Public vs. Private Negotiations in Bilateral Relationships*

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Abstract

The preference between public and private negotiations for a buyer who sequentially visits two suppliers is examined. It is shown that the buyer weakly prefers to conduct private negotiations in order to create strategic uncertainty about the trade history. With substitute goods, such uncertainty is valuable only when price offers have short expiries that prevent a head-to-head supplier competition. With complementary goods, strategic uncertainty is valuable to the extent that price coordination becomes a concern for suppliers, which is likely to be the case when suppliers possess relatively high bargaining powers; price offers have short expiries; and/or goods are weak complements. The effects of mandatory disclosure laws, extended return policies, and purchasing alliance formation on trade efficiency are also discussed.

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1 Introduction

In a variety of bargaining situations a central agent performs bilateral negotiations with several others to acquire goods and services. Examples include a manufacturer procuring inputs from a set of suppliers; a real estate developer buying pieces of land from several different landowners; a firm trying to reach deals with multiple labor unions; a lobbyist seeking the endorsement of various politicians; and an academic department negotiating with multiple faculty candidates.

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An important determinant of trade outcomes in these negotiations is the information flow between the parties. While in many applications, especially those involving government funds, the central player, henceforth the "buyer", is often bound by certain mandatory disclosure laws (also called "sunshine" laws) to conduct "public" negotiations in which the information about the trade history is fully revealed,[1] in many others she is free to engage in "private" negotiations in which this information is kept confidential from the service providers, henceforth the "suppliers".[2] The main objective of this paper is to explore the buyer's preference between public and private negotiations, in particular how this preference depends on the substitutability of goods and players' relative bargaining powers. In doing so, we also identify the social value of mandatory disclosure laws and policies such as the FTC's "cooling-off" rule that enable the buyer to cancel a contract or return a purchase within a certain time frame[3] as well as those that increase buyer power.[4]

Our formal setting consists of three risk-neutral players – two suppliers who each provide a (potentially) differentiated good and a buyer with a unit demand for each good. The buyer negotiates with suppliers sequentially and in the order of her choosing. Under public negotiations this order and the buyer's purchasing history are disclosed to the suppliers, whereas under private negotiations both pieces of information remain known only to the buyer. We model each bilateral negotiation as a one-shot random proposer bargaining in which the buyer makes a price offer with some pre-specified probability that reflects her relative bargaining power. As is common in practice, each supplier's price offer may be subject to an expiration policy; and to capture this, we distinguish between an "exploding" offer that obliges the buyer to reach a purchasing decision before visiting the rival supplier and an "open-ended" offer that lets the buyer decide after visiting both suppliers. We assume expiration policies to be exogenous to the model, perhaps because they are the

[1] According to a recent Wikipedia entry, over seventy countries, including the U.S. and Canada, have implemented some form of sunshine laws that set rules on public access to information about business dealings of government bodies, including participation in actual meetings. [http://en.wikipedia.org/wiki/Freedom_of_Information_Act.]. In the same vein, the Federal Election Campaign Act in the U.S. requires that political candidates file periodic reports disclosing contributions over $200 and the identity of their contributors.

[2] We use the feminine pronoun for the buyer and the masculine ones for suppliers.

[3] The cooling-off rule gives buyers a three-day right to cancel a contract or return a purchase without any penalty, especially if the transaction amount is more than $25 and the agreement is made outside the vendor's permanent place of business [www.consumeraction.gov/caw_shopping_cooling_off.shtml].

[4] For instance, under certain rules and regulations, a group of small consumers may form a "buyer group" to gain bargaining power against sellers [see, e.g., Mathewson and Winter (1997)]. The formation of larger and more powerful industrial buyers have also been on the rise, causing concerns for antitrust authorities in both the U.S. and Europe [see, Inderst and Mazzarotto (2008) for a survey].

industry standards[5] or because they are imposed by certain rules and regulations such as the FTC's cooling-off rule alluded to above.[6] We characterize perfect Bayesian equilibria of both public and private negotiation games. We say that (given our setup) equilibrium trade is efficient if the buyer purchases both goods with probability 1, and inefficient otherwise.

Our analysis shows that, in general, the buyer weakly prefers to perform private negotiations so as to create strategic uncertainty between suppliers. With substitute goods, such strategic uncertainty is valuable to the buyer only when both offers are exploding. To understand this, note that when at least one offer is open-ended, the buyer is able to compare prices by optimally visiting first the supplier with an open-ended offer and initiate an intense Bertrand competition irrespective of the negotiation type. When both offers are exploding, however, such a price comparison is unavailable to the buyer, enabling suppliers to better tailor their prices and extract more surplus from the buyer under public negotiations than they do under private negotiations.

With complementary goods, strategic uncertainty under private negotiations benefits the buyer to the extent that this makes suppliers concerned about price coordination in equilibrium and charge relatively moderate prices as a result. We discover that such concern about price coordination is more pronounced as suppliers gain a greater bargaining power against the buyer; more offers become exploding; and/or goods become weaker complements. Intuitively, when suppliers are powerful, each anticipates the rival to make an aggressive price offer to the buyer and leave little surplus to grab, which, in turn, leads both to set relatively low prices. Said differently, it is the less powerful suppliers who are more likely to behave aggressively. Quite interestingly, this implies that under private negotiations, the buyer may actually prefer to deal with suppliers with greater bargaining powers. With exploding offers, because the buyer pays as she goes, suppliers post moderate prices to entice the buyer to purchase in the upstream without her worrying about the (potential) holdup problem in the downstream. Indeed, in equilibrium, the buyer ends up randomizing over the negotiation sequence, whereby each supplier places some weight on being in the upstream and offers a price discount. Finally, for weak complements, suppliers set relatively low prices because the buyer enjoys little extra surplus by purchasing from both.

[5]For instance, while most job offers in the junior economist market have a two-week deadline and most hotel reservations can be canceled 24 hours in advance at no cost, occasional wear dresses are usually sold under a policy of "all sales are final!", allowing no exchanges or refunds.

[6]Daughety and Reinganum (1992) provide a strategic explanation for the choices of expiration policies in a duopolistic market.

Unlike public negotiations, private negotiations may produce an inefficient trade. With substitutes, inefficiency occurs only when both offers are exploding and strategic uncertainty about the order of negotiations gives rise to equilibrium supply prices other than the low Bertrand prices.[7] With complements, inefficiency is more likely to occur in that it arises for a wider range of parameters. Nevertheless, it turns out that with complements, the factors that moderate supply prices and thus make private negotiations appealing to the buyer are also the ones that are conducive to an efficient trade.

From a policy perspective our results suggest that various mandatory disclosure laws imposed on government buyers are likely to be binding especially for the acquisition of complementary goods. Our results also suggest that the extended return and cancellation policies that effectively cause price offers to be open-ended are most useful for purchases of substitute goods because, as argued above, open-ended offers facilitate a Bertrand competition and let the buyer efficiently purchase both goods. In contrast, for the purchases of complementary goods, such policies may actually hinder an efficient trade by giving suppliers an incentive to price more aggressively. In fact, with complements, our analysis indicates that both the buyer and suppliers could be better off under a policy of "all sales are final!" Finally, our results reveal that the policies that allow for the formation of buyer groups to gain bargaining power against suppliers do not necessarily lower supply prices or benefit the members of such groups.

Related Literature. Our paper is related to a growing literature on one-to-many negotiations. Papers by Cai (2000), Horn and Wolinsky (1988), and Stole and Zwiebel (1996) assume a fixed order of negotiations and public agreements. Arbatskaya (2007), Marx and Shaffer (2007a) and Nagarajan and Bassok (2005) endogenize the order, but they do so in a complementary framework in which suppliers bid for positions. Marshall and Merlo (2004), and Marx and Shaffer (2007b) let the buyer choose the order, but assume contingent and public contracts. Raskovich (2007) uses a costly search model in which all offers are exploding, and the buyer exits upon a successful trade.

The closest paper to ours in this literature is by Noe and Wang (2004). They construct a model much like ours,[8] but assume symmetric products and suppliers. Moreover, they restrict attention to exploding offers and efficient equilibria, and do not consider partially private negotiations. Hence, for substitutes, they find that it is optimal for the buyer to

[7] As we show below, suppliers may end up charging their monopoly prices in this case.

[8] Unlike us, they extend their analysis to the n-supplier case with perfect complements.

conduct private negotiations whereas we find it is generically optimal for her to conduct partially private negotiations precisely because we account for inefficient equilibria. For complements, our conclusion that the buyer is better off conducting private negotiations coincides with Noe and Wang's, but we provide a richer set of conditions under which this is the case.

Our paper is also related to the literature on bilateral contracting with externalities, introduced by Segal (1999), and later generalized by Whinston and Segal (2003). Our model combines some elements of Whinston and Segal's "offering game," in which all offers are made by the buyer, and their "bidding game," in which all offers are made by the suppliers. Unlike Whinston and Segal, we do not assume bilateral contracts are offered simultaneously. Some recent papers, e.g., Genicot and Ray (2006), and Moller (2007), investigate sequential bilateral contracting with endogenous timing to determine when simultaneous contracting may or may not be optimal for the buyer, but they do so by assuming that contracts and agreements are public.

Finally, our paper complements the recent literature on consumer privacy in which it is the suppliers, rather than the buyer, who initiate communication (see, e.g., Calzolari and Pavan (2005), and Taylor (2004)).

The rest of our paper is organized as follows. In the next section, we lay out the basic model. In section 3, we characterize equilibria under public negotiations, followed by equilibrium characterization of private negotiations in Section 4. In Section 5, we determine the buyer's preference between public and private negotiations. In Section 6, we extend our analysis to include partially private negotiations and examine the setting in which the buyer lacks commitment to a disclosure policy. Finally, we conclude in Section 7. The proofs of all formal results appear in the Appendix.

2 The Basic Model

There are three risk-neutral parties: one buyer, b, and two suppliers, s_i, $i = 1, 2$. For instance, the buyer could be an academic department and the suppliers could be two faculty candidates. Each supplier provides a (potentially) differentiated good at *no* cost. The buyer has a unit demand for good i and values it alone by $v_i \in [0, 1]$. Her joint valuation is normalized to 1.[9] Letting $\Delta \equiv 1 - v_1 - v_2$ be its degree, we say that goods are stronger

[9]Note that it is (socially) efficient for the buyer to purchase both goods. Note also that we do not consider the possibility of some good i being individually undesirable for the buyer, i.e., $v_i < 0$. Our results

complements as Δ increases from -1 (perfect substitutes) to $+1$ (perfect complements). More broadly, we say that goods are complements if $\Delta > 0$; substitutes if $\Delta < 0$; and independent if $\Delta = 0$. It is assumed that each party has a no-trade payoff of 0, and that the payoff structure is common knowledge.

The buyer negotiates with suppliers sequentially and only once.[10] We capture each negotiation as a one-shot random proposer bargaining: with a probability $\alpha_i \in (0,1)$, s_i makes the price offer for product i, and with probability $1-\alpha_i$, b does. It is assumed that α_i is common knowledge, but the identity of the proposer is realized at the beginning of each negotiation i and observed only by s_i and b. Note that a greater α_i refers to an *ex ante* more powerful seller vis-à-vis the buyer,[11] and that we eliminate the (uninteresting) cases in which one player has all the bargaining power, though one can take the limits. For simplicity, we ignore discounting between negotiations.

As alluded to in the Introduction, the price offer for product i is associated with an expiration policy, e_i, which may vary across products. In this respect, we consider two types of offers: exploding and open-ended. An offer is said to be *exploding* ($e_i = x$) if it expires as soon as bargaining ends, and *open-ended* ($e_i = o$) if it permits the buyer to decide after visiting both sellers. Let $E \equiv (e_1, e_2)$ represent the expiration vector such that $E \in \{X, O, H_1, H_2\}$ where $X \equiv (x,x)$, $O \equiv (o,o)$, $H_1 \equiv (o,x)$, and $H_2 \equiv (x,o)$. We assume expiration policies to be exogenous – perhaps because they are the industry standards or because they are imposed by certain regulations such as the FTC's cooling-off rule (see Footnotes 3 and 5). Nevertheless, we discuss parties' preferences over expiration policies throughout the analysis.

The information flow between the negotiations is assumed to be controlled by the buyer. In the basic model, we restrict attention to an all-or-nothing type of disclosure for the buyer, engendering public and private negotiations, respectively. Under public negotiations the full history of negotiations is known by all players, whereas under private negotiations only

would, however, go through with a positive cost, c_i, of production as along as: $v_i - c_i \geq 0$ for all i, and $1 - \sum c_i \geq \max_i\{v_i - c_i\}$.

[10] Precluding renegotiation is clearly restrictive and may require some commitment power. Like Moller (2007), and Noe and Wang (2004), we make this assumption to better focus on other aspects of bargaining, but it would be satisfied if, for instance, the buyer has a relatively short time to purchase goods. Sequentiality, however, should not be taken literally, because some sequential negotiations in our framework will be strategically equivalent to simultaneous ones.

[11] For instance, α_i may reflect the likelihood of having other customers for a capacity-constrained seller i. Observe however that our qualitative results would go through if we used a generalized Nash Bargaining solution, where bargaining power is appropriately parameterized (Binmore (1987)).

the buyer possesses this information; in particular, suppliers are not revealed the order of negotiations or the buyer's purchasing history.[12,13] We denote by $P_i^E(\widetilde{\rho}_i)$ and $\overline{P}_i^E(\widetilde{\rho}_i)$ the price offer made by player $\widetilde{\rho}_i \in \{s_i, b\}$ for product i under the expiration vector, E, when negotiations are public and private, respectively. Also, we let $\phi_1 \in \{0, 1\}$ be the upstream trade outcome and $\rho_1 \in \{s_i, b\}$ be the upstream proposer.

The extensive form of our basic negotiation game proceeds in three stages. In stage 0, E is observed by all players. In stage 1, the buyer commits to perform public or private negotiations, and communicates her decision to the suppliers.[14] And, in stage 2, she engages in bilateral negotiations in the order of her choosing. We characterize perfect Bayesian equilibria of this game. We say that (given our setup) trade is *efficient* if, in equilibrium, the buyer purchases both goods with probability 1, and *inefficient* otherwise. In case of indifference, we assume all players break ties in favor of Pareto efficient decisions, i.e., buying and selling more units.

We begin our investigation with public negotiations and then explore private negotiations. In Section 5, we compare their equilibrium outcomes to determine the buyer's preference between them.

3 Public Negotiations

Under public negotiations, the complete history of negotiations – especially price offers and trade outcomes – is known by all players. The analysis of public negotiations is of independent interest as certain mandatory disclosure laws may permit public access to information about the buyer's conduct (see Footnote 1). In what follows, we characterize equilibrium for each pair of expiration policies, and then draw some conclusions in Proposition 1.

Lemma 1. *Suppose negotiations are public and both offers are exploding. Then, in equilib-*

[12] In Section 6.1, we extend the analysis to *partially private* negotiations under which some information about the history such as the order of negotiations and/or the previous price offer is disclosed to the suppliers.

[13] It is worth pointing out that private negotiations exist only if there are more than two contracting periods so that the suppliers cannot infer the order by simply observing the calendar time. Moreover, our assumption of no discounting eliminates the strategic effect of calendar time.

[14] The buyer's ability to commit to a disclosure policy may simply be the result of her negotiation policy or the suppliers' legal requests for confidentiality in their business dealings. For instance, Florida law (Florida Statues 373.139) makes an exception to the sunshine law and allows the state's water management districts to treat negotiations for the acquisition of property for public purpose as confidential to ensure the people of Florida receive the best investment for their tax dollars. Moreover, commitment allows us to better relate our work to the papers with a similar assumption, e.g., Noe and Wang (2004). Nonetheless, in Section 6.2, we determine the equilibrium levels of disclosure when the buyer lacks commitment power, and show that the buyer's preferred disclosure policy can be part of an equilibrium.

rium, trade is efficient; the buyer is indifferent to the order of negotiations; and prices are given by $P_i^X(b) = P_j^X(b|\phi_1 = 1) = 0$*, and*

$$P_i^X(s_i) = 1 - v_j - \alpha_j \Delta \textit{ and } P_j^X(s_j|\phi_1 = 1) = 1 - v_i. \tag{1}$$

Lemma 1 indicates that because the negotiation sequence and the upstream trade outcome are publicly observed, sellers charge prices that induce an efficient trade in equilibrium. Furthermore, the buyer's offer covers only the cost of production, as expected. To understand suppliers' prices in equation (1), note that with exploding offers, any payment made in the upstream negotiation is sunk as far as the downstream supplier is concerned. Hence, following the purchase of good i in the upstream, the buyer is willing to pay up to her marginal utility, $1-v_i$, for good j in the downstream, and this is exactly what seller j proposes. Supplier i, however, has to take the buyer's expectations regarding the downstream negotiation into account. In particular, he knows that the buyer possesses a reservation payoff of $(1-\alpha_j)v_j$ by purchasing good j alone, and that upon acquiring good i, she expects to pay $\alpha_j(1-v_i)$ in the downstream negotiation. Hence, supplier i's price offer, $P_i^X(s_i)$ in equation (1), is the highest that satisfies $1 - P_i^X(s_i) - \alpha_j(1-v_i) \geq (1-\alpha_j)v_j$.

Two remarks are in order. First, with substitutes, i.e. $\Delta < 0$, seller i's price and thus her expected payoff is *increasing* in j's bargaining power. To see this, note that as α_j gets larger, the buyer's reservation payoff diminishes, improving i's bargaining position; but a larger α_j also means the buyer expects to pay more in the downstream negotiation, should she decide to purchase both goods, squeezing seller i's offer. For substitute goods, the former effect dominates because of diminishing marginal value, i.e., $1 - v_i < v_j$. By the same token, with complements, seller i's payoff is decreasing in j's bargaining power because of increasing marginal value.[15] Second, observe that for complements, $P_i^X(s_i) > v_i$ and $P_i^X(s_i) + P_j^X(s_j|\phi_1 = 1) > 1$, where the former shows the upstream seller's incentive to capture the additional surplus, Δ, and the latter indicates a negative payoff for the buyer if both suppliers end up proposing.[16]

When both offers are open-ended, the buyer simply obtains a price quote from the upstream supplier, and makes all her purchasing decisions after bargaining in the down-

[15] The observation that a seller may prefer a stronger rival is consistent with that of Marx and Shaffer (2007a), who also study sequential negotiations, though with fully contingent payments. Lemma 1 shows the importance of substitutability for this observation; and later we also show the importance of the public nature of negotiations.

[16] Of course, the buyer still acquires both goods in this case, because with exploding offers, she reaches a decision in the upstream negotiation in expectation about the downstream.

stream.[17] This means the downstream supplier's price can only depend on the identity of the upstream proposer, ρ_1, as opposed to the trade outcome, ϕ_1.

Lemma 2. *Suppose negotiations are public and both offers are open-ended. Then, in equilibrium, trade is efficient; the buyer is indifferent to the order of negotiations; and prices are given by* $P_i^O(b) = P_j^O(b|\rho_1) = 0$, *and*

$$P_i^O(s_i) = 1 - v_j,$$
$$P_j^O(s_j|\rho_1 = b) = 1 - v_i \text{ and } P_j^O(s_j|\rho_1 = s_i) = \begin{cases} v_j & for \quad \Delta \geq 0, \\ 1 - v_i & for \quad \Delta < 0. \end{cases} \tag{2}$$

Although equilibrium trade is still efficient, and the buyer's price offer is the same, suppliers' pricing strategies with open-ended offers are significantly different from the ones with exploding offers. Most notably, for complements, seller j in the downstream now prices less aggressively to accommodate i's offer; and anticipating this behavior, seller i demands a higher price.[18] For substitutes, however, seller i is forced to set a low "Bertrand" price with an open-ended offer because, without a strict time limit on his offer, he cannot foreclose the competition with the rival. Indeed, for the case of perfect substitutes, open-ended offers drive each price down to the marginal cost 0, whereas with an exploding offer, seller i is able to charge a price of α_j. It is also worth noting that $P_i^O(s_i) + P_j^O(s_j|\rho_1) \leq 1$, implying that unlike with exploding offers, the buyer never receives a negative *ex post* payoff when offers are open-ended.

We complete the investigation of public negotiations with hybrid expiration policies. Note that since only the upstream expiration policy is relevant for the buyer, the equilibrium with hybrid offers reduces to one of the previous two. That is, if the upstream offer is open-ended (resp. exploding), then it is equivalent to having both offers open-ended (resp. exploding).

Lemma 3. *Suppose negotiations are public and expiration policies are hybrid, i.e.,* $E = H_1$ *or* H_2. *Then, in equilibrium, trade is efficient; and the buyer is indifferent to the order of negotiations for complements, whereas, for substitutes, she strictly prefers to negotiate first with the supplier whose offer is open-ended.*

[17] Needless to say, with an open-ended offer, the buyer still has the option of closing the deal in the upstream prior to the downstream negotiation. But this option for the buyer is dominated by the option of waiting to avoid being held up by the downstream supplier.

[18] Indeed, it is easy to verify that for complements, there is a first-mover advantage under open-ended offers, and a second-mover advantage under exploding offers.

An important implication of Lemma 3 is that for complements, the buyer is indifferent between having an exploding and open-ended offer in the first negotiation. In particular, the decision flexibility with an open-ended offer does not make her strictly better off. The reason is as explained above: while a strict deadline in the upstream negotiation may bring an *ex post* negative payoff to the buyer, it also leads to a less aggressive pricing by the upstream supplier. For substitute goods, however, the buyer is strictly better off having open-ended offers in order to create a fiercer competition, leading to Bertrand prices.

Drawing upon the three partial results so far, we reach the following conclusions for public negotiations.

PROPOSITION 1. *In equilibrium with public negotiations,*

(a) *trade is efficient, irrespective of expiration policies;*

(b) *the buyer is indifferent to the order of negotiations for complements; but for substitutes, she strictly prefers to negotiate first with the supplier whose offer is open-ended;*

(c) *if the buyer negotiates with s_1 first whenever indifferent, then s_1 weakly prefers an open-ended (resp. exploding) offer with complements (resp. substitutes), irrespective of the rival's expiration policy.*

Part (a) of Proposition 1 is a consequence of the payoffs and the history of negotiations being common knowledge. Part (b) implies that the equilibrium sequence of negotiations is independent of suppliers' bargaining powers and the heterogeneity of their products. With complements the equilibrium sequence is also independent of the expiration policies, while with substitutes the buyer strictly prefers to visit first the supplier whose offer is open-ended.

Finally, supposing that when indifferent the buyer visits s_1 first, part (c) reveals that s_1 weakly prefers an open-ended offer for complements to further raise his price without running into a coordination problem, and an exploding offer for substitutes to avoid a head-to-head competition with the rival. An implication of this finding is that if s_1 follows a weakly undominated strategy, then it is a best response for s_2 to adopt the same expiration policy as s_1.

4 Private Negotiations

Under private negotiations, the buyer discloses no information about the history of negotiations to suppliers. In particular, suppliers are not told the previous trade outcomes or the order in which they are being approached. Unlike with public negotiations, equilibrium characterization with private negotiations turns out to be significantly different across substitutes and complements due to the fact that equilibrium trade is not always efficient; and with complements, the buyer doesn't have a strict preference over expiration policies. Thus, we start our characterization by substitutes and then proceed to complements.

4.1 Substitutes

Given that they are relatively intuitive, we collect all our observations for substitutes in the following proposition.

PROPOSITION 2. *Suppose goods are substitutes and negotiations are private. Then, in equilibrium* $\overline{P}_i^E(b) = 0$ *for all* i *and* E*. Moreover,*

(a) *if at least one offer is open-ended, i.e.,* $E \neq X$*, then, in equilibrium,*

- *trade is always efficient*;
- *the buyer visits first the supplier with an open-ended offer; and*
- *the suppliers set Bertrand prices:* $\overline{P}_i^E(s_i) = 1 - v_j$.

(b) *If both offers are exploding, i.e.,* $E = X$*, then*

- *there always exists an efficient equilibrium with Bertrand prices as in part (a); but*
- *there also exists an inefficient equilibrium at which* $\overline{P}_i^X(s_i) = v_i$ *for all* i *if and only if* $\underline{\sigma}_i^X < \overline{\sigma}_i^X$*, where the expressions for* $\underline{\sigma}_i^X$ *and* $\overline{\sigma}_i^X$ *are derived in the proof. In an inefficient equilibrium, the buyer visits first* s_i *with probability* $\sigma_i^X \in (\underline{\sigma}_i^X, \overline{\sigma}_i^X)$.

(c) *The buyer weakly prefers to have at least one open-ended offer, whereas each supplier weakly prefers an exploding offer, irrespective of the rival's expiration policy.*

Part (a) of Proposition 2 says that with substitutes, equilibrium trade is always efficient when at least one offer is open-ended, because suppliers charge their Bertrand prices. The reason for such intense competition is obvious when both offers are open-ended: the buyer can compare prices before reaching a purchasing decision. In the case of hybrid offers, the buyer can still compare prices by optimally negotiating first with the supplier whose offer is open-ended.

Part (b) indicates that when both offers are exploding, the efficient equilibrium identified in part (a) still exists, because given that the rival sets a Bertrand price and thus guarantees a sale, the highest a supplier can charge is his Bertrand price, too. However, with all exploding offers, suppliers' posting their "monopoly" prices may form another equilibrium, resulting in an inefficient trade. Intuitively, if seller i expects the rival to set his monopoly price, v_j, and believes his chances of being approached second to be sufficiently high, then it is optimal for him to charge his monopoly price, v_i, too; because, in such an equilibrium, the buyer always rejects the upstream supplier's offer, but accepts the downstream's whenever she hasn't acquired the upstream good by proposing a price of 0 herself. Notice that for a monopoly pricing equilibrium to exist, the buyer has to follow a proper mixed strategy over the sequence so that each seller has a reasonable chance of selling his product by being the second to negotiate. Finally, since Bertrand prices are the lowest the buyer can secure from the suppliers, part (c) reports that the buyer weakly prefers to have at least one open-ended offer; and since Bertrand prices generate the lowest profit for the suppliers, each supplier weakly prefers to have an exploding offer regardless of the rival's expiration policy.

Compared to Proposition 1, Proposition 2 implies that with substitutes, parties' preferences over expiration policies are the same across public and private negotiations. Note, however, that unlike public negotiations, private negotiations enable the buyer to obtain Bertrand prices even when both offers are exploding. This is because the latter create a strategic uncertainty under which each supplier (rationally) believes that a deal has been struck with the rival and prices his product accordingly. Although, as recorded in part (b), such strategic uncertainty may also produce a monopoly pricing equilibrium, in Section 5 we will show that the buyer is still not worse off than in public negotiations.

Proposition 2 also implies that when both offers are exploding, equilibrium sequencing may be "nontrivial" in that the buyer follows a proper mixed strategy over the sequence, which cannot be inferred with certainty by the suppliers. This occurs, however, only at an inefficient equilibrium. Given that the expressions for $\underline{\sigma}_1^X$ and $\overline{\sigma}_1^X$ (derived in the Appendix) are somewhat complicated, we consider two special cases to discern when an inefficient equilibrium is likely to arise; and when it does, how equilibrium sequencing responds to parameters.

Corollary 1. *Suppose both offers are exploding, and negotiations are private. Moreover, suppose goods are identical substitutes, i.e.,* $v_1 = v_2 = v \in (\frac{1}{2}, 1]$. *Then, there is no* *inefficient equilibrium.*

That is, when goods are identical substitutes, there is no equilibrium in which suppliers charge their monopoly prices. Conversely, for an inefficient equilibrium to exist and hence sequencing to be nontrivial, the buyer must possess different valuations for goods.[19] Corollary 1 also provides some robustness check for Noe and Wang's (2004) finding that endogenous sequencing when all offers are exploding is inconsequential for substitute products. Our result shows that their restriction to efficient equilibria is indeed without loss of generality for identical substitute goods (though they also assume symmetric sellers). However, it also shows that when goods are heterogenous, there may exist an inefficient equilibrium at which sequencing is nontrivial. To gain further insight into the latter point, we solve an example.

Example 1. *Suppose both offers are exploding, and negotiations are private. Moreover, suppose* $v_1 \in (\frac{1}{2}, 1)$, $v_2 = 1$, *and* $0 < \alpha_2 < 1 - v_1 < \widehat{\alpha}(v_1) < \alpha_1 < 1$, *where* $\widehat{\alpha}(v_1) \equiv \frac{2v_1 - 1 + \sqrt{1+4v_1}}{2v_1(2-v_1)}(1 - v_1)$. *Then, there exists an inefficient (monopoly pricing) equilibrium. In this equilibrium,* $\sigma_1^X \in (\underline{\sigma}_1^X, 1)$ *where* $\underline{\sigma}_1^X = \frac{1-(1-\alpha_1)v_1}{1-(1-\alpha_1)^2 v_1}$ *such that* $\underline{\sigma}_1^X > \frac{2}{3}$, *and* $\underline{\sigma}_1^X$ *increases in* α_1 *over* $(\widehat{\alpha}(v_1), 1)$.

Given that seller 1 is the stronger seller and his product is individually less valuable, Example 1 indicates that the buyer is more likely to approach him first at a monopoly pricing equilibrium to stay indifferent to the order; otherwise, facing a more powerful seller 1 in the last negotiation, she would be exposed to a holdup problem with a higher probability. By the same token, Example 1 also indicates that the likelihood of seller 1 being negotiated with first goes up as his bargaining power increases.

4.2 Complements

For complements, equilibrium characterization is more involved, leading us to first analyze each case corresponding to a different pair of expiration policies and then provide some general conclusions in Proposition 3. However, since, as mentioned before, expiration policies may be the industry standards each supplier complies with, or they may be imposed by certain regulations, we believe each case is also of independent interest.

[19] Interestingly, having $\alpha_1 = \alpha_2$ doesn't always eliminate the inefficient equilibrium.

Lemma 4. *Suppose goods are complements, and negotiations are private. Moreover, suppose both offers are open-ended, $E = O$. Then, equilibrium prices are given by:* $\overline{P}_i^O(b) = 0$ *and*

$$\overline{P}_i^O(s_i) = \begin{cases} 1 - v_j & \text{if } \sum_k \alpha_k(1-v_k) < \Delta, \\ 1 - v_j; \text{ or } 1 - \overline{P}_j^O(s_j) \text{ for } \overline{P}_j^O(s_j) \in \Omega'_j & \text{if } \sum_k \alpha_k(1-v_k) \geq \Delta \text{ and } \alpha_k(1-v_k) \leq \Delta \text{ for all } k, \\ 1 - \overline{P}_j^O(s_j) \text{ for } \overline{P}_j^O(s_j) \in \Omega''_j & \text{if } \alpha_k(1-v_k) > \Delta \text{ for some } k, \end{cases}$$

for all i where $k = 1, 2$, $\Omega'_j \equiv [(1-\alpha_i)(1-v_i), 1-(1-\alpha_j)(1-v_j)]$ and $\Omega''_j \equiv [v_j, 1-v_i]$.

As with substitutes, suppliers play a simultaneous pricing game when both offers are open-ended, rendering the negotiation sequence inconsequential to the equilibrium outcome. However, with complements, each supplier's pricing strategy trades off charging a (moderate) coordination price, $\overline{P}_i^O(s_i) = 1 - \overline{P}_j^O(s_j)$, thereby guaranteeing a sale against charging a (aggressive) noncoordination price, $\overline{P}_i^O(s_i) = 1 - v_j$, thereby selling only if the buyer proposes and asks for a price of 0 in the other negotiation. Clearly, the latter is a more profitable option if the rival is less powerful against the buyer, and this is exactly what Lemma 4 shows: for sufficiently small α_k's, only the noncoordination equilibrium exists whereas, for sufficiently large α_k's, only the coordination equilibria exist. It is also worth noting that all else equal, a coordination equilibrium is less likely to occure as goods become stronger complements, i.e., a larger Δ. In particular, for perfect complements, there always exists a noncoordination equilibrium. The intuition is that strong complements require the buyer to purchase both goods, and knowing this, each supplier targets the buyer's joint valuation in pricing his product.

Inspecting equilibrium prices in Lemma 4, it is evident that trade is efficient at a coordination equilibrium and inefficient at a noncoordinating equilibrium.[20] It is, however, less evident that when both coordination and noncoordination equilibria exist for some (intermediate) parameter values, the buyer is weakly better off at an efficient equilibrium, because coordination prices are lower.[21] This simple observation points to an interesting trade-off for the buyer: all else equal, she can either deal with weaker sellers who are likely to set higher (noncoordination) prices or deal with stronger sellers who are likely to set lower (coordination) prices. The following example demonstrates that the buyer may indeed resolve such a trade-off in favor of stronger sellers.

[20] Because, at a noncoordination equilibrium, both goods are purchased only if the buyer proposes in at least one negotiation.

[21] A formal statement of this observation and its proof are provided in Lemma A1 in the Appendix.

Example 2. (The buyer may prefer suppliers with higher bargaining powers.) *Suppose both offers are open-ended, and let* $v_1 = 0$, $v_2 \in (\frac{1}{2}, \frac{2}{3})$, *and* $\alpha_2 \in (\frac{2\lambda}{1-\alpha_0+\lambda}, \frac{\lambda}{\alpha_0})$, *where* $\alpha_0 = 1 - v_2$ *and* $\lambda \in (3\alpha_0 - 1, \frac{\alpha_0^2}{1-\alpha_0})$. *Then, all else equal, the buyer is strictly better off dealing with supplier 1 whose bargaining power is* $\alpha_1'' = \alpha_0 + \lambda$ *than the one with* $\alpha_1' = \alpha_0 - \lambda$.[22]

An important implication of Example 2 is that when procuring complementary goods, a purchasing alliance or buyer group may not necessarily obtain lower prices (or a higher payoff) as its size and hence relative bargaining power vis-à-vis the suppliers increases.

Next, we characterize equilibrium for the case with hybrid offers.

Lemma 5. *Suppose goods are complements, and negotiations are private. Moreover, suppose supplier i's offer is exploding whereas j's offer is open-ended, i.e.* $E = H_j$. *Then, in equilibrium* $\overline{P}_i^{H_j}(b) = 0$ *for all* i. *Moreover,*

- *there exist efficient equilibria all with* $\overline{P}_i^{H_j}(s_i) + \overline{P}_j^{H_j}(s_j) = 1$ *if and only if* $\alpha_i(1 - v_i) + \alpha_j(1 - v_j) \geq \Delta - \frac{\theta}{1-\theta}v_i$, *where* $\theta = \frac{\sqrt{1+4\alpha_j^2}-1}{\sqrt{1+4\alpha_j^2}+1}$. *In an efficient equilibrium, the buyer visits first* s_i *with probability* $\sigma_i^{H_j} \in [\underline{\sigma}_i^{H_j}, \overline{\sigma}_i^{H_j}]$, *where* $\underline{\sigma}_i^{H_j} = \frac{\Delta-\alpha_i(1-v_i)-\alpha_j(1-v_j)}{1-v_j-\alpha_i(1-v_i)-\alpha_j(1-v_j)}$ *and* $\overline{\sigma}_i^{H_j} \in [\theta, 1]$;
- *there also exists an inefficient equilibrium at which* $\overline{P}_i^{H_j}(s_i) = 1 - v_j$ *for all* i *if and only if* $\alpha_i(1-v_i) \leq \Delta$ *for all* i. *In an inefficient equilibrium, the buyer visits first* s_i *with probability* $\sigma_i^{H_j} \in [0, \overline{\sigma}_i^{H_j}]$, *where* $\overline{\sigma}_i^{H_j} = \min\{\frac{\alpha_j(1-\alpha_j)}{1-v_j-\alpha_j^2\Delta}\Delta, \frac{\Delta-\alpha_j(1-v_j)}{(1-\alpha_j)(1-v_j)}\}$.

Lemma 5 is perhaps best understood in conjunction with Lemma 4. First, note that the same equilibrium prices as the ones with open-ended offers are sustainable under hybrid offers. The difference is that while the parameter conditions for an inefficient noncoordination equilibrium in the two cases coincide, the condition for an efficient coordination equilibrium is weaker under hybrid offers (by the term $\frac{\theta}{1-\theta}v_i$). This means an efficient trade

[22] This follows because, according to Lemma 4, the negotiation game with parameter values $(v_1, v_2, \alpha_1', \alpha_2)$ possesses a unique and inefficient equilibrium at which the buyer's expected payoff is $\pi'(b) = (1 - \alpha_1')(1 - \alpha_2) + \alpha_1'(1 - \alpha_2)v_2$, whereas the game with parameter values $(v_1, v_2, \alpha_1'', \alpha_2)$ possesses only the efficient coordination equilibria at which $\pi''(b) = (1 - \alpha_1'')(1 - \alpha_2) + \alpha_1''(1 - \alpha_2)(1 - P_1''(s_1)) + (1 - \alpha_1'')\alpha_2 P_1''(s_1)$, where $0 \leq P_1''(s_1) \leq 1 - v_2$. Comparing the two payoffs and noting $\alpha_1'' > \alpha_2$, we have

$$\pi''(b) - \pi'(b) \geq \alpha_0[1 - \alpha_1'' - (1 - \alpha_2)(1 - \alpha_1')] > 0,$$

where the last inequality follows because $\alpha_2 > \frac{2\lambda}{1-\alpha_0+\lambda}$.

is easier to obtain when offers are hybrid than when both are open-ended. The intuition is that the supplier with an exploding offer prices less aggressively in order to ensure a sale when approached first *and* an exploding offer requires the buyer to undertake one decision at a time, ignoring previous payments. Second, unlike with open-ended offers, equilibrium sequencing with hybrid offers is nontrivial, because $(\underline{\sigma}_i^{H_j}, \overline{\sigma}_i^{H_j}) \neq (0,1)$. This implies that unlike with substitutes, the buyer has no strict preference for an open-ended offer. As with public negotiations, the trade-off comes from the fact that although an open-ended offer allows the buyer to receive a nonnegative payoff *ex post*, the supplier with an exploding offer prices less aggressively. These two observations with hybrid offers extend to the case with both exploding offers, as the following lemma shows.

LEMMA 6. *Suppose goods are complements, and negotiations are private. Moreover, suppose both offers are exploding. Then, in equilibrium* $\overline{P}_i^X(b) = 0$ *for all* i*. Moreover,*

- *there always exists an efficient equilibrium at which* $\overline{P}_i^X(s_i) = \frac{1}{1-\alpha_i\alpha_j}[(1-\alpha_j)(1-v_j) + \alpha_j(1-\alpha_i)v_i]$ *for all* i*. In an efficient equilibrium, the buyer visits first* s_i *with probability* $\sigma_i^X \in [1 - \frac{\overline{P}_i^X(s_i)}{1-v_j}, \frac{\overline{P}_j^X(s_j)}{1-v_i}] \neq \{\}$*;*
- *there also exists an inefficient equilibrium at which* $\overline{P}_i^X(s_i) = 1 - v_j$ *for all* i *if and only if* $\alpha_i(1-v_i) \leq \Delta$ *and* $\underline{\sigma}_i^X \leq \overline{\sigma}_i^X$ *for all* i*, where* $\underline{\sigma}_i^X = 1 - \min\{\frac{\alpha_i(1-\alpha_i)}{1-v_i-\alpha_i^2\Delta}\Delta, \frac{\Delta-\alpha_i(1-v_i)}{(1-\alpha_i)(1-v_i)}\}$ *and* $\overline{\sigma}_i^X = \overline{\sigma}_i^{H_j}$*. In an inefficient equilibrium, the buyer visits first* s_i *with probability* $\sigma_i^X \in [\underline{\sigma}_i^X, \overline{\sigma}_i^X]$*.*

Lemma 6 confirms our previous intuition: efficient trade is even easier to obtain when both offers are exploding, since, unlike with hybrid offers, there always exists an efficient equilibrium and the parameter condition for an inefficient equilibrium is more stringent. Lemma 6 also reveals that equilibrium sequencing with both exploding offers is nontrivial, too. To extract further insight into equilibrium sequencing and contrast it with the case of substitutes in Example 2, we consider a special case with perfect complements.

EXAMPLE 3. *Suppose goods are perfect complements, i.e.,* $v_i = 0$ *for all* i*, and negotiations are private. Moreover, suppose both offers are exploding. Then, by Lemma 6, there only exist efficient equilibria where* $\overline{P}_i^X(s_i) = \frac{1-\alpha_j}{1-\alpha_i\alpha_j}$ *and* $\sigma_i^X \in [\alpha_j\frac{1-\alpha_i}{1-\alpha_i\alpha_j}, \frac{1-\alpha_i}{1-\alpha_i\alpha_j}]$.

Inspecting Example 3, note that unlike with substitutes, equilibrium sequencing with complements involves randomization even at an efficient equilibrium.[23] Note also that as α_i increases, both the upper and lower bounds of the interval for σ_i^X decrease, meaning that the buyer is more likely to negotiate first with the less powerful supplier. This is somewhat surprising because, leaving the more powerful supplier to the end, the buyer seems to be subjecting herself to a more severe holdup problem. However, anticipating this problem and being worried about coordination at an efficient equilibrium, it is the weaker supplier who is willing to give a larger discount in the upstream negotiation. In fact, $\overline{P}_i^X(s_i) \to 0$ as $\alpha_j \to 1$.

Now, combining the results of Lemmas 4-6, we reach some general conclusions about private negotiations with complements.

PROPOSITION 3. *In equilibrium with private negotiations and complements,*

(a) *trade is more likely to be efficient as more offers become exploding;*

(b) *if $\alpha_i(1 - v_i) > \Delta$ for some i, then trade is always efficient, irrespective of expiration policies; and*

(c) *the buyer is better off when $E = X$ than when $E = O$ if and only if $\sum_i \alpha_i(1 - v_i) < \Delta$. Moreover, if $\sum_i \alpha_i < 1$ and $v_i = 0$ for all i, then each supplier is also better off when $E = X$ than when $E = O$.*

The intuition behind part (a) is as explained above: all else equal, the supplier with an exploding offer prices less aggressively than the one with an open-ended offer to guarantee a sale whenever approached first and exploding offers compel the buyer to make one purchasing decision at a time by ignoring previous payments. Part (b) says that trade is also likely to be efficient if at least one supplier is sufficiently powerful against the buyer and/or goods are sufficiently weak complements.[24] This makes sense because, as alluded to above, it is the more powerful suppliers who, anticipating a demanding offer by the rival, are more concerned about coordinating prices. In addition, price coordination is less of a problem for weaker complements due to the buyer's diminished need for a joint purchasing decision.

[23] Otherwise, if the suppliers could perfectly predict the order in an efficient equilibrium, they would charge the same prices under public negotiations; in particular the downstream supplier would raise his to 1.

[24] Note that for identical complements, i.e., $v_i = v < 1/2$, the ratio $\frac{\Delta}{1-v_i}$ is strictly decreasing in v, relaxing the condition $\alpha_i(1 - v_i) > \Delta$.

Finally, part (c) shows that the buyer benefits from the increased efficiency by exploding offers whenever inefficient trade is the only outcome under open-ended offers, and we know from Lemma 4 is that the latter is the case for sufficiently weak suppliers and/or sufficiently strong complements. Perhaps more interestingly, part (c) also shows that suppliers might also benefit from the increased efficiency for, while they have to lower prices when offers are exploding, they are more likely to sell their products.

In contrast to public negotiations, note that private negotiations with complements may result in an inefficient trade owing to the strategic uncertainty the suppliers are exposed to. Moreover, with private negotiations, the buyer is unlikely to be indifferent between having exploding and open-ended offers, because the strategic uncertainty affects supplier competition differently under different pairs of expiration policies. Finally, with private negotiations, suppliers' preferences for expiration policies seem to be more aligned with those of the buyer's. In particular, to the extent that price coordination is hard to achieve in equilibrium with open-ended offers, suppliers are as likely to opt for exploding offers as is the buyer.

5 Public vs. Private Negotiations

Armed with the equilibrium characterizations and insights from public and private negotiations, we now determine the buyer's preference between the two, beginning with substitutes.

Proposition 4. *With substitutes, if at least one offer is open-ended, then the buyer is indifferent between public and private negotiations; otherwise if both offers are exploding, she weakly prefers private negotiations.*

Proposition 4 is a direct consequence of Propositions 1 and 2. With substitutes, when at least one offer is open-ended, the buyer is able to uniquely generate the Bertrand competition in both public and private negotiations by approaching first the supplier whose offer is open-ended. When both offers are exploding, however, the buyer's inability to compare prices may allow suppliers to avoid such head-to-head competition; and absent any uncertainty about the trade history, this is the case with public negotiations. In contrast, with private negotiations, Bertrand prices still emerge in equilibrium when each supplier (correctly) anticipates a successful trade in the other negotiation, irrespective of its proposer. As alluded to in Proposition 2, such strategic uncertainty in private negotiations may also lead to a monopoly pricing equilibrium, resulting in an inefficient trade. Nonetheless, in this equilibrium, the buyer receives an expected payoff equal to the one in public negotiations, explaining the weak preference in Proposition 4. In light of Corollary 1, we note one sufficient condition under which the buyer's preference for private negotiations is strict.

Corollary 2. *With substitutes, if both offers are exploding, and $v_i = v$ for all i, then the buyer strictly prefers private negotiations.*

With complements, the buyer's preference depends not only on expiration policies but also on suppliers' relative bargaining powers and strength of complementarity, because, as explained in the previous section, all these factors affect trade efficiency.

Proposition 5. *With complements,*

(a) *if both offers are open-ended, then the buyer weakly prefers private negotiations whenever $\sum_i \alpha_i(1-v_i) \geq \Delta$; and she is indifferent between public and private negotiations whenever $\sum_i \alpha_i(1-v_i) < \Delta$;*

(b) *if supplier i's offer is exploding and j's offer is open-ended, then the buyer weakly prefers private negotiations whenever $\sum_i \alpha_i(1-v_i) \geq \Delta - \frac{\theta}{1-\theta}v_i$; and she is indifferent between public and private negotiations whenever $\sum_i \alpha_i(1-v_i) < \Delta - \frac{\theta}{1-\theta}$, where $\theta = \frac{\sqrt{1+4\alpha_j^2}-1}{\sqrt{1+4\alpha_j^2}+1}$;*

(c) *if both offers are exploding, then the buyer strictly prefers private negotiations.*

Inspecting Proposition 5, it is clear that with complements, the buyer is more likely to prefer private negotiations as: (1) suppliers become more powerful vis-à-vis the buyer; (2) more offers become exploding; and/or (3) goods become weaker complements. Recall from Proposition 3 that these are also the conditions under which an efficient equilibrium with private negotiations is more likely to come about. Although achieving efficiency is not the buyer's objective per se, with private negotiations increased efficiency is an indication of a less aggressive pricing by the suppliers who compete under the veil of strategic uncertainty. It is thus this uncertainty that lets the buyer claim a larger share of the trade surplus under private negotiations.

Combining Propositions 4 and 5, we can say that the buyer is weakly better off keeping the negotiations private. Moreover, given the prevalence of parameter conditions, we can also say that the buyer is more likely to prefer private negotiations when purchasing complements. Since, unlike public negotiations, private negotiations may result in an inefficient trade, our results may provide an explanation for the presence of mandatory disclosure laws (see Footnote 1), and why such laws are likely to be binding for the buyer. In the absence of mandatory disclosure laws, or when they do not apply to the buyer, it appears that the extended return and cancellation policies such as the FTC's cooling-off rule that essentially require suppliers' price offers to be open-ended are most conducive to achieving efficiency for the purchases of substitute goods because open-ended offers create a Bertrand competition. For the purchases of complementary goods, however, such policies may actually interfere with an efficient trade by giving suppliers an incentive to price more aggressively. In fact, in light of part (a) of Proposition 3 with complements, it seems an alternative policy of "all sales are final!" could be socially more desirable.

6 Extensions

In this section, we first augment our previous analysis to include "partially" private negotiations in which some but not all information about the history of trade is disclosed to suppliers. Then, we consider situations in which the buyer lacks commitment to a disclosure policy, and we provide a characterization of equilibrium disclosure levels, with an emphasis on whether or not her preferred disclosure policies under commitment can be sustained as equilibria.

6.1 Partially Private Negotiations

With respect to the suppliers' knowledge of the negotiation history, public and private negotiations correspond to two extreme forms of information structure, one in which the upstream price offer (hence, the order of negotiations) and the buyer's purchasing decision in case of an exploding offer are made public, and one in which both pieces of information are private to the buyer. Here we extend the analysis to partially private negotiations whose information structure lies in-between the two extremes: while the (endogenous) order of negotiations becomes public before they commence, the buyer's upstream trade decision remains confidential. In many applications, the upstream price also remains confidential, but in some others it may be publicly available. If both offers are open-ended, then clearly a confidential (resp. public) upstream price turns partially private negotiations into private (resp. public) ones, which we have analyzed above. If, on the other hand, both offers are exploding, an upstream price that is public may provide the downstream supplier with valuable information about the upstream trade. We find, however, that such information has no value in equilibrium, as trade is always efficient.

LEMMA 7. *Suppose negotiations are partially private, and both offers are exploding. Then, in equilibrium, trade is efficient whether the upstream price is public or confidential; the buyer is indifferent to the order of negotiations; and prices are given by:* $\widehat{P}_i^X(b) = \widehat{P}_j^X(b) = 0$, *and*

$$\widehat{P}_i^X(s_i) = \begin{cases} 1 - v_j & if \quad \Delta < 0 \\ 1 - v_j - \alpha_j \Delta & if \quad \Delta \geq 0, \end{cases} \quad and \ \widehat{P}_j^X(s_j) = 1 - v_i,$$

where, w.l.o.g., the buyer negotiates with supplier i first.

Lemma 7 says that with substitutes the buyer can actually secure the Bertrand prices as the unique equilibrium prices *even* when both offers are exploding. This makes sense, because the reason behind an inefficient (monopoly pricing) equilibrium under private negotiations was that both suppliers put a sufficient weight on being the last to negotiate with the buyer. Lemma 7 also says that with complements equilibrium prices under partially private negotiations are exactly the same as those under public negotiations, because $1 - v_i > v_j$, which means that the buyer purchases from supplier j only if she makes the offer.

With the extension of partially private negotiations, we reach the following conclusion for the buyer's preference over negotiation form:

PROPOSITION 6. *Among public, private, and partially private negotiations, the buyer weakly prefers partially private negotiations for substitutes and private negotiations for complements.*

The intuition for substitutes is rather clear: when at least one offer is open-ended, then the buyer can uniquely engender the Bertrand prices, irrespective of the negotiation form; and when both offers are exploding, we know from Lemma 7 that she can still do so under partially private negotiations, irrespective of the upstream price being public or private. To understand the buyer's preference with complements, recall from the previous section that the buyer weakly prefers private negotiations over public ones. Next note that when both offers are exploding, Lemma 7 implies that public and partially private negotiations are strategically equivalent. As explained before, with exploding offers the buyer is subject to a severe holdup problem in the downstream if her purchasing history is perfectly predicted in equilibrium. She partially avoids this problem under private negotiations by being able to randomize over the negotiation sequence. When both offers are open-ended, depending on whether the upstream price is public or private, partially private negotiations are strategically equivalent to public or private negotiations respectively. Finally, when offers are hybrid, the buyer is weakly better off commencing partially private negotiations with the supplier whose offer is open-ended. But then, partially private and private negotiations are strategically equivalent.

REMARK. In some applications, it may be feasible for supplier i to make a lottery offer, $(q_i, P_i(s_i))$, where $P_i(s_i)$ is the lottery price and q_i is the probability that the good is delivered to the buyer. In Appendix C we briefly investigate this possibility when both offers are exploding, and show that the upstream supplier may indeed use a lottery in equilibrium under partially private negotiations with publicly observable offers. Compared to the case without lotteries, we find that with complements, the buyer's payoffs and thus her preference among negotiation types remains as in Proposition 6. In contrast, with substitutes the buyer may strictly prefer private negotiations over partially private ones, though public negotiations are still dominated by both. The intuition for substitutes is that by committing to not deliver the good with some probability, the upstream supplier induces the downstream rival to set a monopoly price, which, in turn, raises the buyer's willingness to pay in the upstream.

6.2 Negotiating without Commitment to a Disclosure Policy

Up to now we have considered the cases in which, prior to visiting suppliers, the buyer can credibly commit to her disclosure policy, resulting in public, private, or partially private negotiations. In this subsection we consider the cases in which the buyer lacks such commitment power, and determine the equilibrium levels of disclosure. There are, however, a plethora of equilibria owing to different equilibrium beliefs that may be held by suppliers. To highlight the role of noncommitment, we show two such equilibria: one that yields the same level of disclosure the buyer would choose under commitment, and the other that yields full disclosure leading to public negotiations.

Let $I_1 = \{s\}$ and $I_2 \subseteq \{s, \phi_1, P_1\}$ denote the buyer's information sets prior to the upstream and downstream negotiations, respectively, where s represents the negotiation sequence, and as before, P_1 denotes the upstream price and ϕ_1 the purchasing decision. Clearly, the buyer can at most reveal the sequence, s in the upstream whereas in the downstream, she can reveal either the full history (namely, P_1 if $e_1 = o$, and both ϕ_1 and P_1 if $e_1 = x$), only the sequence, or nothing. Let $d_1 \in \{\{s\}, \{\}\}$ and $d_2 \in \{I_2, \{s\}, \{\}\}$ be the buyer's respective disclosure choices in each negotiation. Specifically, the extensive form game proceeds as follows. First, all players observe the expiration policies. Next, the buyer chooses the negotiation sequence. Then, she decides on her disclosure action d_i in negotiation i, and the bargaining over the price begins. The following proposition is the main result in this subsection.

Proposition 7. *Under noncommitment,*

(a) *there exist equilibria at which* $d_1^* = d_2^* = \{s\}$ *for substitutes, and* $d_1^* = d_2^* = \{\}$ *for complements;*

(b) *there also exist equilibria at which* $d_1^* = \{s\}$ and $d_2^* = I_2$ *for both substitutes and complements.*

Proposition 7 indicates that, in the absence of commitment, the buyer's most preferred disclosure policy, namely partially private for substitutes and private for complements, as well as her least preferred one, namely public negotiations, are sustainable as equilibrium outcomes.[25] The intuition is most transparent under exploding offers. With substitutes, recall from the previous section that the buyer obtains the Bertrand prices under partially private negotiations. This means that she has a strict incentive to disclose the sequence in the upstream, and cannot gain by disclosing the trade history (i.e., the purchasing decision or the price) in the downstream. In turn, it is an equilibrium strategy for the upstream supplier to offer his Bertrand price, and anticipating this, the downstream supplier to offer his. It is, however, also an equilibrium for the buyer to fully disclose in each negotiation, if suppliers expect her to do so and hold (off-equilibrium) beliefs such that whenever the buyer discloses less, each believes to be in the upstream. With complements, as explained in Lemma 6, any deviation from nondisclosure subjects the buyer to a more severe holdup problem in the downstream. Thus, if suppliers expect nondisclosure, then the buyer has no incentive to deviate. However, as with substitutes, if each supplier believes full disclosure

[25] Our equilibrium constructions are based on *passive* beliefs: upon observing a deviation by the buyer, each supplier still anticipates the buyer to follow her equilibrium disclosure strategy with the rival.

in both negotiations, and a failure to do so by the buyer would mean he is in the upstream, then public negotiations also arise in equilibrium.

Proposition 7 is significant in two respects. First, since the buyer's most preferred disclosure policy is sustained in equilibrium, the lack of commitment does not necessarily diminish her payoff. Second, since the full disclosure is also sustained in equilibrium, mandatory disclosure laws may be less binding for those buyers who lack credible means to commit to a disclosure policy.

7 Concluding Remarks

In closing, we mention two future research avenues. The first one is to draw more upon the literature on bilateral contracting with externalities. In particular, the externality between the suppliers in our model comes only from the buyer's demands, which will indeed be the case if one has in mind products, political endorsements, pieces of land, workers in different production lines, etc. However, if one has in mind two skilled workers, say two faculty members, who may enjoy some direct externality by learning from each other, then our analysis may have to be modified. Some recent papers, e.g., Genicot and Ray (2006), and Moller (2007), endogenize the timing of bilateral negotiations with such externalities, but all assume contracts and agreements to be public. The second research avenue is to make expiration policies strategic choices of suppliers, much like in Daughety and Reinganum (1992). Our analysis would be useful here as it has already provided insights into parties' preferences for expiration policies.

Appendix A

This appendix provides the proofs of the formal results in Sections 3,4 and 5.

Proof of Lemma 1. Let negotiations be public, and $E = X$. Clearly, the buyer proposes a price of 0 in each negotiation. Now, assume s_i is approached first. If $\phi_1 = 0$, then $P_j^X(s_j|\phi_1 = 0) = v_j$, yielding an expected payoff of $(1 - \alpha_j)v_j$ to the buyer. If, on the other hand, $\phi_1 = 1$, then $P_j^X(s_j|\phi_1 = 1) = 1 - v_i$, in which case she acquires both goods. Anticipating these two continuation outcomes, s_i sets the highest price subject to $1 - P_i(s_i) - \alpha_j(1 - v_i) \geq (1 - \alpha_j)v_j$, whose unique solution is $P_i^X(s_i) = 1 - v_j - \alpha_j\Delta$. The buyer then acquires both goods with probability 1, resulting in an efficient trade.

The buyer's *ex ante* payoff from negotiating with seller i first is

$$\pi^X(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_i + \alpha_j(1 - \alpha_i)v_j. \tag{A-1}$$

which, by re-labeling, is equal to her expected payoff from negotiating with seller j first, proving her indifference to the order. ■

Proof of Lemma 2. Let negotiations be public, and $E = O$. Assume for now, s_i is approached first. It is straightforward to show that s_j's best response to an upstream price offer, $\widetilde{P}_i$ is:

$$P_j^*(s_j|\widetilde{P}_i) = \begin{cases} 1 - \widetilde{P}_i & if \quad v_i < \widetilde{P}_i \leq 1 - v_j \\ v_j & if \quad \widetilde{P}_i > \max\{v_i, 1 - v_j\} \\ 1 - v_i & if \quad \widetilde{P}_i \leq \min\{v_i, 1 - v_j\} \\ v_j - v_i + \widetilde{P}_i & if \quad 1 - v_j < \widetilde{P}_i \leq v_i. \end{cases} \tag{A-2}$$

To find equilibrium, consider now the first negotiation. Clearly, $P_i(b) = P_j(b|\rho_1) = 0$ in any equilibrium. To determine $P_i(s_i)$, note that if s_i anticipates j to propose in the downstream negotiation, then according to (A-2), s_i earns a nonnegative payoff whenever $P_i(s_i) \leq 1 - v_j$, leading him to post $P_i(s_i) = 1 - v_j$. If, on the other hand, s_i anticipates the buyer to propose in the downstream negotiation, then conjecturing $P_j(b|\rho_1) = 0$, he sets a price such that $1 - P_i(s_i) - 0 = v_j$, or $P_i(s_i) = 1 - v_j$. Hence, $P_i(s_i) = 1 - v_j$ irrespective of whether goods are substitutes or complements. The equilibrium response of s_j, however, depends on Δ.

Let $\Delta \geq 0$. If s_i proposes in the first negotiation so that $P_i = 1 - v_j$, then $P_j(s_j|\rho_1 = s_i) = v_j$ by (A-2). If, on the other hand, the buyer proposes in the first negotiation so that $P_i = 0$, then $P_j(s_j|\rho_1 = b) = 1 - v_i$. Similarly, for $\Delta < 0$, (A-2) reveals that $P_j(s_j|\rho_1 = s_i) = P_j(s_j|\rho_1 = b) = 1 - v_i$.

Given these equilibrium prices, the buyer purchases both goods with probability 1, giving rise to an efficient trade and yielding an expected payoff:

$$\pi^O(b) = \begin{cases} (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)v_i + \alpha_j(1-\alpha_i)v_j & for \quad \Delta \geq 0, \\ (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)v_i + \alpha_j(1-\alpha_i)v_j + \alpha_i\alpha_j(-\Delta) & for \quad \Delta < 0. \end{cases} \tag{A-3}$$

By re-labeling i and j, (A-3) implies that the buyer is indifferent to the negotiation sequence for all Δ. ■

Proof of Lemma 3. Let negotiations be public, and $E = H_i$, i.e., $e_i = o$ and $e_j = x$. Note that public negotiations with $\rho_1 = s_i$ (resp. $\rho_1 = s_j$) are strategically equivalent to the ones with $E = O$ (resp. $E = X$). Thus, equilibrium outcomes are as characterized in Lemmas 2 and 1, respectively. This means equilibrium trade with $E = H_i$ is efficient. Next, using (A-1) and (A-3), we have $\pi^X(b) - \pi^O(b) = 0$ for $\Delta \geq 0$, and $\pi^X(b) - \pi^O(b) = \alpha_i\alpha_j\Delta < 0$ for $\Delta < 0$. In words, the buyer is indifferent to the order of negotiations for complements, whereas, for substitutes, she strictly prefers to negotiate first with the supplier whose offer is open-ended. ■

Proof of Proposition 1. Parts (a) and (b) are immediate consequences of Lemmas 1-3. To prove part (c), suppose the buyer negotiates with s_1 first whenever she is indifferent. Note that the expected payoff for s_1 is $\pi^E(s_1) = \alpha_1 P_1^E(s_1)$, which using (1) and (2) implies $\pi^X(s_1) - \pi^O(s_1) = \alpha_1\alpha_2(-\Delta) =^{sign} -\Delta$. This means that s_1 weakly prefers an open-ended (resp. exploding) offer with complements (resp. substitutes) irrespective of the rival's expiration policy. ■

Proof of Proposition 2. Let negotiations be private, and $\Delta < 0$. It is clear that $\overline{P}_i^E(b) = 0$ for all E. Moreover, if $E \neq X$, i.e., at least one offer is open-ended, the buyer could engender Bertrand competition with unique prices, $\overline{P}_i^E(s_i) = 1 - v_j$, by approaching first the supplier with an open-ended offer.

Now consider $E = X$. We first argue that in equilibrium, either $\overline{P}_i^X(s_i) = 1 - v_j$ for all i, j or $\overline{P}_i^X(s_i) = v_i$ for all i, j. Supressing X to save on notation, note that in any equilibrium, $\overline{P}_i(s_i) \leq v_i$. We exhaust several regions.

If $\overline{P}_i(s_i) \leq 1 - v_j$ for both suppliers, then each sells with certainty, leaving $\overline{P}_i(s_i) = 1 - v_j$ the only candidate in this region. Next consider the region in which $\overline{P}_i(s_i) \leq 1 - v_j$ and $\overline{P}_j(s_j) > 1 - v_i$. Then, s_i sells with certainty, independent of the order. As for s_j, if he is approached second, his offer is rejected with certainty, since, upon purchasing good i, the buyer's marginal valuation for good j is $1-v_i$. If, on the other hand, j is approached first, his offer will be accepted only if $\max\{v_j - \overline{P}_j(s_j), 1 - \overline{P}_j(s_j) - \alpha_i \overline{P}_i(s_i)\} \geq v_i - \alpha_i \overline{P}_i(s_i)$. Since $\overline{P}_j(s_j) > 1 - v_i$ by hypothesis, this requires $v_j - \overline{P}_j(s_j) \geq v_i - \alpha_i \overline{P}_i(s_i)$, or equivalently $\overline{P}_j(s_j) \leq v_j - v_i + \alpha_i \overline{P}_i(s_i)$, which, by using $\overline{P}_i(s_i) \leq 1 - v_j$, implies $\overline{P}_j(s_j) \leq 1 - v_i$, contradicting $\overline{P}_j(s_j) > 1 - v_i$. This means that in this region, s_j's offer is rejected with certainty, independent of the order, giving him a strict incentive to lower his price. Hence, prices such that $\overline{P}_i(s_i) \leq 1 - v_j$ and $\overline{P}_j(s_j) > 1 - v_i$ cannot be sustained as an equilibrium.

Now, suppose $1 - v_j < \overline{P}_i(s_i) < v_i - v_j + \alpha_j \overline{P}_j(s_j)$ and $\overline{P}_j(s_j) > 1 - v_i$. We argue by the following two cases that no price pair in this region can be equilibrium candidates, because the buyer always rejects s_j's offer, giving j a strict incentive to lower his price.

- s_i *is approached first.* Then, the buyer would purchase good i if and only if $\max\{v_i - \overline{P}_i(s_i), 1 - \overline{P}_i(s_i) - \alpha_j \overline{P}_j(s_j)\} \geq v_j - \alpha_j \overline{P}_j(s_j)$, which, given the hypothesis, is equivalent to $v_i - \overline{P}_i(s_i) \geq v_j - \alpha_j \overline{P}_j(s_j)$ and thus satisfied. Since good i is purchased with certainty, the buyer's marginal valuation for good j is $1 - v_i$, which is strictly less than $\overline{P}_j(s_j)$ by hypothesis. Hence, s_j's offer will be rejected.

- s_i *is approached second.* In this case, s_j sells his good in the first negotiation if and only if $\max\{v_j - \overline{P}_j(s_j), 1 - \overline{P}_j(s_j) - \alpha_i \overline{P}_i(s_i)\} \geq v_i - \alpha_i \overline{P}_i(s_i)$. Since $\overline{P}_j(s_j) > 1 - v_i$, we have $1 - \overline{P}_j(s_j) - \alpha_i \overline{P}_i(s_i) < v_i - \alpha_i \overline{P}_i(s_i)$. Therefore, in order for good j to be purchased, it is necessary that $v_j - \overline{P}_j(s_j) \geq v_i - \alpha_i \overline{P}_i(s_i)$, or equivalently $\overline{P}_i(s_i) \geq \frac{v_i - v_j + \overline{P}_j(s_j)}{\alpha_i}$. Note, however, that because $v_i - v_j + \overline{P}_j(s_j) \geq 0$, we have $v_i - v_j + \alpha_j \overline{P}_j(s_j) < v_i - v_j + \overline{P}_j(s_j) \leq \frac{v_i - v_j + \overline{P}_j(s_j)}{\alpha_i}$, which, using the hypothesis, implies $\overline{P}_i(s_i) < \frac{v_i - v_j + \overline{P}_j(s_j)}{\alpha_i}$. Hence, s_j's offer will be rejected.

Finally, let prices be such that $\overline{P}_i(s_i) > \max\{1 - v_j, v_i - v_j + \alpha_j \overline{P}_j(s_j)\}$ for both suppliers. Consider s_i. Since $\overline{P}_i(s_i) > 1 - v_j$, an exact argument to that above reveals that if s_i is approached first, the buyer will reject his offer with certainty. If, on the other hand, he is approached second, he successfully sells only if j does not. (The latter happens whenever s_j is the first to negotiate and makes an offer.) This leads s_i to set the highest price, $\overline{P}_i(s_i) = v_i$.

Next, we find conditions under which the surviving price pairs $\{\overline{P}_i^X(s_i) = 1 - v_j\}$ and $\{\overline{P}_i^X(s_i) = v_i\}$ can each be sustained as an equilibrium. Since s_i who charges $\overline{P}_i^X(s_i) = 1 - v_j$ guarantees to sell his good independent of the order, the best s_j can do is to charge

$\overline{P}_j(s_j) = 1 - v_i$. Hence, the price pair, $\{\overline{P}_i^X(s_i) = 1 - v_j\}$ is always an equilibrium. As for the pair, $\{\overline{P}_i^X(s_i) = v_i\}$, note that given $\overline{P}_j^X(s_j) = v_j$, s_i's expected payoff from setting $\overline{P}_i^X(s_i) = v_i$ is $(1 - \sigma_i^X)\alpha_j v_i$, where σ_i^X is the probability that the buyer visits s_i first. But, for $\overline{P}_i^X(s_i) = v_i$ to be a best response for s_i, there are two possible deviations we have to discourage. The first is $\overline{P}_i^X(s_i) = 1 - v_j$, leading to a payoff of $1 - v_j$, and the second is $\overline{P}_i^X(s_i) = v_i - (1 - \alpha_j)v_j$, leading to a payoff of $[\sigma_i^X + (1 - \sigma_i^X)\alpha_j][v_i - (1 - \alpha_j)v_i]$. Hence, it must be that $(1 - \sigma_i^X)\alpha_j v_i \geq \max\{1 - v_j, [\sigma_i^X + (1 - \sigma_i^X)\alpha_j][v_i - (1 - \alpha_j)v_i]\}$. Letting $f^+ \equiv \max\{0, f\}$ and $f_{[k>0]} \equiv \begin{cases} f, \text{ if } k > 0 \\ 1, \text{ if } k \leq 0, \end{cases}$, this inequality is satisfied if and only if $\sigma_i^X \in (\underline{\sigma}_i^X, \overline{\sigma}_i^X)$, where $\underline{\sigma}_i^X \equiv 1 - \min\left\{ \left(\frac{\alpha_i(1-\alpha_i)v_i}{v_j-(1-\alpha_i)^2 v_i}\right)_{[v_j-(1-\alpha_i)v_i>0]}, \left(1 - \frac{1-v_i}{\alpha_i v_j}\right)^+ \right\}$ and $\overline{\sigma}_i^X \equiv \min\left\{ \left(\frac{\alpha_j(1-\alpha_j)v_i}{v_i-(1-\alpha_j)^2 v_j}\right)_{[v_i-(1-\alpha_j)v_j>0]}, \left(1 - \frac{1-v_j}{\alpha_j v_i}\right)^+ \right\}$. ■

Proof of Corollary 1. Let $v_1 = v_2 = v > \frac{1}{2}$. Then, using Proposition 2 and simplifying terms we obtain $\underline{\sigma}_1^X \equiv 1 - \min\left\{\frac{1-\alpha_1}{2-\alpha_1}, \left(1 - \frac{1-v}{\alpha_1 v}\right)^+\right\}$ and $\overline{\sigma}_1^X \equiv \min\left\{\frac{1-\alpha_2}{2-\alpha_2}, \left(1 - \frac{1-v}{\alpha_2 v}\right)^+\right\}$. Again, by Proposition 2, an inefficient equilibrium exists if and only if $\underline{\sigma}_1^X < \overline{\sigma}_1^X$. Simple algebra shows that this condition is never satisfied if $v_1 = v_2 = v > \frac{1}{2}$. ■

Proof of Lemma 4. Suppose negotiations are private, and $\Delta > 0$. Moreover, suppose $E = O$. This means the buyer makes her purchasing decisions after obtaining a private price quote from each supplier, inducing them to play a simultaneous pricing game. Hence, equilibrium supply prices lie at the intersection of suppliers' best response functions. Noting that for $i, j = 1, 2$ and $i \neq j$, $\overline{P}_i^O(b) = 0$ in any equilibrium and exhausting several cases, s_i's best response $\overline{P}_i^*(s_i|\overline{P}_j(s_j))$ to j's price offer $\overline{P}_j(s_j)$ is found to be:

$$\overline{P}_i^*(s_i|\overline{P}_j(s_j)) = \begin{cases} 1 - v_j & if \quad \overline{P}_j(s_j) \leq v_j \\ 1 - \overline{P}_j(s_j) & if \quad v_j < \overline{P}_j(s_j) \leq 1 - (1 - \alpha_j)(1 - v_j) \\ 1 - v_j & if \quad 1 - (1 - \alpha_j)(1 - v_j) \leq \overline{P}_j(s_j) \leq 1 - v_i \\ 1 - v_j & if \quad \overline{P}_j(s_j) > 1 - v_i \text{ and } \alpha_j(1 - v_j) \leq \Delta \\ v_i & if \quad \overline{P}_j(s_j) > 1 - v_i \text{ and } \alpha_j(1 - v_j) \geq \Delta. \end{cases} \tag{A-4}$$

Using (A-4), it is straightforward to verify that equilibrium supply prices, $\overline{P}_i^O(s_i)$ are exactly as described in text. ■

Lemma A1. *Suppose negotiations are private, $\Delta > 0$ and $E = O$. Moreover, suppose $\alpha_i(1 - v_i) \leq \Delta$ for all i and $\sum_i \alpha_i(1 - v_i) \geq \Delta$ so that, as identified in Lemma 4, efficient and inefficient equilibria co-exist. Then, the buyer is weakly better off in every efficient equilibrium.*

Proof of Lemma A1. Under the hypotheses of Lemma A1, Lemma 4 reveals that supply prices at an efficient equilibrium satisfy: $\overline{P}_i^O(s_i) + \overline{P}_j^O(s_j) = 1$ and $\overline{P}_j^O(s_j) \in [(1-\alpha_i)(1-v_i), 1-(1-\alpha_j)(1-v_i)]$. Together with $\alpha_i(1-v_i) \leq \Delta$ and $\alpha_j(1-v_j) \leq \Delta$ by hypothesis, it follows that $\overline{P}_i^O(s_i) \geq v_i$ and $\overline{P}_j^O(s_j) \geq v_j$. Next, note that the buyer's expected payoffs at efficient and inefficient equilibria are given, respectively, by: $\overline{\pi}^O(b|\text{eff.}) = (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)\overline{P}_j^O(s_j) + \alpha_j(1-\alpha_i)\overline{P}_i^O(s_i)$, and $\overline{\pi}^O(b|\text{ineff.}) = \overline{\pi}^O(b|\text{eff.})\big|_{\overline{P}_i^O(s_i)=v_i, \overline{P}_j^O(s_j)=v_j}$. Hence, $\overline{\pi}^O(b|\text{eff.}) \geq \overline{\pi}^O(b|\text{ineff.})$. ■

Proof of Lemma 5. Suppose negotiations are private, and $\Delta > 0$. Moreover, suppose $E = H_j$, i.e., $e_i = x$ and $e_j = o$. It is easily verified that for $i, j = 1, 2$ and $i \neq j$, $\overline{P}_i^{H_j}(b) = 0$ and $\overline{P}_i^{H_j}(s_i) \in [v_i, 1-v_j]$ in equilibrium.[26] Given this, we next exhaust several regions to search for equilibrium prices. To save on notation, we suppress H_j.

- $\overline{P}_i(s_i) + \overline{P}_j(s_j) < 1$: Such prices *cannot* be sustained in equilibrium because at least one supplier would have a strict incentive to deviate to a different price without affecting the probability of a sale.

- $\overline{P}_i(s_i) + \overline{P}_j(s_j) = 1$: These prices *can* be sustained as an equilibrium. First, note that under these prices, each supplier realizes a sale with certainty, and the buyer is indifferent to the order of negotiations. Second, if s_j is approached first and some supplier increases his price, then the probability of sale for j will decrease, since neither good is purchased if both suppliers end up making the offers. However, if s_i is approached first, he can increase his price to $\overline{P}_i(s_i) = 1 - \overline{P}_j(s_j) + (1-\alpha_j)[\overline{P}_j(s_j) - v_j]$ without affecting his probability of a sale. Hence, a necessary condition for $\overline{P}_i(s_i) + \overline{P}_j(s_j) = 1$ to be part of an equilibrium is: $[\sigma_i + (1-\sigma_i)(1-\alpha_j)](1 - \overline{P}_j(s_j) + (1-\alpha_j)(\overline{P}_j(s_j) - v_2) \leq 1 - \overline{P}_j(s_j)$. However, s_i could also deviate to $1 - v_j$, in which case he would sell only if j were negotiated first and buyer made an offer. To prevent this deviation, we also require: $1 - \overline{P}_j(s_j) \geq (1-\sigma_i)(1-\alpha_j)(1-v_j)$. s_j, on the other hand, could only deviate to $\overline{P}_j(s_j) = 1 - v_i$. To curb this incentive, it must be true that $[\sigma_i + (1-\sigma_i)(1-\alpha_i)](1-v_i) \leq \overline{P}_j(s_j)$. Combining and simplifying these three conditions, a pair of prices that satisfy $\overline{P}_i(s_i) + \overline{P}_j(s_j) = 1$ if and only if $\alpha_i(1-v_i) + \alpha_j(1-v_j) \geq \Delta - \frac{\theta}{1-\theta}v_i$, where $\theta = \frac{\sqrt{1+4\alpha_j^2}-1}{\sqrt{1+4\alpha_j^2}+1}$.

[26] The latter follows because s_i guarantees a sale for any price less than or equal to v_i, while his probability of selling for any price strictly above $1 - v_j$ is 0.

- $1 < \overline{P}_i(s_i) + \overline{P}_j(s_j) \leq 1 + (1-\alpha_j)[\overline{P}_j(s_j) - v_j]$: These prices *cannot* be part of an equilibrium: if s_j is approached first, then i sells only if the buyer proposes against j. If, on the other hand, s_i is approached first, he sells with certainty, and so does j. Since the probability of a sale does not change up to the upper bound of $\overline{P}_i(s_i)$, it must be that in equilibrium, $\overline{P}_i(s_i) = 1 - \overline{P}_j(s_j) + (1-\alpha_j)[\overline{P}_j(s_j) - v_j]$. Together with the fact that $\overline{P}_i(s_i) + \overline{P}_j(s_j) > 1$, it follows that the buyer would strictly prefer to first negotiate with s_j whose offer is open-ended. But then, s_i is strictly better off by increasing his price to $\overline{P}_i(s_i) = 1 - v_j$, yielding a contradiction.

- $\overline{P}_i(s_i) + \overline{P}_j(s_j) > 1 + (1-\alpha_j)[\overline{P}_j(s_j) - v_j]$: These prices *can* be part of an equilibrium. Under this price structure, if s_j is approached first, each supplier sells only if the buyer proposes against the rival. If s_i is approached first, then i's offer is always rejected whereas j's offer is accepted only if the buyer makes an offer in the previous negotiation. Since the probability of a sale does not change as prices increase in this region, the only equilibrium candidates are $\overline{P}_i(s_i) = 1 - v_j$ and $\overline{P}_j(s_j) = 1 - v_i$. Note that under these prices the buyer is indifferent to the order of negotiation. A possible deviation for s_j is to reduce his price to v_j, in which case he sells with certainty. Such deviation is unprofitable if and only if $v_j \leq (1-\alpha_i)(1-v_i)$, or equivalently $\alpha_i(1-v_i) \leq \Delta$. s_i, however, has two possible deviations: (1) Reducing his price to $\overline{P}_i(s_i) = (1-\alpha_j)(1-v_j) + \alpha_j v_i$, in which case he sells if approached first *or* if approached second and the buyer proposes in the previous negotiation. (2) Charging v_i, which guarantees a sale. To prevent these deviations for i, we thus require: $[\sigma_i + (1-\sigma_i)(1-\alpha_j)]((1-\alpha_j)(1-v_j) + \alpha_j v_i) \leq (1-\sigma_i)(1-\alpha_j)(1-v_j)$ *and* $v_i \leq (1-\sigma_i)(1-\alpha_j)(1-v_j)$. Combining these two conditions, we conclude that $\overline{P}_i(s_i) = 1 - v_j$ and $\overline{P}_j(s_j) = 1 - v_i$ constitute an equilibrium if and only if $\alpha_i(1-v_i) \leq \Delta$ for all i. In this equilibrium, $\sigma_i^{H_j} \in [0, \overline{\sigma}_i^{H_j}]$, where $\overline{\sigma}_i^{H_j} = \min\{\frac{\alpha_j(1-\alpha_j)}{1-v_j-\alpha_j^2\Delta}\Delta, \frac{\Delta - \alpha_j(1-v_j)}{(1-\alpha_j)(1-v_j)}\}$.
■

Proof of Lemma 6. Suppose negotiations are private, and $\Delta > 0$. Moreover, suppose $E = X$. Analogous to Lemma 5, it is clear that $\overline{P}_i^X(b) = 0$ and $\overline{P}_i^X(s_i) \in [v_i, 1-v_j]$ in equilibrium. Suppressing X, note that when approached first, s_i sells his good with certainty if and only if $1 - \overline{P}_i(s_i) - \alpha_j \overline{P}_j(s_j) \geq (1-\alpha_j)v_j$, or equivalently $\overline{P}_i(s_i) \leq 1 - v_j - \alpha_j[\overline{P}_j(s_j) - v_j]$. Thus, for both suppliers' offers to be accepted irrespective of the order we must have $\overline{P}_i(s_i) \leq 1 - v_j - \alpha_j[\overline{P}_j(s_j) - v_j]$ for both suppliers. Since both suppliers are guaranteed a sale in this region, they would maximize their payoff by making the inequality binding. It follows that the only equilibrium price candidates in this region are $\overline{P}_i(s_i) = \frac{1}{1-\alpha_i\alpha_j}[(1-\alpha_j)(1-v_j)+\alpha_j(1-\alpha_i)v_i]$. It is straightforward to show that these prices are part of an equilibrium if and only if the buyer visits s_i with probability $\sigma_i \in [1-\frac{\overline{P}_i(s_i)}{1-v_j}, \frac{\overline{P}_j(s_j)}{1-v_i}]$. Since $1-\frac{\overline{P}_i(s_i)}{1-v_j} \leq \frac{\overline{P}_j(s_j)}{1-v_i}$ holds for all parameter values, the equilibrium always exists.

Exhausting other regions where $\overline{P}_i(s_i) > 1-v_j-\alpha_j[\overline{P}_j(s_j)-v_j]$ for some s_i, we observe that the only other equilibrium candidate lies in the region where $\overline{P}_i(s_i) > 1-v_j-\alpha_j[\overline{P}_j(s_j)-v_j]$ for both suppliers. For such prices, each supplier sells only if he is approached second *and* the buyer proposes in the previous negotiation, leading each supplier to set $\overline{P}_i(s_i) = 1-v_j$. For these prices to be an equilibrium, we need to prevent two possible deviations for s_i. The first one is to set $\overline{P}_i(s_i) = v_i$, in which case i's offer would be accepted with probability 1. Such a deviation would not be profitable to s_i if and only if the buyer mixes over the order with probability σ_i such that $\sigma_i \leq 1-\frac{v_i}{(1-\alpha_j)(1-v_j)}$. The second deviation is to a price $\overline{P}_i(s_i) = (1-\alpha_j)(1-v_j)+\alpha_j v_i$, which is accepted with probability $\sigma_i+(1-\sigma_i)(1-\alpha_j)$. This deviation is unprofitable to s_i if and only if $\sigma_i \leq \frac{\alpha_j(1-\alpha_j)\Delta}{1-v_j-\alpha_j^2\Delta}$. Combining the two requirements, we see that $\overline{P}_i(s_i) = 1-v_j$ is a best response for s_i to $\overline{P}_j(s_j) = 1-v_i$ if and only if $\sigma_i \leq \min\left\{\frac{\alpha_j(1-\alpha_j)\Delta}{1-v_j-\alpha_j^2\Delta}, \frac{\Delta-\alpha_j(1-v_j)}{(1-\alpha_j)(1-v_j)}\right\} \equiv \overline{\sigma}_i^X$. Switching labels and recalling that $\sigma_i+\sigma_j = 1$, we conclude that the same is also true for s_j if and only if $\sigma_i \geq 1-\min\{\frac{\alpha_i(1-\alpha_i)}{1-v_i-\alpha_i^2\Delta}\Delta, \frac{\Delta-\alpha_i(1-v_i)}{(1-\alpha_i)(1-v_i)}\} \equiv \underline{\sigma}_i^X$. Note that if $\alpha_i(1-v_i) > \Delta$ for some i, then $[\underline{\sigma}_i^X, \overline{\sigma}_i^X] = \{\}$. Hence, the price pair, $\overline{P}_i(s_i) = 1-v_j$ is supported as an equilibrium if and only if $\alpha_i(1-v_i) \leq \Delta$ for all i *and* $\underline{\sigma}_i^X \leq \overline{\sigma}_i^X$. ■

Proof of Proposition 3. Suppose negotiations are private, and $\Delta > 0$. To prove part (a), fix some parameter values, α_i, α_j, v_i, and v_j. Note that if there exists an efficient equilibrium under $E = O$, then by Lemma 4 it must be true that $\alpha_i(1-v_i)+\alpha_j(1-v_j) \geq \Delta$. Using Lemma 5, this condition implies that for the same parameter values, there also exists an efficient equilibrium under $E = H_j$. Since by Lemma 6 an efficient equilibrium always exists under $E = X$, we say that an efficient equilibrium is easier to support as more offers become exploding. A similar argument reveals that an inefficient equilibrium, however, is harder to obtain as more offers become exploding. Together these two observations prove part (a): all else equal, equilibrium trade is more likely to be efficient as more offers become exploding.

Part (b) easily follows from Lemmas 4-6. To show part (c), we consider two cases.

- $\alpha_i(1-v_i)+\alpha_j(1-v_j)<\Delta$: by Lemma 4 there is a unique inefficient equilibrium under $E=O$, yielding the following payoff to the buyer: $\overline{\pi}^O(b)=(1-\alpha_i)(1-\alpha_j)+\alpha_i(1-\alpha_j)v_j+\alpha_j(1-\alpha_i)v_i$. Under $E=X$, both efficient and inefficient equilibria are possible. It is easy to verify that the buyer's payoff at the inefficient equilibrium is equal to $\overline{\pi}^O(b)$. Using the prices from part (a) of Lemma 6 and simplifying terms, we have $\overline{\pi}^X(b)=1-\alpha_i\overline{P}_i^X(s_i)-\alpha_j\overline{P}_j^X(s_j)$, implying that $\overline{\pi}^X(b)-\overline{\pi}^O(b)=\alpha_i\alpha_j(1-\alpha_i)(1-\alpha_j)\Delta>0$. Hence, the buyer is better off under $E=X$ than under $E=O$.

- $\alpha_i(1-v_i)+\alpha_j(1-v_j)\geq\Delta$: under $E=O$ there are efficient equilibria such that $\overline{P}_i^O(s_i)+\overline{P}_j^O(s_j)=1$, at which the buyer's payoff is: $\overline{\pi}^O(b)=1-\alpha_i\overline{P}_i^O(s_i)-\alpha_j\overline{P}_j^O(s_j)$. Comparing $\overline{\pi}^O(b)$ with $\overline{\pi}^X(b)=1-\alpha_i\overline{P}_i^X(s_i)-\alpha_j\overline{P}_j^X(s_j)$, the buyer's payoff at an efficient equilibrium under $E=X$, it follows that $\overline{\pi}^O(b)\geq\overline{\pi}^X(b)$. Although inefficient equilibria are still possible under both $E=O$ and $E=X$, the buyer's payoffs are equal at those. Therefore, the buyer is better off under $E=O$ than under $E=X$.

Combining the two cases, we conclude that the buyer is better off when $E=X$ than when $E=O$ if and only if $\sum_i\alpha_i(1-v_i)<\Delta$.

To complete the proof of part (c), suppose $v_i=v_j=0$ and $\alpha_i+\alpha_j<1$. By Lemma 4, there is only inefficient equilibrium under $E=O$, and s_i's payoff is $\pi^O(s_i)=\alpha_i(1-\alpha_j)$. Under $E=X$, on the contrary, equilibrium is always efficient by Lemma 6, and s_i's payoff is $\pi^X(s_i)=\alpha_i\frac{(1-\alpha_j)}{1-\alpha_i\alpha_j}$. Comparing the two payoffs, it is clear that $\pi^X(s_i)>\pi^O(s_i)$. ■

Proof of Proposition 4. Let $\Delta<0$. If $E\neq X$, we know from Propositions 1 and 2 that the buyer can always secure the Bertrand prices under both public and private negotiations by visiting first the supplier whose offer is open-ended, making her indifferent between the two types of negotiations. Now, suppose $E=X$. Under public negotiations, the buyer's payoff, $\pi^X(b)$ is given in (A-3). Under private negotiations, the buyer's payoff at an efficient equilibrium is equal to the one under public negotiations with open-ended offers, i.e., $\overline{\pi}^X(b)=\pi^O(b)$; because both generate Bertrand prices in equilibrium. Since $\pi^O(b)>\pi^X(b)$ for $\Delta<0$, we have $\overline{\pi}^X(b)>\pi^X(b)$. If the equilibrium under private negotiations is inefficient, then it is easily verified that $\overline{\pi}^X(b)=\pi^X(b)$. Hence, when $E=X$, the buyer is weakly better off under private negotiations. ■

Proof of Corollary 2. This directly follows from Corollary 1 and the proof of Proposition 4. ■

Proof of Proposition 5. Let $\Delta>0$. For $\sum_i(1-\alpha_i)v_i<\Delta$, only the inefficient equilibrium is possible under private negotiations, and Propositions 1 and 3 reveal that $\pi^O(b) = \overline{\pi}^O(b|\text{ineff})$. For $\sum_i (1-\alpha_i)v_i \geq \Delta$, both efficient and inefficient equilibria can arise under private negotiations, and by Lemma A1, we have $\overline{\pi}^O(b|\text{eff}) \geq \overline{\pi}^O(b|\text{ineff}) = \pi^O(b)$, proving part (a). Together with Proposition 1, parts (b) and (c) follow from Lemmas 5 and 6, respectively. ■

Appendix B

This appendix provides the proofs of the formal results in Section 6.

Proof of Lemma 7. Suppose negotiations are partially private, and $E = X$.

- *The upstream price offer, P_i is public.*

Let $\phi_1(P_i)$ be the equilibrium probability that the buyer purchases good i. Clearly, $\widehat{P}_i(b) = \widehat{P}_j(b) = 0$ in any equilibrium. Suppose $\Delta < 0$. Setting $\widehat{P}_j(s_j) \in (1 - v_i, v_j)$ would not change the probability of a trade in the downstream, and hence $\widehat{P}_j(s_j) = 1 - v_i$ or v_j in equilibrium. As with private negotiations, the Bertrand prices, $\widehat{P}_i(s_i) = 1 - v_j$ and $\widehat{P}_j(s_j) = 1 - v_i$, form an equilibrium, because it is a dominant strategy for the buyer to purchase each good at this price irrespective of the other's. Next, suppose that $\widehat{P}_j(s_j) = v_j$ is part of an equilibrium. Then, we must have $\phi_1(P_i) \in (0, 1)$ whenever $P_i \neq 0$, which requires the buyer to be indifferent in the upstream purchase:

$$\alpha_j[v_i - \widehat{P}_i] + (1 - \alpha_j)[1 - \widehat{P}_i] = (1 - \alpha_j)v_j,$$

implying $\widehat{P}_i(s_i) = 1 - v_j - \alpha_j \triangle$. To show that the price pair $\widehat{P}_i(s_i) = 1 - v_j - \alpha_j \triangle$ and $\widehat{P}_j(s_j) = v_j$ cannot be part of an equilibrium, we consider two possible best replies for j whenever he observes a deviation to $P_i(s_i) < 1 - v_j - \alpha_j \triangle$ by s_i. The first is $\widehat{P}_j(s_j) = 1 - v_i$, in which case the buyer would expect to purchase from j with certainty and thus reject i's offer, i.e., $\phi_1(P_i(s_i)) = 0$, contradicting $\phi_1(P_i) \neq 0$. The second is $\widehat{P}_j(s_j) = v_j$, in which case s_i would have a strict incentive to charge $P_i(s_i) = 1 - v_j - \alpha_j \triangle - \varepsilon$ in order to guarantee a sale, contradicting $\phi_1(P_i) \neq 1$. Hence, $\widehat{P}_j(s_j) = v_j$ cannot be part of an equilibrium.

Next, suppose $\Delta \geq 0$. Similar to $\Delta < 0$, there are only two possible prices for s_j in equilibrium: $\widehat{P}_j(s_j) = 1 - v_i$, which is accepted only if good i is purchased, or $\widehat{P}_j(s_j) = v_j$, which is accepted regardless. If $\widehat{P}_j(s_j) = 1 - v_i$, then good i is purchased whenever $1 - P_i(s_i) - \alpha_j(1 - v_i) \leq (1 - \alpha_j)v_j$ for the buyer, or equivalently $P_i(s_i) \leq 1 - v_j - \alpha_j \triangle$.

This means that the price pair $\widehat{P}_i(s_i) = 1 - v_j - \alpha_j\triangle$ and $\widehat{P}_j(s_j) = 1 - v_i$ is part of an equilibrium, and trade is efficient in this equilibrium. $\widehat{P}_j(s_j) = v_j$, on the other hand, cannot be part of an equilibrium: expecting good j to be purchased with certainty, s_i would then set $\widehat{P}_i(s_i) = 1 - v_j$. If s_i deviated to a lower price, then s_j would either respond by $P_j(s_j) = 1 - v_i$, in which case $\phi_1(P_i) = 0$ giving s_j a strict incentive to lower his price to v_j, or by $P_j(s_j) = v_j$, in which case s_i would have a strict incentive to reduce his price to $1 - v_j - \varepsilon$.

- *The upstream price offer, P_i is confidential.*

In this case, the downstream supplier, s_j can only conjecture ϕ_1. First, let $\Delta < 0$. Given equilibrium ϕ_1, s_j can either charge $1 - v_i$ and sell his good with probability 1 or charge v_j and sell it with probability $1 - \phi_1$. Thus, the best response of s_j is

$$P_j(s_j|\phi_1) = \begin{cases} 1 - v_i & if \quad \phi_1 > -\frac{\Delta}{v_j}, \\ v_j & if \quad \phi_1 \leq -\frac{\Delta}{v_j}. \end{cases} \tag{B-1}$$

If $P_j(s|\phi_1) = v_j$ in equilibrium, then

$$\phi_1|(P_j = v_j) = \begin{cases} 1 & if \quad P_i < 1 - v_j \\ [0,1] & if \quad P_i = 1 - v_j \\ 0 & if \quad P_i > 1 - v_j \end{cases}$$

Knowing buyer's purchasing strategy, s_i would propose a price of $1 - v_j - \varepsilon$, inducing $\phi_1 = 1$. However, s_j would then deviate to $1 - v_i \neq v_j$, yielding a contradiction. Hence, we must have $P_j(s_j|\phi_1) = 1 - v_i$ in equilibrium, which, in turn, implies $\phi_1 > -\frac{\Delta}{v_j}$ by (B-1). Note that if the buyer proposes in the upstream, then $P_i(b) = 0$; and good i is acquired with certainty. If, on the other hand, s_i proposes, then the buyer accepts if and only if $1 - P_i(s_i) - \alpha_j P_j(s_j|\phi_1) \geq v_j - \alpha_j P_j(s_j|\phi_1)$, which implies $P_i(s_i) = 1 - v_j$, and hence $\phi_1 = 1$. Anticipating this, $P_j(s_j|\phi_1 = 1) = 1 - v_i$. As a result, the same equilibrium with Bertrand prices is uniquely obtained for $\Delta < 0$ irrespective of the upstream price being public or confidential. And we know from Proposition 2, trade is efficient in this case and the buyer is indifferent to the order.

Next, let $\Delta \geq 0$. We first show that the price pair $\widehat{P}_i(s_i) = 1 - v_j - \alpha_j\triangle$ and $\widehat{P}_j(s_j) = 1 - v_i$ is part of an equilibrium. Similar to (B-1), s_j's best reply is

$$P_j(s_j|\phi_1) = \begin{cases} 1 - v_i & if \quad \phi_1 > 1 - \frac{\Delta}{1-v_i}, \\ v_j & if \quad \phi_1 \leq 1 - \frac{\Delta}{1-v_i}. \end{cases}$$

Note that if the buyer makes the offer over good i, she purchases it with certainty. Depending on α_1, there are two parametric cases to consider:

- $\alpha_i < \frac{\Delta}{1-v_i}$: Then, since $\phi_1 \geq 1-\alpha_i$, we have $P_j(s_j|\phi_1) = 1-v_i$, independent of $P_i(s_i)$. Thus, s_i sells his item if and only if $1-P_i(s_i)-\alpha_j(1-v_i) \geq (1-\alpha_j)v_j$, which implies $P_i(s_i) = 1-v_j-\alpha_j\Delta$. Given $P_i(s_i)$, $P_j(s_j|\phi_1)$, and $P_i(b) = 0$, it follows that $\phi_1 = \phi_2 = 1$.

- $\alpha_i \geq \frac{\Delta}{1-v_i}$: In this case, given $P_i(s_i) = 1-v_j-\alpha_j\Delta$, we have $\phi_1 = 1$, which implies $P_j(s_j|\phi_1) = 1-v_i$.

Next, we show that this equilibrium is unique. Clearly, by the above argument, this is so for $\alpha_i < \frac{\Delta}{1-v_i}$. For $\alpha_i \geq \frac{\Delta}{1-v_i}$, suppose $\phi_1 \leq 1-\frac{\Delta}{1-v_i}$ in equilibrium. If $\phi_1 = 0$, then $P_j(s_j|\phi_1) = v_j$. Then, s_i would offer a price such that $1-P_i(s_i)-\alpha_j v_j > (1-\alpha_j)v_j$, or equivalently $P_i(s_i) < 1-v_j$ and induce the purchase of good i, contradicting $\phi_1 = 0$. If, on the other hand, $0 < \phi_1 \leq 1-\frac{\Delta}{1-v_i}$, then, the buyer must be indifferent between accepting and rejecting seller's offer in the first negotiation. Such indifference, however, can occur only if $P_i(s_i) = 1-v_j-\alpha_j\Delta$. Below this price, good i is acquired with certainty, giving a strict incentive to s_i to lower his price by ε. Hence, $0 < \phi_1 \leq 1-\frac{\Delta}{1-v_i}$ cannot be part of an equilibrium. As a result, $\phi_1 > 1-\frac{\Delta}{1-v_i}$ generating the same equilibrium prices as described above. Moreover, it follows that $\phi_1 = \phi_2 = 1$ in equilibrium, generating an efficient trade.

Finally, given the efficiency of trade and prices in Lemma 7, the buyer's expected payoffs are exactly the same as $\pi^O(b)$ and $\pi^X(b)$ for substitutes and complements, respectively, implying that the buyer is indifferent to the order in each case.$\blacksquare$

Proof of Proposition 6. With substitutes, the result follows from Proposition 4 and Lemma 7; and with complements it follows from Proposition 5 and Lemma 7. $\blacksquare$

Proof of Proposition 7. Let $D_i^j(\widehat{d}_i)$ denote s_i's belief of the buyer's disclosure to s_j given that he is disclosed, $\widehat{d}_i$. In what follows, suppose both suppliers hold "passive" beliefs: $D_i^j(\widehat{d}_i) = d_j^*$ for all $\widehat{d}_i$ and $i \neq j$. Note that with substitutes, $d_i^* = \{s\}$ for all i is sustained in equilibrium because, by Lemma 7, the buyer uniquely obtains Bertrand prices under partially private negotiations, leaving her no strict incentives to deviate. To show $d_i^* = \{\}$ for all i is part of an equilibrium with complements, consider first the case with $E = X$. Clearly, the buyer has no incentive to deviate $\widehat{d}_2 = \{\phi_1\}$ in the downstream, because then she would lose all the surplus. If the buyer expected an efficient equilibrium, then she would not deviate to $\widehat{d}_2 = \{s\}$ either; because, knowing the order, the downstream supplier would strictly increase his price. If, instead, the buyer expected an inefficient equilibrium, from the proof of lemma 6 we know that only the downstream seller would realize a sale, and thus would not change his price. As for the upstream negotiation, under the efficient pricing equilibrium, a deviation to $\widehat{d}_1 = \{s\}$ would not result in any price change, because s_i would then set $P_i(s_i)$ that satisfies $1 - P_i(s_i) - \alpha_j \overline{P}_j^X(s_i) = (1-\alpha_j)v_j$, yielding exactly the same price as the efficient equilibrium in Lemma 6. Under an inefficient equilibrium, a deviation to $\widehat{d}_1 = \{s\}$ would result in $P_i(s_i) = 1 - v_j - \alpha_j \triangle$, under which the buyer would simply be indifferent between accepting and rejecting the offer. Next, consider the case with $E = O$. Note that a deviation to $\widehat{d}_2 = \{P_1\}$ would not make the buyer better off because then the downstream supplier would extract the remaining surplus. In addition, under $E = O$, since the order is inconsequential, a deviation to $\widehat{d}_2 = \{s\}$ is not profitable in the downstream either, and by the same token, neither is a deviation to $\widehat{d}_1 = \{s\}$ in the upstream. Similar arguments for $E = X$ and $E = O$ also apply to the case with H_j. Thus, $d_i^* = \{\}$ for all i, i.e., private negotiations, is part of an equilibrium with complements, completing the proof of part (a).

To prove part (b), consider equilibrium disclosure $d_1^* = \{s\}$ and $d_2^* = I_2$, corresponding to public negotiations. Suppose $E = X$. Since $D_i^j(\widehat{d}_i) = d_j^*$, deviating to $\widehat{d}_2 = \{s\}$ would not change equilibrium prices because, knowing the sequence, the downstream supplier would conjecture $\phi_1 = 1$. Next, define $\widehat{\sigma}_i(\widehat{d}_i)$ to be s_i's belief about being in the upstream when he is disclosed $\widehat{d}_i$; in particular $\widehat{\sigma}_i(\{\}) = 1$ if $\Delta < 0$ and $\widehat{\sigma}_i(\{\}) = 0$ if $\Delta \geq 0$. Since, by Lemmas 1 and 2, $P_i(s_i|\widehat{d}_i = \{\}) = \begin{cases} 1 - v_j - \alpha_j \triangle & if \quad \triangle < 0 \\ 1 - v_j & if \quad \triangle \geq 0 \end{cases}$, the buyer has no incentive to deviate to $\widehat{d}_i = \{\}$. Now, suppose $E = O$. If $\triangle < 0$, then a deviation $\widehat{d}_i \neq d_i^*$ is not profitable to the buyer because $D_i^j(\widehat{d}_i) = d_j^*$ implies that s_i anticipates a sale only if $P_i(s_i) \leq 1 - v_j$. For $\triangle \geq 0$, $\widehat{d}_i = \{\}$ is analogous to $E = X$: if $\widehat{\sigma}_i(\{\}) = 1$, then $P_i(s_i|\widehat{d}_i = \{\}) = 1 - v_j$, leaving no incentive to the buyer to deviate. A deviation to $\widehat{d}_2 = \{s\}$ would also prove unprofitable to the buyer as long as s_i conjectures that the upstream offer was made by the buyer. More formally, define $\widehat{\rho}_i(\widehat{d}_i)$ to be s_i's belief about the identity of the proposer in negotiation j. Under $\widehat{\rho}_i(\widehat{d}_2 = \{s\}) = b$, the buyer would not deviate to $\widehat{d}_2 = \{s\}$ because $P_i(s_i|\widehat{d}_2 = \{s\}) = 1 - v_j$. In a similar fashion, we can find beliefs that support $d_1^* = \{s\}$ and $d_2^* = I_2$ in equilibrium under $E = H_j$. ■

Appendix C

In this appendix, we briefly explore the possibility of lotteries and re-examine the buyer's preference between public, private and partially private negotiations. Suppose, whenever it is his turn, s_i makes an offer, $(q_i, P_i(s_i))$, where $P_i(s_i)$ is the lottery price and q_i is the probability that the good is delivered to the buyer. That is, if the buyer accepts s_i's offer, she pays $P_i(s_i)$ but obtains the good with probability q_i. To save space, we only consider the cases with $E = X$. It is easy to verify that it is optimal for suppliers not to use lotteries in equilibrium under public and private negotiations; hence equilibrium prices and payoffs remain unchanged in these cases. Thus, the following sequence of results (except for Proposition C1) provide an equilibrium characterization for partially private negotiations where the upstream offer is publicly observed, but the buyer's acceptance decision is private.

Lemma C1. *Suppose goods are complements. Then, the buyer is indifferent to the order and prices are given by $\widehat{P}_i(b) = 0$ for all i and*

$$\left\{\begin{array}{|c|c|c|}\hline \begin{array}{c}(\widehat{q}_i, \widehat{P}_i(s_i)) = (1, 1 - v_j - \alpha_j \Delta) \\ \widehat{P}_j(s_j|\widehat{q}_i) = 1 - v_i\end{array} & \textit{if} & \alpha_j \leq \frac{1-v_j}{1-v_i} \\ \hline \begin{array}{c}(\widehat{q}_i, \widehat{P}_i(s_i)) = (\frac{v_j}{1-v_i}, \frac{v_j(1-v_j)}{1-v_i}) \\ \widehat{P}_j(s_j|\widehat{q}_i) = v_j\end{array} & \textit{if} & \alpha_j > \frac{1-v_j}{1-v_i}. \\ \hline \end{array}\right. \tag{C-1}$$

Proof. Suppose the buyer visits first s_i. If $q_i \leq \frac{v_j}{1-v_i}$, then s_j sets $P_j(s_j) = v_j$, in which case s_i chooses the highest price that satisfies $P_i(s_i) \leq q_i(1 - v_j)$. Then, the optimal offer is $\widehat{q}_i = \frac{v_j}{1-v_i}$ and $\widehat{P}_i(s_i) = \frac{v_j(1-v_j)}{1-v_i}$. If $q_i > \frac{v_j}{1-v_i}$ instead, then $P_j(s_j) = 1 - v_i$, in which case s_i's offer must satisfy $P_i(s_i) \leq q_i(1 - v_j - \alpha_j \triangle)$, resulting in the optimal offer: $\widehat{q}_i = 1$ and $\widehat{P}_i(s_i) = 1 - v_j - \alpha_j \triangle$. Comparing s_i's profits (or simply his prices) across, the result in (C-1) follows. Using (C-1), it is easy to show that the buyer's expected payoff is $\widehat{\pi}(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + \alpha_j(1 - \alpha_i)v_i$, regardless of who is visited first, proving her indifference in the order. ■

Lemma C2. S*uppose goods are substitutes. Then, $\widehat{P}_i(b) = 0$ for all i, and*

- *if $\alpha_i > \frac{(1-v_i)(1-v_j)}{\Delta^2}$ for all i, then the buyer is indifferent to the order; and $q_i = -\frac{\Delta}{v_j}$, $\widehat{P}_i(s_i) = -\frac{\Delta}{v_j}(1 - v_j - \alpha_j \Delta)$ and $\widehat{P}_j(s_j|q_i) = v_j$;*

- *if $\alpha_i \leq \frac{(1-v_i)(1-v_j)}{\Delta^2}$ for some i, then the buyer negotiates first with s_i whenever $\alpha_i \leq \alpha_j$, in which case $q_i = 1$, $\widehat{P}_i(s_i) = 1 - v_j$, and $\widehat{P}_j(s_j|q_i) = 1 - v_i$.*

PROOF. Suppose the buyer visits first s_i. If $q_i \leq -\frac{\triangle}{v_j}$, then $P_j(s_j) = v_j$, in which case s_i's optimal offer is $\widehat{q}_i = -\frac{\triangle}{v_j}$ and $\widehat{P}_i(s_i) = -\frac{\Delta}{v_j}(1 - v_j - \alpha_j \Delta)$.

If $q_i > -\frac{\triangle}{v_j}$, then $P_j(s_j) = 1 - v_i$, in which case $\widehat{q}_i = 1$ and $\widehat{P}_i(s_i) = 1 - v_j$. Comparing his prices, note that s_i prefers $\widehat{q}_i = -\frac{\triangle}{v_j}$ to $\widehat{q}_i = 1$ if $\alpha_i \geq \frac{(1-v_i)(1-v_j)}{\triangle^2}$. In this case, $\widehat{\pi}(b) = (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)v_j + \alpha_j(1-\alpha_i)v_i$, which is strictly lower than the payoff the buyer obtains when both suppliers charge their Bertrand prices, i.e., $\widehat{P}_i(s_i) = 1 - v_j$. Thus, if $\alpha_i \leq \frac{(1-v_i)(1-v_j)}{\triangle^2}$ for some i, then it is optimal for the buyer to negotiate first the supplier whose bargaining power is smaller so that she can engender Bertrand prices for both. ■

LEMMA C3. *With complements, the buyer obtains the same expected payoff with or without lotteries. With substitutes, the buyer is strictly worse off with lotteries than without them if* $\alpha_i > \frac{(1-v_i)(1-v_j)}{\Delta^2}$ *for all* i*; otherwise, she is indifferent between the two.*

PROOF. With complements, the result follows from Lemmas 7 and C1. With substitutes, it follows from Lemmas 7 and C2. ■

PROPOSITION C1. *Suppose lotteries are feasible, and the upstream offer is publicly observed under partially private negotiations. Then, with complements, the buyer weakly prefers private negotiations as in the case without lotteries. With substitutes, if* $v_i = v$ *and* $\alpha_i > \frac{(1-v)^2}{(1-2v)^2}$ *for all* i*, then, unlike the case without lotteries, the buyer strictly prefers private negotiations.*

PROOF. With complements, the result follows from Lemma C3 and Proposition 6. With substitutes, suppose $v_i = v$ and $\alpha_i > \frac{(1-v)^2}{(1-2v)^2}$ for all i. Then, under private negotiations, Corollary 1 reveals that the unique equilibrium is efficient and $\overline{P}_i(s_i) = 1 - v$ for all i, and by Lemma C2, these prices are strictly more favorable for the buyer than the ones under partially private negotiations. ■

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