambda (18) :

$$\hat{\Lambda}_t = \gamma_1 \hat{r}_t + \hat{w}_t - \hat{p}_t + \hat{h}_{i,t} - \hat{y}_{i,t} \quad (58)$$

6 The Phillips curve:

$$\hat{\pi}_t = \beta E_t (\hat{\pi}_{t+1}) + \kappa \hat{\Lambda}_t \quad (59)$$

where  $\kappa = (1 - \omega)(1 - \omega\beta)/\omega$ , and  $\omega$  is the Calvo parameter. We can use the labour-share to measure the real marginal cost.  $S_t = \frac{WH}{PY}$

$$\hat{\Lambda}_t = \gamma_1 \hat{r}_t + \hat{w}_t - \hat{p}_t + \hat{h}_t - \hat{y}_t = \gamma_1 \hat{r}_t + \hat{s}_t \quad (60)$$

7. Credit market:

$$\frac{m_{t+1}}{Y_t} - \frac{C_t}{Y_t} - \gamma_2 \frac{I_t}{Y_t} = \gamma_1 S_t \quad (61)$$

as  $S_t = \frac{W_t H_t}{P_t Y_t}$ . The Log-linearisation of that is given by

$$\gamma_1 \widehat{s}_t = \frac{m/Y}{\varpi} \widehat{m}_{t+1} - \frac{s_c}{\varpi} \widehat{c}_t - \frac{\gamma_2 s_i}{\varpi} \widehat{i}_t - y_t \text{ for } \gamma_1 \neq 0 \quad (62)$$

$$\widehat{m}_{t+1} = \frac{s_c}{(s_c + \gamma_2 s_i)} \widehat{c}_t + \frac{\gamma_2 s_i}{(s_c + \gamma_2 s_i)} \widehat{i}_t \text{ for } \gamma_1 = 0 \quad (63)$$

where  $\varpi = m/Y - (s_c + \gamma_2 s_i)$ . Given the data on the velocity of money we set  $\frac{m}{Y} = 1/0.44$  or the elasticity when  $\gamma_2 = 1$  is  $\epsilon_v = \frac{m}{m-Y} = \frac{\frac{m}{Y}}{\frac{m}{Y} - 1} = \frac{1/0.44}{1/0.44 - 1} = 1.7857$ .

We can use the real marginal cost to measure the labour-share  $S_t = \frac{WH}{PY}$ :

$$\widehat{\Lambda}_t = \gamma_1 \widehat{r}_t + \widehat{w}_t - \widehat{p}_t + \widehat{h}_t - \widehat{y}_t = \gamma_1 \widehat{r}_t + \widehat{s}_t \quad (64)$$

$$\widehat{s}_t = \widehat{\Lambda}_t - \gamma_1 \widehat{r}_t \quad (65)$$

## 8. Money Process

Nominal Money balances evolve as follows  $M_{t+1} = M_t + V_t$ . Dividing by  $P_t$  and log linearizing gives (note that  $m_t = \frac{M_t}{P_{t-1}}$ ):

$$\widehat{m}_{t+1} = \widehat{m}_t - \widehat{\pi}_t + v_t$$

## 7.5 Appendix 3: Additional Figures

Insert Figures 7, 8, 9 and 10 here.

# Figures - Main Text

Figure 1: Full and Small Cost Channels

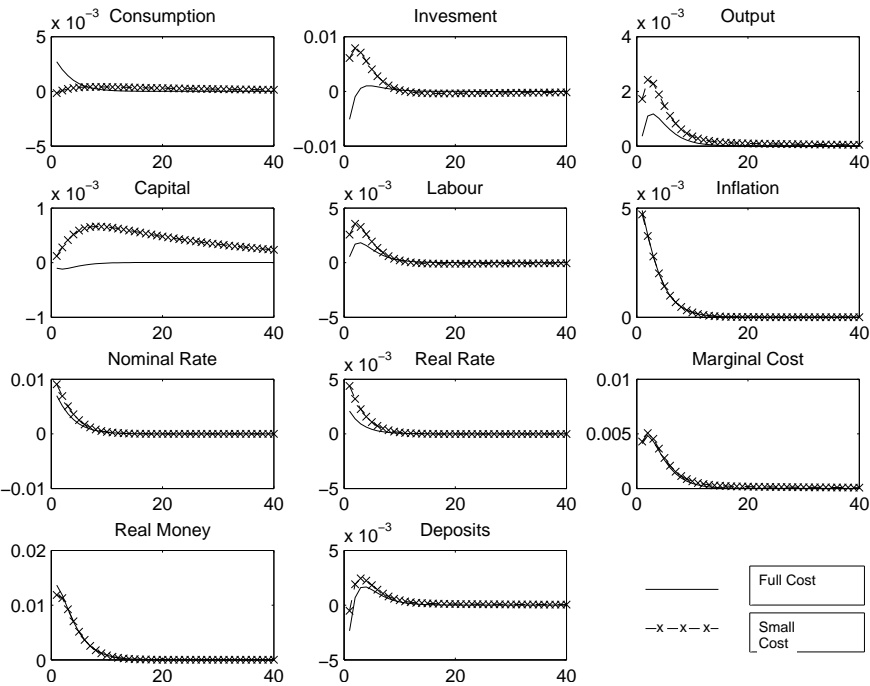


Figure 2: Full, Investment and Labour Cost Channels

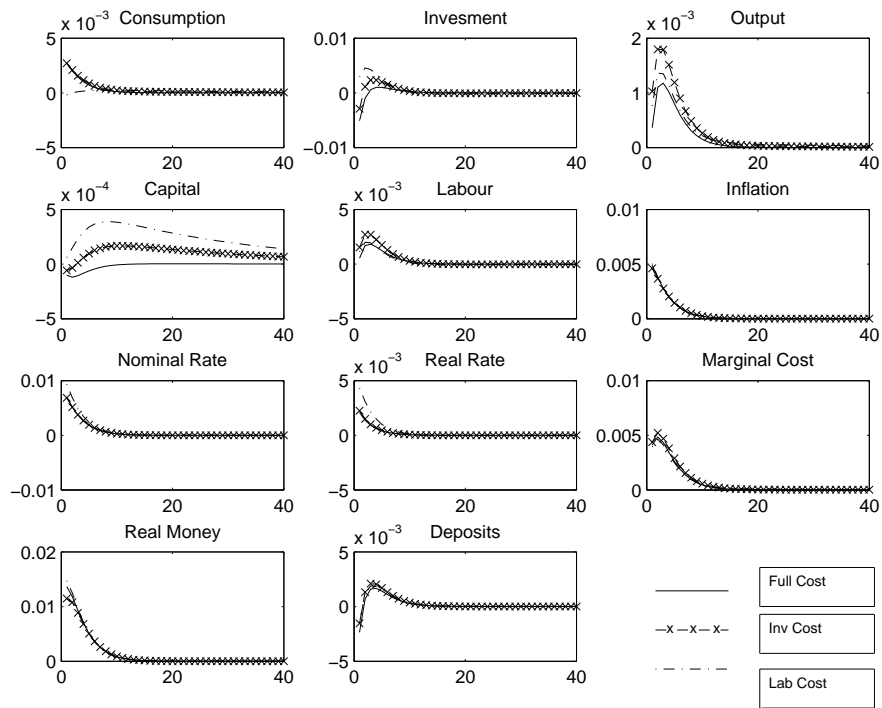


Figure 3: CIA Model Full Cost vs. Small Cost Channel - Output Response

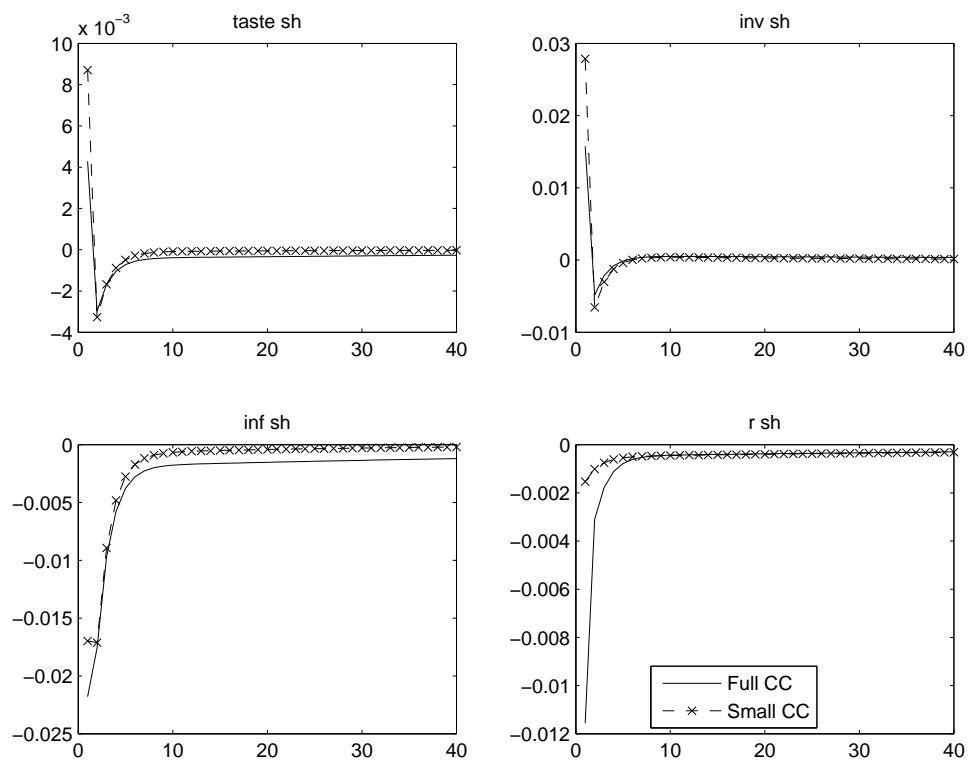




Figure 4: CIA Model Full Cost vs. Small Cost Channel - Inflation Response

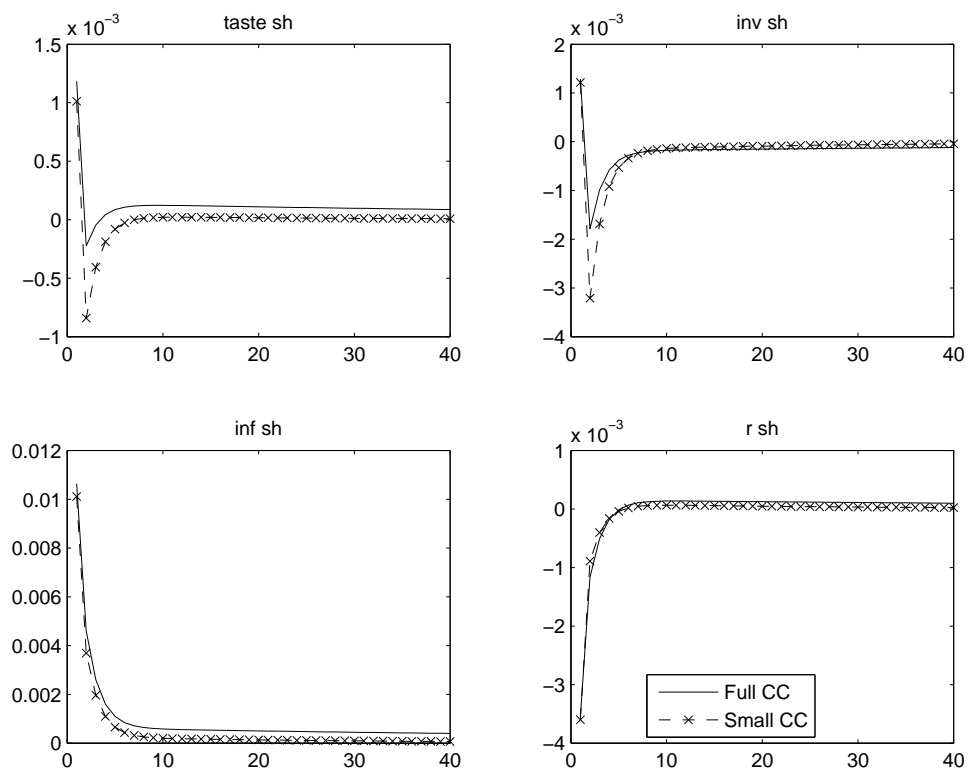


Figure 5: CIA Model - Liquidity Effect

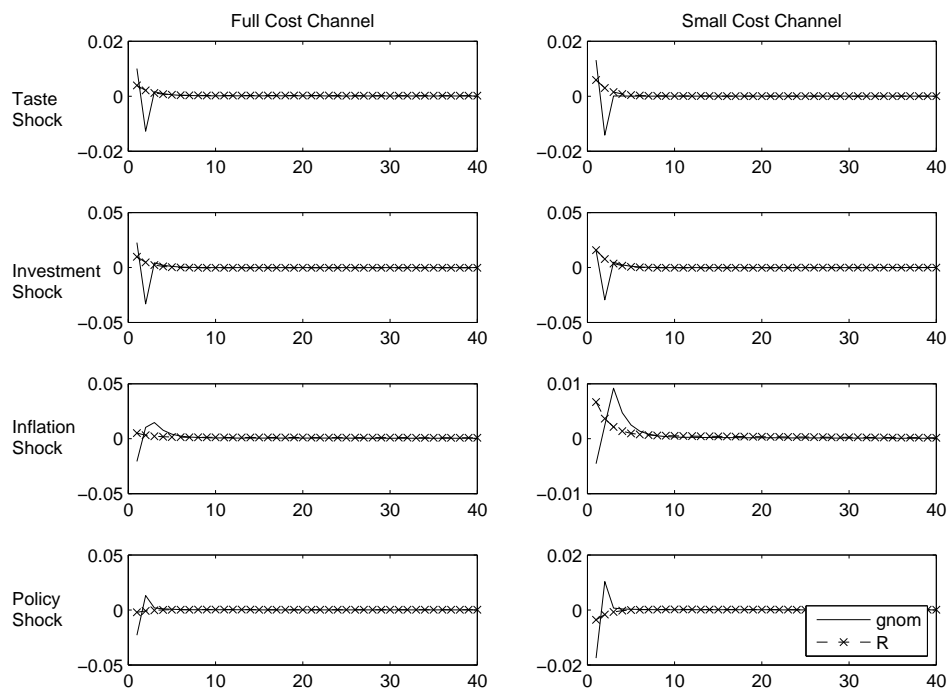
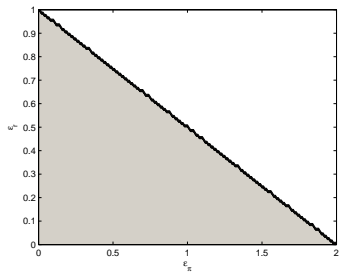
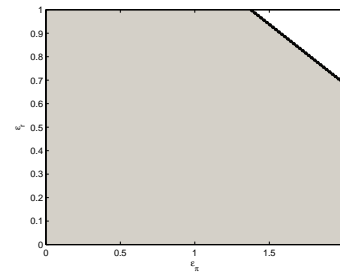


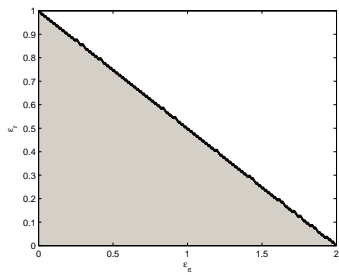
Figure 6: CIA model - Indeterminacy Analysis



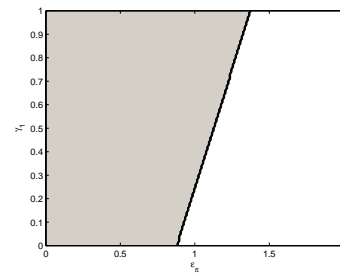
(a) Full Cost Channel -  $\epsilon_y = 0$



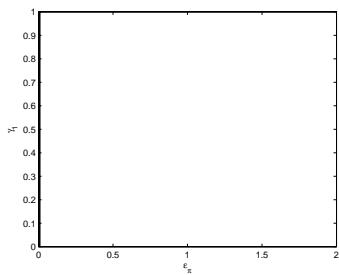
(b) Full Cost Channel -  $\epsilon_y = 0.5$



(c) Inv. Cost Channel -  $\epsilon_y = 0.5$



(d) Inv. Cost Channel -  $\epsilon_y = 0.5$  and  $\epsilon_r = 1$



(e) Inv. Cost Channel -  $\epsilon_y = 0$  and  $\epsilon_r = 1$

# Figures - Appendix

Figure 7: CIA Model - Full Cost vs. Small Cost Channel - Consumption Response

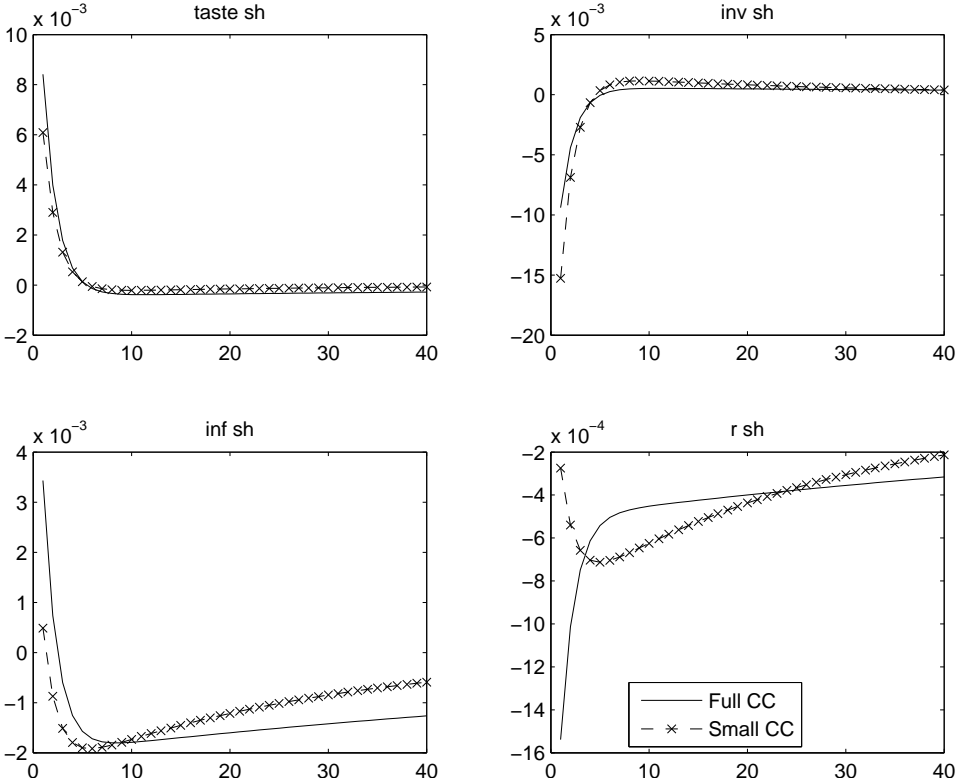


Figure 8: CIA Model Full Cost vs. Small Cost Channel - Investment Response

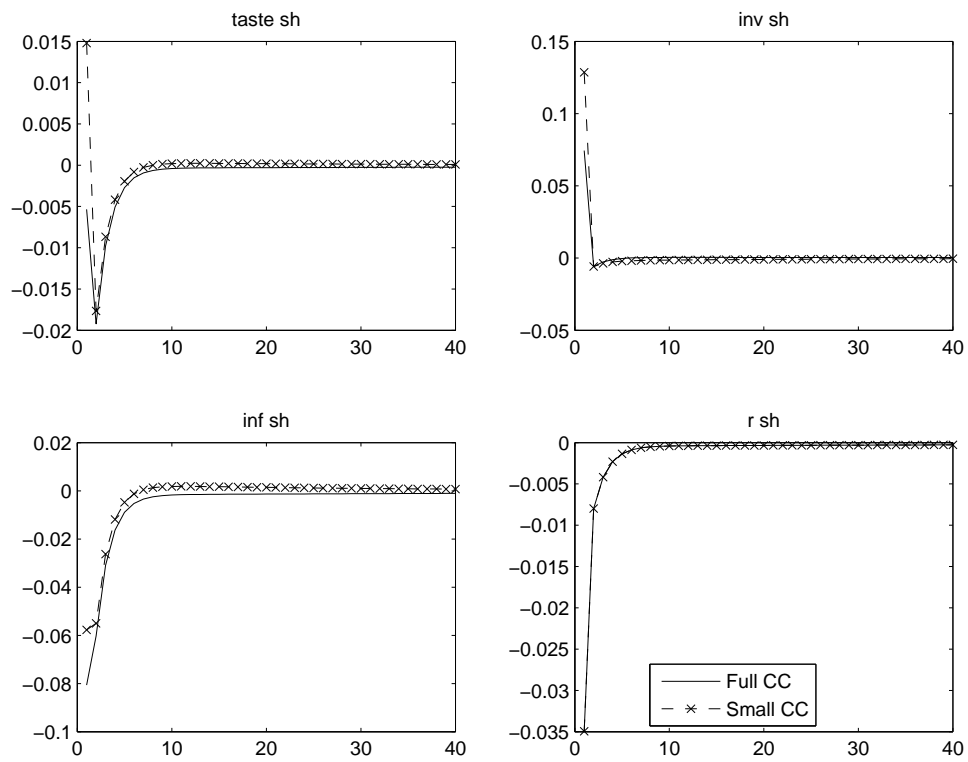


Figure 9:  $NK_i$  Model - Liquidity Effect

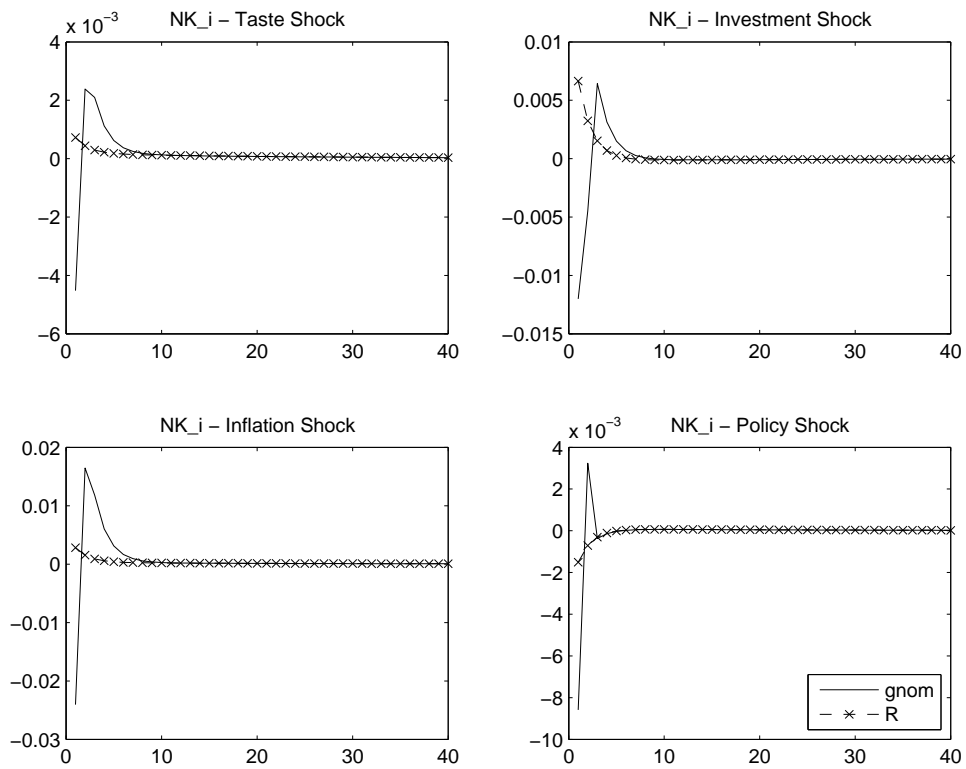
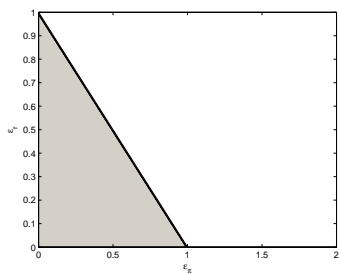
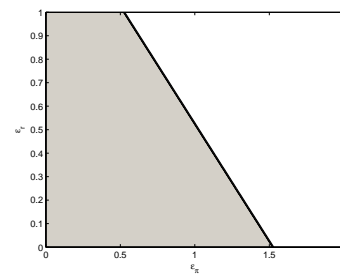


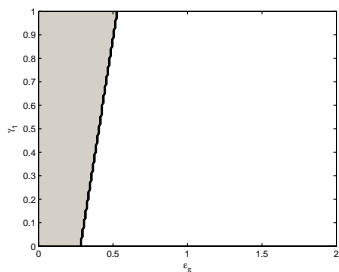
Figure 10:  $NK_i$  model with Cost Channel - Indeterminacy Analysis



(a) Full Cost Channel -  $\epsilon_y = 0$



(b) Full Cost Channel -  $\epsilon_y = 0.5$



(c) Inv. Cost Channel -  $\epsilon_y = 0.5$  and  $\epsilon_r = 1$