Technology Adoption and Fuzzy Patent Rights

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(Very preliminary. Please do not circulate.)

Abstract

This paper considers why a patent-holder would have low incentives to reduce the uncertainty of patent boundary. Clearer patent rights, i.e., when the patent office examination results are more informative about subsequent court decisions, will provide better guidance to technology-specific investment and encourage technology adoption. However, under mild conditions the patent-holder's post-adoption payoff is decreasing in clarity. The patent-holder, then, prefers to maintain "fuzzy" patent rights when she wants to monopolize the use of the technology, or when promoting technology adoption is not a strong concern for her. The latter happens when the patent-holder, as a pure licensor, has a low (*ex ante*) quality invention. The paper also shows that early licensing can eliminate the patent-holder's efforts to clarify patent boundary without jeopardizing technology adoption. In its absence, however, public decision-makers (the patent office or court) should assume the role to provide certainty of patent rights.

Keywords: Fuzzy Patents, Public Notice, Technology Adoption.

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1 Introduction

Uncertainty clouds the patent system (Lemley and Shapiro, 2005). When preparing for the patent application, an inventor cannot be sure whether she will be granted the patent protection.[1] After filing the application, a lengthy bargaining with the patent examiner renders uncertain both the time to complete a patent office decision and the allowed claims should a patent be issued (Gans *et al.*, 2007). And even after issuance, it is extremely difficult for anyone, including the patent-holder, to know for sure whether the patent is valid and what exactly is the technology territory it covers (Jaffe and Lerner, 2004; Meurer and Bessen, 2008).

This paper concerns uncertainty of the last sort, i.e., uncertainty surrounding patent scope or patent boundary. Because of the exclusive power attached to a valid and infringed patent, lingering uncertain patent boundary causes serious concerns. It may hinder technology progress, as downstream technology users and future inventors along the chain of cumulative innovation process may find it too difficult to survive in the "patent thicket" and prefer not to use new inventions without clearing the patent "mine field" (Shapiro, 2001; Jaffe and Lerner, 2004). Indeed, some scholars even suggest that under current situation patents do not deserve to be called intellectual *property rights* because they fail to provide the important "public notice" function that any meaningful property entitlement should deliver (Meurer and Bessen, 2008; Risch, 2007). That is, issued patents do not convey precise information about the technology boundary of patented inventions, and it is very difficult to evaluate whether a patent would be infringed by simply reading its claims, which are supposed to delineate the patent coverage on the technology space.

This uncertainty may come from several sources. Technological features may cause scientific knowledge in some fields (e.g. software) more difficult to be codified into written records than others (e.g. chemistry and pharmaceutical industry). Agency problem within the patent office may contaminate the "quality" of its decision, and so the informativeness of an issued patent. In this paper, we consider private incentives. We examine a patent applicant's willingness, or unwillingness, to help reduce the uncertainty associated with her future patent rights, and its impact on technology adoption by the other party. The patent applicant, either as the inventor of the technology or

[1] Ayres and Klemperer (1999) suggests using uncertainty as a policy instrument, i.e., granting the owner of a valid patent the exclusive power with probability smaller than one in order to reduce the monopoly loss associated with patent rights.

a party closely related to the inventor, should possess relevant knowledge about the patentability issues and technological features of the invention. Furthermore, in most jurisdictions patent examination has the *ex parte* feature, i.e., it is administered as a bargaining process between the inventor/patent applicant and patent examiner. For these reasons, the patent applicant is a natural candidate one would turn to when seeking private efforts to reduce uncertainty.

Although the probabilistic nature of patent rights has been introduced into the economic analysis in the past decade,[2] previous studies are mostly concerned with the infringement or validity probability at the litigation stage, where patent disputes are ultimately resolved, without addressing the patent examination stage. This perhaps reflects a by and large neglected role played by the patent office in the literature. In this paper, we join some recent efforts to fill this gap and include patent examination as a possible way to mitigate uncertainty.[3] To the best of our knowledge, the discussion about "fuzzy" patent rights and the lack of public notice function is mostly provided by law school professors. Their efforts, nevertheless, are more directed toward whether certain legal rules should be enacted or abolished to alleviate this problem.[4] In this paper, we take one step back and investigate why a patent applicant may lack incentives to help reduce the uncertainty associated with her patent rights. Understanding one important source of the problem, in our view, is a necessary step toward finding solutions.

In Section 2, we introduce a two-player model with an inventor/patent applicant/patent-holder and a developer/technology-adopter. The developer wishes to incorporate a new technology into his investment project. After adoption, he has to exert an non-observable, technology-specific investment effort (hold-up). Using the new technology, however, will expose the developer to the risk of infringing on the inventor's patent rights. The inventor's patent boundary is resolved in a two-stage process. The patent office first issues its opinion on whether the patent scope is broad enough to cover the new technology, and the court has the final say. The patent office decision serves as

[2]See, e.g., Lemley and Shapiro (2005) and papers cited there.

[3]Our analysis is complementary to recent works on the patent examination *per se* (Caillaud and Duchên, 2005; Langinier and Marcoul, 2003; Prady, 2008; Shuett, 2008). Chiou (2008) also consider a two-tier structure with both patent examination and litigation, but focuses on the problems of relying on post-grant patent challengers to improve patent quality.

[4]For instance, Meurer and Nard (2005) and Lichtman (2005) debate about the "doctrine of equivalents," on which the court relies to expand patent scope beyond what is specified in patent claims; and Risch (2007) attacks the "broadest reasonable constructing rule" used by patent examiners in interpreting patent claims.

a signal about the court judgement. The patent-holder, as the one of the players (if not the only one) most familiar with her invention, can exert some refinement effort to help the patent examiner make a decision more in line with the future court judgement should a patent dispute arise. One key assumption is that when deciding whether to use the new technology, the developer observes the refinement effort chosen by the inventor. Technology adoption therefore depends on the expectation of how informative the patent office decision will be concerning later infringement judgement of the court.

In Section 3 we show that the technology-specific effort, and thus hold-up concerns, will induce the developer's adoption payoff to be decreasing and *convex* in infringement probability, or patent power. More informative patent office decisions better guide the developer's investment decision in the presence of infringement risk, and increase the developer's payoff from adopting the new technology. In other words, reducing uncertainty encourages technology adoption.

More informativeness, however, also implies that the developer's investment will respond more to the patent office decision. For instance, suppose that *ex ante* the developer believes that with probability 1/2 the court will find infringement, and consider two extreme cases. If the patent office decision is uninformative at all, then whatever the examination outcome the developer will make an investment decision according to this probability; investment level is not affected by the patent office decision. But if patent examination predicts perfectly the court ruling, i.e., the two actually make the same decision, then according to the patent office decision the developer will make an investment either as if there is no infringement risk (when the patent office issues a narrow patent indicating no infringement), or as if infringement will happen for sure (when the patent office issues a broad patent indicating infringement).

A more diverse investment adjustment may not be in the patent-holder's interest. Indeed, even when risk neutrality is imposed, we get a fairly mild condition under which the patent-holder's payoff, when the developer adopts the technology, is *concave* in infringement probability.[5] Given adoption, to reduce the magnitude of investment adjustment, the patent-holder prefers less informative patent office decisions and fuzzier patent rights,

The opposite preferences on the informativeness of patent decisions suggest that the inventor's incentives to clarify the patent rights are driven by her interests in promoting

[5]The condition requires that the second-order effect of hold-up (the impact of infringement probability on investment) is not too large.

technology adoption. No private refinement efforts will be exerted when the inventor wants to monopolize the use of the new technology; she will choose the lowest possible refinement level in order to discourage technology diffusion. And even when, say, the inventor intends to extract some licensing payment from the developer, she still has no incentives to engage in refinement as long as adoption is guaranteed, or will happen with significant probability. The second scenario applies for a patent-holder with a sufficiently low *ex ante* quality invention, i.e., without information from the patent office examination the patent is very unlikely to be held valid in court, because in this case hold-up is less a threat and the developer will have high probability to adopt the new technology.

In Section 4, we consider whether licensing can solve the hold-up problem and how it affects refinement incentives.[6] It turns out that if licensing takes place early enough, especially if a license is negotiated before the patent applicant makes refinement decision, it can perfectly substitute for refinement efforts without jeopardizing technology diffusion. This implies that if we rely on early licensing to alleviate the hold-up problem, then fuzzy patents are the side effect we have to live with.

Before concluding the paper in Section 6, we extend the analysis to a simple multiple patent-holder case in Section 5. For simplicity, we consider two symmetric patent-holders, and ignore earlier licensing opportunities. We first show that even when the two patent-holders' technologies exhibit some degree of complementarity, as long as each has stand-alone value, technological synergy (i.e., the additional value generated by combining the two technologies) by itself cannot guarantee that the developer will use both technologies. Refinement efforts from both patentees are also required. And under the extreme cases where the two technologies are either perfect complements or perfect substitutes, we show that refinement efforts tend to be strategic complements. In spite of its restrictive assumptions, this result suggests that it may be worthwhile to design an incentive scheme to encourage some, if not all, patent-holders to exert refinement efforts, for it will have a snowballing effect to increase the overall refinement level.

[6] Gans *et al.* (2007) empirically tests the impact of uncertainty on the timing of patent licensing. They found that the likelihood of licensing increases after the patent office decision and thus uncertainty is reduced. In this paper, however, we do not consider the players' optimal timing to licensing.

2 Model

In the basic model, we consider the simplest case of one patent-holder/inventor (she) and one developer/technology adopter (he). The inventor seeks patent protection for her invention, and is granted a patent with uncertain boundary covering a technology field. The developer wishes to develop a product in order to enter the downstream market related to that field.

To successfully build a product, the developer has to incur an "entry fee," or a technology adoption cost $c_f \geq 0$ to learn the fundamental knowledge of the field. Based on the knowledge he acquired, the developer then exerts a technology-specific effort, or the investment effort $e \in [0,1]$, at a private cost $c_E(e)$. This effort is referred to as the probability that the developer successfully commercializes a final product and brings it into the downstream market. It may be technological in nature, such as the successful development of a new functionality; or a pure marketing strategy, such as the advertisement expense; or both, such as the efforts exerted in finding a profitable market niche. Assume that $c_E(e)$ and $c_E'(e) \geq 0$, $c_E''(e) > 0$, and $c_E'''(e) \geq 0$, $\forall e \in [0,1]$. Also assume that $c_E(0) = c_E'(0) = 0$, and $c_E'(1)$ is large enough to guarantee interior solutions. The investment effort and the associated cost $c_E(e)$ is the developer's private information (moral hazard).

We also assume that the adoption cost c_f is only known to the developer (adverse selection). The inventor holds a belief that c_f is distributed on $[0, \infty)$, with $F(\cdot)$ as the CDF. For simplicity, we will assume that $F(\cdot)$ is continuously differentiable and $F'(v) > 0$ over the relevant range. To gain more insights, in several accounts we will consider the special case of quadratic investment cost, $c_E(e) = e^2/2K$, and uniform distribution for c_f, i.e., $F(c_f) = \nu c_f$ for $c_f \in [0, 1/\nu]$, with both K and ν strictly positive and small enough. We assume that the developer will adopt the technology if and only if his expected adoption return is strictly higher than the cost c_f.

As stated above, a successfully built product may fall into the inventor's patent claims. The resolution of patent boundary consists of two steps: patent office examination a^P and court verdict a^C. Both authorities take binary decisions, a^P and $a^C \in \{0, 1\}$. When $a^P = 1$, the patent office issues a favorable decision supporting the inventor's claim over the new technology; and if $a^P = 0$, the patent-holder's claim is not approved by the patent office.[7] Similarly, when $a^C = 1$ ($a^C = 0$) the court up-

[7] Thus the patent office examination outcomes can have a broader interpretation than simply issuance or rejection. When $a^P = 0$, a patent can be granted, but an important claim in the original application has

holds (strikes down, respectively) the patent-holder's rights over the technology under dispute.

For simplicity, let the court be the final decision-maker, i.e., the patent-holder or technology adopter can appeal the patent office decision a^P to the court; but the issue is fully settled with the court ruling. If the court upholds her rights, the patent-holder is granted the injunction power; otherwise the technology adopter can freely use the technology. Assume away any legal or management cost to obtain either decisions. Denote $\alpha \in (0,1)$ as the common *ex ante* belief that the court will uphold the inventor's rights, $\alpha = Pr(a^C = 1)$. This probability measures the general strength of the patent protection, or the inventor's technological contribution in terms of patent law requirement. In the following we shall refer to it as the invention quality.

The decision of patent office a^P provides some information about the final resolution of patent boundary a^C. The informativeness of patent office decision, however, is affected by the inventor's "refinement effort" r. This effort includes any actions from the inventor to help the patent examination result more predictive of a^C. For instance, the inventor can search and disclose prior arts to the patent examiner, communicate with the examiner to help the latter better understand the invention, and more carefully draft of the patent application (including patent claims and specifications) *etc.* (Meurer and Nard, 2005; Risch, 2007).

Formally, assume that

$$Pr(a^P = a^C) = r \in [\underline{r}, 1], \quad \text{for both } a^P = a^C \in \{0,1\}.$$

That is, r is the probability that the patent office will make the same decision as the court. We assume that the minimal possible refinement effort $\underline{r}$ is strictly greater than but sufficiently close to $1/2$. This minimal level $\underline{r}$ may come from the patent examiner's effort or the degree to which the court will defer to the patent office decision.[8] Exerting an effort r entails a cost c_R, with c_R and $c'_R \geq 0$, and $c''_R > 0$, $c_R(\underline{r}) = c'_R(\underline{r}) = 0$, and $c'_R(1) = \infty$. We will also assume that c''_R is large enough to "force" the concavity of patent-holder's payoff function when deciding the optimal refinement effort. For simplicity we will assume that the effort r chosen by the inventor is observable to the developer. For instance, the patent application and examination record is made public

been trimmed by the examiner.

[8] Although introduced for technical reason (see footnote 13), we argue that it would be too pessimistic to say that patent examiners have no input at all and so without the patent applicant's effort the examination results will be totally uninformative.

after certain period of application (generally 18 months), and via these documents third parties can roughly figure out the "quality" of patent application and thus patent office decisions. We discuss the case of unobservable private refinement effort in Remark 3 in the end of Section 3.

Contrary to Meurer and Bessen (2006), where a higher refinement effort is assumed to always improve the patent-holder's prospect in court, in our model refinement's impact is symmetric.[9] Higher refinement effort will increase the predictive power of the patent office decision both when $a^P = 0$ and 1, i.e., whether the patent office is favorable to the inventor. As a justification, current U.S. patent prosecution procedure does not require an applicant to conduct prior art search, but does impose "a duty of candor and good faith" (Rule 56), which obliges the disclosure of any information known to the application that may be material to the issue of patentability. Its violation may render an issued patent unenforceable. Hence an applicant may have questionable incentives to exert r and search prior arts, and may be even more hesitant to lie and hide gathered information.

Given r and α, the probability distribution of the patent office decision is

$$\begin{aligned}
Pr(a^P = 1|\alpha, r) &\equiv q = \alpha r + (1-\alpha)(1-r), \quad \text{and} \\
Pr(a^P = 0|\alpha, r) &= 1 - q = \alpha(1-r) + (1-\alpha)r.
\end{aligned}$$

By observing α and r, after patent office decision, the technology adopter updates his beliefs about the court decision:

$$\hat{\alpha}^1 \equiv Pr(a^C = 1|\alpha, r, a^P = 1) = \frac{\alpha r}{q} \quad \text{and} \quad \hat{\alpha}^0 \equiv Pr(a^C = 1|\alpha, r, a^P = 0) = \frac{\alpha(1-r)}{1-q}.$$

We also call these updated beliefs as "patent power," for it measures the probability of infringing an issued patent. ' It is easy to verify that when $r = 1/2$, $\hat{\alpha}^0 = \hat{\alpha}^1 = \alpha$, the patent office decision is totally uninformative; and $\forall r > 1/2$, $\hat{\alpha}^0 < \alpha < \hat{\alpha}^1$, and

$$\frac{\partial \hat{\alpha}^1}{\partial r} = \frac{\alpha(1-\alpha)}{q^2} > 0 \quad \text{and} \quad \frac{\partial \hat{\alpha}^0}{\partial r} = -\frac{\alpha(1-\alpha)}{(1-q)^2} < 0.$$

A higher refinement effort will increase (decrease) the patent power when thee patent office sides with (against, respectively) the inventor.

Referring to Figure 1, the timing of the game is as follows:

[9] One way to reconcile the assumption in Meurer and Bessen (2006) and our model is to assume that the game ends after the patent office issues a decision $a^P = 0$. For instance, the patent office rejects the patent application and there is no further appeal by the applicant.

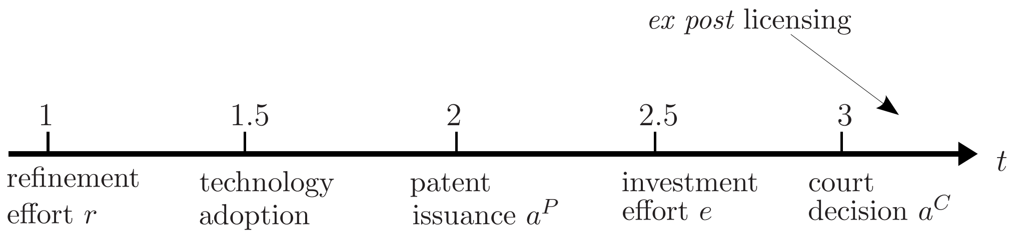

Figure 1: Timing

- At time 1 the patent-holder exerts an observable refinement effort r;
- at time 1.5, after learning r but before the patent office decision is known, the developer learns the value of adoption cost c_f and decides whether to enter into the field;
- at time 2, the patent office issues its decision $a^P \in \{0, 1\}$;
- at time 2.5, after adopting the new technology, the developer makes technology-specific investment effort e; and
- at time 3, if provoked, the court makes a final decision $a^C \in \{0, 1\}$.

The key assumption in the timing is that the technology adoption decision is made *before* the patent office decision. This scenario captures a common situation in high-tech, fast-moving industries where important decisions, such as which technologies to be incorporated into a standard or which standard to adopt for product development, to name a few, have to be made in the clouds of future patent disputes and cannot wait for the lengthy patent examination to reduce (but may not fully eliminate) the uncertainty. By contrast, the investment effort is exerted after the patent office decision in order to keep the hold-up element. As stated above, this effort may capture more than technological choices. For our purpose, the key point is that the developer can make some adjustment according to the patent office decision. (In Appendix B, we show that our main results hold in a later timing of adoption, time 2.5, after learning the patent office decision.)

Concerning payoffs, we assume that the patent-holder, as a pioneering inventor, may already operate in the downstream market. Her payoff, however, may be affected by the entry of the developer. If the developer doesn't incur the technology adoption cost, the inventor receives a revenue $u_{10} \geq 0$ and the developer receives zero; this pair of revenues also applies when the developer spends c_f and adopts the new technology, but fails to develop a product and so cannot enter the downstream market, which happens with probability $1 - e$. On the other hand, with probability e the developer successfully conducts the investment project and can be present in the market. In this case, the revenues accrued to each player are determined by the prevailing market participation profile. Denote u_{ij} and v_{ji} as the patent-holder's and the developer's revenues, respectively, where $i = 1$ ($j = 1$) means that the patent-holder (the developer, respectively) is operating on the market; and when $i = 0$ ($j = 0$), she (he, respectively) exits the market. Assume there is no exit cost, and

Assumption 1. For all i, $j \in \{0, 1\}$, u_{1j} and $v_{1i} \geq 0$, $u_{0j} = v_{0i} = 0$, and $v_{11} > 0$.

That is, revenues from downstream market operation are non-negative, and is zero upon exit. Define $\pi \equiv \max_{ij} u_{ij} + v_{ji} \in \{u_{10}, v_{10}, u_{11} + v_{11}\}$ as the maximal joint revenue from the downstream market. When $\pi \in \{u_{10}, v_{10}\}$, it is privately efficient to let one player exit the market. (The case of $\pi = u_{00} + v_{00} = 0$ is obviously uninteresting.)

To close the model, notice that licensing can take place in several occasions: after the court decision (*ex post* licensing), after patent issuance but before the adopter makes technology-specific investment (*interim* licensing), and before patent issuance (*ex ante* licensing). We assume that *ex post* licensing is always available, and takes place only when the developer's investment project succeeds and the court upholds the patent-holder's claim, $a^C = 1$.[10] In this case, the court grants the patent-holder the injunction power to shut down the developer, which serves as the threat point at bargaining. Other licensing opportunities are considered only in Section 4. Whenever negotiating a license, we assign the whole bargaining power to the patent-holder.

We also assume that both players are protected by limited liability. The developer thus cannot be punished when receiving no revenue.

3 Adopter's Convex Preferences in Patent Power

This section illustrates the basic trade-offs underlying the inventor's refinement decision. Suppose that only *ex post* licensing is available. By backward induction, let us start with the subgame where the technology adopter uses the new technology and has successfully conducted his investment project; otherwise the game ends and the patent-holder gets a revenue u_{10} and developer gets zero.

Consider the outcomes of court decision. When $a^C = 0$, the court rejects the patent-holder's claim over the invention and so the developer can freely use the new

[10]When $a^C = 0$, without supports of patent rights the antitrust authority will challenge any attempts to monopolize the market.

technology. The returns from market are u_{11} for the patent-holder and v_{11} for the developer, respectively. When $a^C = 1$, the court upholds the patent-holder's claim. With injunction and full bargaining power the patent-holder can realize the maximal return π by either shutting down the developer (when $\pi = u_{10}$) or offering a license which fully extracts the surplus and leaves zero return to the developer (as in his outside option of exiting).

At time 2.5, with the updated belief $\hat{\alpha}$, the developer's optimal investment effort $\hat{e}$ is determined by

$$\hat{e} \equiv \arg\max_{e} \quad (1-\hat{\alpha})ev_{11} - c_E(e) \quad \Rightarrow \quad FOC: \quad (1-\hat{\alpha})v_{11} \equiv c_E'(\hat{e}).$$

Denote the optimal investment efforts $\hat{e}^0 \equiv \hat{e}(\hat{\alpha}^0)$ and $\hat{e}^1 \equiv \hat{e}(\hat{\alpha}^1)$, corresponding to the patent office decision, and define $\hat{v} \equiv (1-\hat{\alpha})\hat{e}v_{11} - c_E(\hat{e})$. The developer's expected payoff from adopting the new technology is

$$\begin{aligned} V(\alpha, r) &= q\hat{v}(\hat{\alpha}^1) + (1-q)\hat{v}(\hat{\alpha}^0) \\ &= q\big[(1-\hat{\alpha}^1)\hat{e}^1 v_{11} - c_E(\hat{e}^1)\big] + (1-q)\big[(1-\hat{\alpha}^0)\hat{e}^0 v_{11} - c_E(\hat{e}^0)\big]. \end{aligned}$$

At time 1.5, after observing α and r, the developer will opt for the new technology when $V(\alpha, r) > c_f$.

For the patent-holder, she knows that after the refinement choice r the probability that the developer will adopt her invention is $F(V(\alpha, r))$. Upon adoption, given the patent power $\hat{\alpha}$, at time 2.5 the patent-holder's expected payoff (gross of refinement effort) is

$$\hat{u}(\hat{\alpha}) = (1-\hat{e})u_{10} + \hat{e}\left[\hat{\alpha}\pi + (1-\hat{\alpha})u_{11}\right] = u_{10} + \hat{e}\left[\hat{\alpha}(\pi - u_{11}) + (u_{11} - u_{10})\right].$$

When the developer's project fails, the inventor receives u_{10}; and when the developer succeeds, the inventor receives π if her patent rights are upheld in court, and u_{11} otherwise. The term $\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}$ reflects the effect of developer's entry on the patent-holder. Besides a direct impact $u_{11} - u_{10} \gtrless 0$, the patent endows its owner another source of revenue $\hat{\alpha}(\pi - u_{11})$. Without further assumptions the term $\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}$ can be either positive or negative. For instance, it is negative when the two players compete fiercely on the market so that $u_{11} < u_{10}$ and the patent power $\hat{\alpha}$ is low. By contrast, if the patent-holder does not participate in the downstream market and only gathers licensing income, $u_{11} = u_{10} = 0$, it is always positive.

The patent-holder's expected payoff (gross of refinement cost) at time 1.5, given that the developer adopts the new technology, is

$$u_\tau = q\hat{u}(\hat{\alpha}^1) + (1-q)\hat{u}(\hat{\alpha}^0),$$

and at time 1, when making the refinement decision, the patent-holder's expected payoff (net of refinement cost) is

$$U = [1 - F(V)]\, u_{10} + F(V)u_\tau - c_R.$$

With these payoffs in hand, we first show the two players' preferences toward the refinement effort r. This will be the driving force of the subsequent analysis.

It is easy to see that, because of the technology-specific investment effort, the technology adopter has a decreasing and strictly *convex* preference over patent power:

$$\frac{d\hat{v}}{d\hat{\alpha}} = -\hat{e}v_{11} \quad \text{and} \quad \frac{d^2\hat{v}}{d\hat{\alpha}^2} = -v_{11}\frac{d\hat{e}}{d\hat{\alpha}} = \frac{v_{11}^2}{c_E^{''}(\hat{e})} > 0,$$

for $d\hat{e}/d\hat{\alpha} = -v_{11}/c_E^{''} < 0$. Together with the fact that $q\hat{\alpha}^1 + (1-q)\hat{\alpha}^0 = \alpha r + \alpha(1-r)$, a strictly convex $\hat{v}$ implies that, for all $r > 1/2$,

$$V(\alpha, r) = q\hat{v}(\hat{\alpha}^1) + (1-q)\hat{v}(\hat{\alpha}^0) > \hat{v}(q\hat{\alpha}^1 + (1-q)\hat{\alpha}^0) = \hat{v}(\alpha) = V(\alpha, r = \frac{1}{2}).$$

In other words, the technology adopter will prefer some clarity, $r > 1/2$, to totally uninformative patent office decisions, $r = 1/2$. Intuitively, more informative patent office decisions provide better guidance about infringement risk $\hat{\alpha}$, and so the developer can take better responses and adjust his investment accordingly. Fixing the level of invention quality α, reducing uncertainty boosts incentives to adopt the new technology.

For the patent-holder, the impact of patent power $\hat{\alpha}$ is

$$\begin{aligned}
\frac{d\hat{u}}{d\hat{\alpha}} &= [\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}]\frac{d\hat{e}}{d\hat{\alpha}} + (\pi - u_{11})\hat{e}, \quad \text{and} \\
\frac{d^2\hat{u}}{d\hat{\alpha}^2} &= 2(\pi - u_{11})\frac{d\hat{e}}{d\hat{\alpha}} + [\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}]\frac{d^2\hat{e}}{d\hat{\alpha}^2}, \quad \text{where} \quad \frac{d^2\hat{e}}{d\hat{\alpha}^2} = \frac{v_{11}}{(c_E^{''})^2}c_E^{'''}\frac{d\hat{e}}{d\hat{\alpha}} \leq 0.
\end{aligned}$$

Because $\pi > u_{11}$,[11] the payoff $\hat{u}$ is increasing in patent power if $\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}$ is negative, or, when it is positive, if the absolute size of $[\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}]\frac{d\hat{e}}{d\hat{\alpha}}$ is not too large.

[11]If $\pi = u_{11}$, which implies that $v_{11} = v_{01} = 0$, the technology adopter will never use the new technology.

More importantly, $\hat{u}$ is strictly concave in $\hat{\alpha}$ as long as the second-order effect of patent power on specific investment, $d^2\hat{e}/d\hat{\alpha}^2$, is not too large. If fact, $d^2\hat{u}/d\hat{\alpha}^2$ is non-negative at some $\hat{\alpha}$ only when

$$[\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}]\frac{d^2\hat{e}}{d\hat{\alpha}^2} \geq -2(\pi - u_{11})\frac{d\hat{e}}{d\hat{\alpha}} > 0,$$

or, by replacing $d^2\hat{e}/d\hat{\alpha}^2$,

$$[\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}]\frac{v_{11}}{(c_E^{''})^2}c_E^{'''} \leq -2(\pi - u_{11}) < 0. \tag{1}$$

A necessary condition is that $[\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}]c_E^{'''} < 0$. It will not hold, and thus the patent-holder will have a concave $\hat{u}$, when $c_E^{'''} = 0$, i.e, the investment cost c_E takes a quadratic form.[12]

When condition (1) fails so that $\hat{u}$ is globally strictly concave in $\hat{\alpha}$, for all $r > 1/2$,

$$u_\tau = q\hat{u}(\hat{\alpha}^1) + (1-q)\hat{u}(\hat{\alpha}^0) < \hat{u}(q\hat{\alpha}^1 + (1-q)\hat{\alpha}^0) = \hat{u}(\alpha).$$

Contrary to the technology adopter, the patent-holder will prefer totally uninformative patent office decisions ($r = 1/2$) to some clarity ($r > 1/2$). This also implies that, when adoption is not a concern, $F(V) = 1$, the patent applicant would be strictly better off if she could undermine the examiner's effort, i.e., reduce $\underline{r}$ to 1/2, at little cost.

Proposition 1. *(Preference conflict). When the technology adopter has to exert a non-contractible, technology-specific investment effort, and only ex post licensing is available, the technology adopter has decreasing and strictly convex preferences toward patent power, but when condition (1) fails, the patent-holder has strictly concave preferences over patent power.*

This conflict in preferences establishes the basic trade-off in our model regarding the patent refinement, or the uncertainty of patent boundary. To gain more insights, we consider the special case of a quadratic investment cost function, $c_E(e) = e^2/2K$, and if necessary, the uniform distribution for the adoption cost c_f with support $[0, 1/\nu]$. Assume that both $1/K$ and $1/\nu$ are strictly larger than v_{11}, so that the optimal $\hat{e}(\hat{\alpha}) < 1$ for all $\hat{\alpha}$, and $F(V) < 1$ for all possible α and r. Note that under the quadratic cost function, $c_E^{'''} = 0$, which busts condition (1). The patent-holder has a strictly concave preference toward patent power.

[12]If $c_E^{'''} > 0$, the necessary condition fails when $\hat{\alpha}(\pi - u_{11}) + u_{11} - u_{10}$ is greater than zero for all $\hat{\alpha}$, which is true when the patent-holder is a pure licensor, $u_{11} = u_{10} = 0$. And in the case of $\pi = u_{10}$, it requires that $v_{11}c_E^{'''} < 2(c_E^{''})^2$.

This cost function allows us to solve analytically the developer's optimal investment effort and his expected payoff from adopting the new technology:

$$\begin{aligned}
\hat{e} &= (1-\hat{\alpha})Kv_{11}, \quad \hat{v} = (1-\hat{\alpha})^2 v_{11}^2 \frac{K}{2}, \qquad \text{and} \\
V(\alpha, r) &= q\hat{v}(\hat{\alpha}^1) + (1-q)\hat{v}(\hat{\alpha}^0) = [q(1-\hat{\alpha}^1)^2 + (1-q)(1-\hat{\alpha}^0)^2]v_{11}^2 \frac{K}{2} \\
&= \left\{ q \left[\frac{(1-\alpha)(1-r)}{q} \right]^2 + (1-q) \left[\frac{(1-\alpha)r}{1-q} \right]^2 \right\} v_{11}^2 \frac{K}{2} \\
&= \left[\frac{(1-r)^2}{q} + \frac{r^2}{1-q} \right] (1-\alpha)^2 v_{11}^2 \frac{K}{2}.
\end{aligned}$$

This cost function also gives us the nice property that, given technology adoption, the overall probability that the adopter will successfully conduct his project is not affected by refinement effort:

$$q\hat{e}^1 + (1-q)\hat{e}^0 = [q(1-\hat{\alpha}^1) + (1-q)(1-\hat{\alpha}^0)]Kv_{11} = (1-\alpha)Kv_{11}.$$

We can then measure *ex ante* technology diffusion by considering only the adoption probability $F(V)$.

For convenience, define

$$\Psi \equiv \frac{(1-r)^2}{q} + \frac{r^2}{1-q},$$

which summarizes the impact of refinement effort r on technology adoption incentives. We can confirm that the developer strictly prefers a higher r: $\forall r > 1/2$ and $\alpha \in (0,1)$,

$$\begin{aligned}
\frac{\partial \Psi}{\partial r} &= \frac{\partial}{\partial r} \left[\frac{(1-r)^2}{q} + \frac{r^2}{1-q} \right] = \frac{rq - (1-r)(1-q)}{q^2(1-q)^2} \left\{ 2q(1-q) + [rq + (1-r)(1-q)] \frac{\partial q}{\partial r} \right\} \\
&= \frac{\alpha^2[r^2 - (1-r)^2][r + (1-r)]^2}{q^2(1-q)^2} = \alpha \frac{\alpha[r-(1-r)]}{q^2(1-q)^2} > 0, \\
\frac{\partial^2 \Psi}{\partial \alpha \partial r} &= \frac{\partial}{\partial \alpha} \frac{\alpha^2[r-(1-r)]}{q^2(1-q)^2} = \frac{2\alpha[r-(1-r)]}{q^3(1-q)^3} \left\{ q(1-q) - \alpha[(1-q)-q] \frac{\partial q}{\partial \alpha} \right\} \\
&= \frac{2\alpha[r-(1-r)]}{q^3(1-q)^3} \left\{ r(1-r) + \alpha^2[r-(1-r)]^2 \right\} > 0.
\end{aligned}$$

A higher refinement effort encourages technology adoption, and this effect is larger when the *ex ante* invention quality is better (α higher).

Concerning the impact of invention quality α:

$$
\begin{aligned}
\frac{\partial V}{\partial \alpha} &= v_{11}^2 \frac{K}{2}\left[-2(1-\alpha)\Psi + (1-\alpha)^2 \frac{\partial \Psi}{\partial \alpha}\right] = (1-\alpha) v_{11}^2 \frac{K}{2}\left[(1-\alpha)\frac{\partial \Psi}{\partial \alpha} - 2\Psi\right] \\
&= -\frac{(1-\alpha) v_{11}^2 \frac{K}{2}}{q^2(1-q)^2}\Big\{2\alpha^3 r(1-r)[r^3 + (1-r)^3] + 2(1-\alpha)^3 r^2 (1-r)^2 \\
&\quad + \alpha^2(1-\alpha)\big[[r-(1-r)][r^4 - (1-r)^4] + 2r(1-r)[1-r(1-r)]\big] \\
&\quad + \alpha(1-\alpha)^2 2r(1-r)\big[(1-r)^3 + 2r(1-r) + r^3\big]\Big\} < 0, \quad \text{and} \\
\frac{\partial^2 V}{\partial \alpha \partial r} &= (1-\alpha) v_{11}^2 \frac{K}{2}\left[(1-\alpha)\frac{\partial^2 \Psi}{\partial \alpha \partial r} - 2\frac{\partial \Psi}{\partial r}\right] \\
&= 2\frac{\alpha(1-\alpha) r(1-r) v_{11}^2 \frac{K}{2}}{q^3(1-q)^3}\big[\alpha - (1-\alpha)\big]\big[r - (1-r)\big].
\end{aligned}
$$

Intuitively, the hold-up threat is stronger when the invention quality α is higher. Fixing r, the adoption payoff is decreasing in α. And although a higher refinement effort increases adoption payoff, $\partial V/\partial r > 0$, the impact of invention quality on this benefit, $\frac{\partial}{\partial \alpha}(\frac{\partial V}{\partial r})$ is non-monotonic in α. It is strictly positive (strictly negative) when α is strictly higher (strictly lower, respectively) than $1/2$, and the impact is negligible when α approaches to zero or one: $\alpha(1-\alpha) \to 0$ as $\alpha \to 0$ or 1.

For the patent-holder, under quadratic investment cost, the expected payoff from technology adoption is

$$
\begin{aligned}
u_\tau &= u_{10} + q\hat{e}^1\left[\hat{\alpha}^1(\pi - u_{11}) + u_{11} - u_{10}\right] + (1-q)\hat{e}^0\left[\hat{\alpha}^0(\pi - u_{11}) + u_{11} - u_{10}\right] \\
&= u_{10} + (\pi - u_{11})\left[q\hat{\alpha}^1\hat{e}^1 + (1-q)\hat{\alpha}^0\hat{e}^0\right] + (u_{11} - u_{10})\left[q\hat{e}^1 + (1-q)\hat{e}^0\right] \\
&= u_{10} + (1-\alpha) v_{11} K\left[\alpha(\pi - u_{11})\frac{r(1-r)}{q(1-q)} + u_{11} - u_{10}\right].
\end{aligned}
$$

Define

$$
\Lambda \equiv \frac{r(1-r)}{q(1-q)},
$$

through which the refinement effort r exerts its effect on the patent-holder's post-adoption payoff. Similarly, for all $r > 1/2$ and $\alpha \in (0,1)$,

$$
\begin{aligned}
\frac{\partial \Lambda}{\partial r} &= \frac{1}{q^2(1-q)^2}\left[(1-r-r)q(1-q) - r(1-r)(1-q-q)\frac{\partial q}{\partial r}\right] = \frac{\alpha(1-\alpha)[(1-r)^2 - r^2]}{q^2(1-q)^2} \\
&= -(1-\alpha)\frac{\alpha[r-(1-r)]}{q^2(1-q)^2} < 0,
\end{aligned}
$$

a higher refinement effort reduces the patent-holder's post-adoption payoff. Furthermore,

$$\begin{aligned}\frac{\partial^2 u_\tau}{\partial\alpha\partial r} &= v_{11}(\pi - u_{11})K\frac{\partial}{\partial\alpha}\left[\alpha(1-\alpha)\frac{\partial\Lambda}{\partial r}\right] \\ &= 2\frac{\alpha(1-\alpha)r(1-r)v_{11}(\pi-u_{11})K}{q^3(1-q)^3}\left[\alpha-(1-\alpha)\right]\left[r-(1-r)\right].\end{aligned}$$

This term has similar qualitative properties as $\partial^2 V/\partial\alpha\partial r$: It is positive when $\alpha \geq 1/2$ and negative when $\alpha \leq 1/2$, with negligible magnitude when α is close to zero or one.

Assume that the patent-holder's expected payoff at the refinement stage,

$$U = [1-F(V)]\,u_{10} + F(V)u_\tau - c_R(r) = u_{10} + F(V)(u_\tau - u_{10}) - c_R(r),$$

is strictly concave in r over the range $[\underline{r}, 1]$.[13] By $\partial u_\tau/\partial r < 0$, we immediately obtain that when $F(V) = 1$, i.e., when technology adoption concern is absent, the inventor has no refinement incentives. Consider the range over when $F(V) < 1$ and $F'(V) > 0$. The first-order condition to determine the optimal refinement effort is

$$\frac{\partial U}{\partial r} = F'(V)(u_\tau - u_{10})\frac{\partial V}{\partial r} + F(V)\frac{\partial u_\tau}{\partial r} - c'_R = F(V)\left[\frac{F'}{F}(V)\frac{\partial V}{\partial r}(u_\tau - u_{10}) + \frac{\partial u_\tau}{\partial r}\right] - c'_R.$$

For all $r > 1/2$, $\partial V/\partial r$ is strictly positive and $\partial u_\tau/\partial r$ strictly negative. When choosing the refinement effort, the trade-off, if any, lies between higher adoption probability and larger post-adoption revenue. But whether there is such a trade-off crucially depends on the patent-holder's downstream strategy.

Let us consider two market structures. In the first scenario, we assume that $u_{11} = u_{10} = 0$, and so $\pi = v_{11}$, i.e., the patent-holder is a pure licensor and does not participate in downstream market. This is a special case of $\pi = u_{11} + v_{11}$, where after infringement ($a^C = 1$) the patent-holder will grant an *ex post* license and extracts a licensing fee v_{11} from the developer. In the second scenario, we consider $\pi = u_{10}$ and so when obtains an injunction the patent-holder will shut down the developer.

When the patent-holder is a pure licensor, her post-adoption payoff is

$$u_\tau = q\hat{u}(\hat{\alpha}^1) + (1-q)\hat{u}(\hat{\alpha}^0) = \left[q\hat{e}^1\hat{\alpha}^1 + (1-q)\hat{e}^0\hat{\alpha}^0\right]v_{11}.$$

[13] It can be "forced" to hold by assuming c''_R large enough. Notice that at $r = 1/2$, both $\partial\Psi/\partial r = \partial\Lambda/\partial r = 0$. If we let $\underline{r} = 0$ while maintaining $c'_R(1/2) = 0$, $r = 1/2$ will become a critical point. Together with the concavity of the payoff function, the optimal refinement is always $r = 1/2$.

With quadratic investment cost, the first-older condition to determine the optimal r is

$$\begin{aligned}\frac{\partial U}{\partial r} &= F'(V)\frac{\partial V}{\partial r}[q\hat{u}(\hat{\alpha}^1)+(1-q)\hat{u}(\hat{\alpha}^0)-u_{10}]+F(V)\frac{\partial}{\partial r}[q\hat{u}(\hat{\alpha}^1)+(1-q)\hat{u}(\hat{\alpha}^0)-u_{10}]-c_R' \\ &= \alpha(1-\alpha)v_{11}^2 KF(V)\left\{\frac{F'}{F}(V)\frac{V}{\Psi}\Lambda\frac{\partial \Psi}{\partial r}+\frac{\partial \Lambda}{\partial r}\right\}-c_R' \\ &= \alpha^2(1-\alpha)^2 v_{11}^2 KF(V)\frac{[r-(1-r)]}{q^2(1-q)^2}\left\{\frac{F'}{F}(V)V\frac{\alpha\Lambda}{(1-\alpha)\Psi}-1\right\}-c_R'.\end{aligned}$$

A necessary and sufficient condition for the patent-holder to be willing to exert any refinement effort is, when evaluated at $r=\underline{r}>1/2$,

$$\frac{F'}{F}(V)V\frac{\alpha\Lambda}{(1-\alpha)\Psi}>1 \quad\Rightarrow\quad \frac{F'}{F}(V)V>\frac{(1-\alpha)\Psi}{\alpha\Lambda}. \tag{2}$$

It can be shown that the right-hand side of the condition is decreasing in α, and when $\alpha\to 0$, $\Psi/\Lambda\to 1$. Therefore, as long as $F'V/F$ is uniformly bounded,[14] this condition is less likely to hold when α is small enough. For instance, under uniform distribution, $F'(V)V/F(V)=1$, this condition becomes

$$\alpha\Lambda>(1-\alpha)\Psi \Leftrightarrow \left[\alpha-(1-\alpha)^2\right]\underline{r}(1-\underline{r})>\alpha(1-\alpha)\left[\underline{r}^3+(1-\underline{r})^3\right],$$

which won't be true when $\alpha\leq(1-\alpha)^2$. In addition, because $\underline{r}>1/2$, it is less likely to satisfy when $\underline{r}$ becomes larger. Higher efforts from the patent examiner thus will crowd out, or substitute, the patent applicant's refinement efforts.

When $\pi=u_{10}$, the patent is used to excluded any competitors from the downstream market. Technology adoption does not serve the patent-holder's interest: $\forall\hat{\alpha}\in[0,1)$,

$$\hat{u}(\hat{\alpha})=u_{10}+\hat{e}[\hat{\alpha}(u_{10}-u_{11})-(u_{10}-u_{11})]=u_{10}-(1-\hat{\alpha})\hat{e}(u_{10}-u_{11})<u_{10},$$

and so

$$u_\tau=q\hat{u}(\hat{\alpha}^1)+(1-q)\hat{u}(\hat{\alpha}^0)=u_{10}-(u_{10}-u_{11})[q\hat{e}^1(1-\hat{\alpha}^1)+(1-q)\hat{e}^0(1-\hat{\alpha}^0)]<u_{10}.$$

To find the optimal refinement effort:

$$\begin{aligned}\frac{\partial U}{\partial r} &= -F'(V)(u_{10}-u_{11})[q\hat{e}^1(1-\hat{\alpha}^1)+(1-q)\hat{e}^0(1-\hat{\alpha}^0)]\frac{\partial V}{\partial r} \\ &\quad -F(V)(u_{10}-u_{11})\frac{\partial}{\partial r}[q\hat{e}^1(1-\hat{\alpha}^1)+(1-q)\hat{e}^0(1-\hat{\alpha}^0)]-c_R' \\ &= -F(V)(u_{10}-u_{11})\left\{\frac{F'}{F}(V)[q\hat{e}^1(1-\hat{\alpha}^1)+(1-q)\hat{e}^0(1-\hat{\alpha}^0)]\frac{\partial V}{\partial r}\right. \\ &\quad \left.+\frac{\partial}{\partial r}[q\hat{e}^1(1-\hat{\alpha}^1)+(1-q)\hat{e}^0(1-\hat{\alpha}^0)]\right\}-c_R'.\end{aligned}$$

[14] This can be interpreted as the probability elasticity, i.e., $F'V/F=(dF/dV)/(F/V)$. Roughly speaking, uniformly bounded requirement means that there is no sudden jump in percentage of probability density.

With quadratic investment cost c_E, $\partial V/\partial r > 0$ and

$$\begin{aligned}
& \frac{\partial}{\partial r}[q\hat{e}^1(1-\hat{\alpha}^1)+(1-q)\hat{e}^0(1-\hat{\alpha}^0)] = (1-\alpha)\frac{\partial}{\partial r}[(1-r)\hat{e}^1+r\hat{e}^0] \\
= \;& (1-\alpha)Kv_{11}\frac{\partial}{\partial r}[(1-r)(1-\hat{\alpha}^1)+r(1-\hat{\alpha}^0)] = -(1-\alpha)Kv_{11}\frac{\partial}{\partial r}[(1-r)\hat{\alpha}^1+r\hat{\alpha}^0] \\
= \;& -\alpha(1-\alpha)Kv_{11}\frac{\partial\Lambda}{\partial r} > 0, \quad \forall r > \frac{1}{2}.
\end{aligned}$$

Therefore, the patent-holder has no incentives to refine the patent application, the optimal $r = \underline{r}$, *whatever her invention quality* α. This result doesn't require further assumptions on the distribution $F(\cdot)$.

Proposition 2. *Suppose that the technology adopter's technology-specific investment cost is quadratic,* $c_E(e) = e^2/2K$. *Consider two special cases.*

When the inventor is a pure licensor ($u_{11} = u_{10} = 0$ *and so* $\pi = v_{11}$*), the inventor will exert some refinement effort, the optimal* $r > \underline{r}$ *if and only if condition (2) holds at* $r = \underline{r}$. *When* $F'V/F$ *is uniformly bounded, this condition will not hold when* α *is low enough. In the case of uniform distribution, a patent-holder with* α *low enough such that* $\alpha \le (1-\alpha)^2$ *will not exert any refinement effort.*

When $\pi = u_{10}$ *and so the patent-holder prefers to monopolize the market, then* $\forall\alpha \in (0,1)$ *no refinement efforts will be exerted.*

Remark 1. (IPR strategy and business model). Here we assume that the patent-holder is already operating in the market. If the patent-holder still has an interest in obtaining the monopoly position, $\pi = u_{10}$, but also needs to make some investment in order to enter the downstream market, then the same reasoning suggests that the patent-holder would not want to engage in refinement at all. Again, a higher refinement effort will attract more entry and reduce the prospect to maintain the monopoly, for the patent-holder will have to rely on the court's injunction grant to exclude competitors. This suggests a relationship between a patent-holder's refinement policy and business strategy. At the aggregate level, it would be interesting to investigate the prevailing market structure and the degree of patent rights uncertainty across industries. ‖

Remark 2. (The third case). The case of $\pi = v_{10}$ is somewhat between the two cases $\pi = u_{10}$ and $\pi = u_{11} + v_{11}$. In this case, post infringement, the patent-holder will exit the market and grant the developer a license in exchange for some payment. If $u_{11} \ge u_{10}$, or even when the developer's entry has a negative impact on the patent-holder, $u_{11} < u_{10}$, but its magnitude is not too large, then $u_\tau > u_{10}$ and the patent-holder faces the same trade-off between encouraging adoption $F(V)$ and raising post-adoption payoff u_τ as in the case of $\pi = u_{11} + v_{11}$.

However, if the competition is so fierce that $u_\tau \leq u_{10}$, as in the case of $\pi = u_{10}$, then the patent-holder will against not exert any refinement effort. Despite the assumption that post infringement the patent-holder can extract the full surplus v_{10} via licensing, this would happen because *ex post* licensing only takes place when the court upholds the patent rights. To see this, note that with quadratic cost function,

$$u_\tau - u_{10} = (1-\alpha)v_{11}K\left[\alpha\Lambda(v_{10} - u_{11}) + (u_{11} - u_{10})\right],$$

which is negative when α is small enough. $\|$

Remark 3. (Non-observable refinement). When the refinement effort r is not observable to the technology adopter, a rational expectation equilibrium requires that, along the equilibrium path, the developer guess right the equilibrium refinement effort exerted by the patent-holder, and the latter won't secretly deviate from the equilibrium level.

Let $\tilde{r} \in [\underline{r}, 1]$ be a rational expectation equilibrium, with corresponding $\tilde{q} \equiv \alpha\tilde{r} + (1-\alpha)(1-\tilde{r})$, $\tilde{\alpha}^1 \equiv \alpha\tilde{r}/\tilde{q}$ and $\tilde{\alpha}^0 \equiv \alpha(1-r)/(1-\tilde{q})$. At the technology adoption stage (time 1.5), the adopter then expects a payoff $\tilde{V} \equiv \tilde{q}\hat{v}(\tilde{\alpha}^1) + (1-\tilde{q})\hat{v}(\tilde{\alpha}^0)$. The adoption probability is $F(\tilde{V})$ and upon adoption, the developer will exert an effort $\tilde{e}^1 \equiv \hat{e}(\tilde{\alpha}^1)$ when $a^P = 1$ and $\tilde{e}^0 \equiv \hat{e}(\tilde{\alpha}^0)$ when $a^P = 0$, respectively. The patent-holder's equilibrium payoff, then, is $U = u_{10} + F(\tilde{V})(u_\tau - u_{10}) - c_R$, where

$$\begin{aligned} u_\tau - u_{10} &= (\pi - u_{11})\left[\tilde{q}\tilde{\alpha}^1\tilde{e}^1 + (1-\tilde{q})\tilde{\alpha}^0\tilde{e}^0\right] + (u_{11} - u_{10})\left[\tilde{q}\tilde{e}^1 + (1-\tilde{q})\tilde{e}^0\right] \\ &= \alpha(\pi - u_{11})\left[\tilde{r}\tilde{e}^1 + (1-\tilde{r})\tilde{e}^0\right] + (u_{11} - u_{10})\left[\tilde{q}\tilde{e}^1 + (1-\tilde{q})\tilde{e}^0\right]. \end{aligned}$$

Suppose that $\tilde{r} > \underline{r}$, the patent-holder exerts some refinement effort in the equilibrium, and consider if she deviates from the equilibrium. Because the refinement effort is not observable, after the deviation the developer still believes that the prevailing refinement is $\tilde{}$ and so $\tilde{V}$ as well as $\tilde{e}^1$ and $\tilde{e}^0$ are unchanged. A secret deviation thus the patent-holder's payoff to change by:

$$\begin{aligned} & \frac{\partial U}{\partial r}\Big|_{secret\ deviation} = F(\tilde{V})\frac{\partial(u_\tau - u_{10})}{\partial r}\Big|_{secret\ deviation} - c_R' \\ = \quad & F(\tilde{V})(\tilde{e}^1 - \tilde{e}^0)\left\{\alpha(\pi - u_{11}) + [\alpha - (1-\alpha)](u_{11} - u_{10})\right\} - c_R' \\ = \quad & F(\tilde{V})(\tilde{e}^1 - \tilde{e}^0)\left[\alpha(\pi - u_{10}) - (1-\alpha)(u_{11} - u_{10})\right] - c_R'. \end{aligned}$$

Because $\tilde{e}^1 < \tilde{e}^0$, the patent-holder will have an incentive to deviate to a lower refinement effort as long as $\alpha(\pi - u_{10}) \geq (1-\alpha)(u_{11} - u_{10})$, which is true for both $\pi = u_{10}$ and

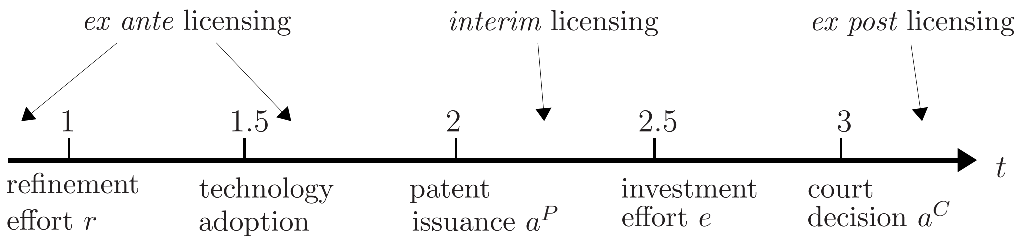

Figure 2: Licensing opportunities

$\pi = v_{11}$ while $u_{11} = u_{10} = 0$. In these cases, the only rational expectation equilibrium is $r = \underline{r}$.

Furthermore, a necessary condition of no secret deviation is

$$\frac{\alpha}{1-\alpha} < \frac{u_{11} - u_{10}}{\pi - u_{10}},$$

which could be satisfied only when the presence of the developer on the downstream market exerts a positive benefit to the patent-holder, $u_{11} > u_{10}$. And given that is true, the invention quality should be low enough. ||

4 Will Licensing Help? (To be revised)

Previous results are obtained under the assumption that only *ex post* licensing is available. This section considers how earlier licensing opportunities would affect the patent-holder's refinement incentives. We separately consider *interim* and *ex ante* licensing (see Figure 2). *Ex post* licensing remains as an alternative if no agreement is reached at an earlier stage. However, to simplify the analysis, we do not allow *interim* licensing when considering *ex ante* licensing. We let the patent-holder make licensing offers when the opportunity arises.

Note that when $\pi = u_{10}$, the patent-holder prefers the technology adopter to stay off the market even if the latter successfully conducts the investment project. Licensing, when taking place, will involve the highly controversial reverse payment where the patent-holder pays the technology adopter in exchange of the latter's exit, or not using the new technology in the first place.

Even if reverse payment can escape skepticism and challenges from antitrust authority, by assigning the whole bargaining power to the patent-holder, the technology adopter's expected payoff under *en ante* or *interim* licensing will be the same as in the previous case. The patent-holder will still have no incentive to engage in any refinement in order to dissuade the developer's from using the new technology, and thus avoid the event to "buy out" the developer.[15] For this reason, in the remaining of this section we consider only the case where the patent-holder is a pure licensor, $\pi = v_{11}$, with $u_{10} = u_{11} = 0$.

☐ *Interim* **licensing:** Referring to Figure 2, *interim* licensing takes place before the technology adopter's specific investment but after the patent office issues its decision. Under the assumptions that the technology adopter is protected by limited liability and the investment effort e is not contractible, the type of licenses the patent-holder can offer is a royalty $l \in [0, 1]$ such that when the developer's project succeeds, the patent-holder gets a share $l \cdot v_{11}$. Let l^0 and l^1 be the licensing terms offered by the patent-holder, given the patent office decision $a^P = 0$ and $a^P = 1$, respectively. To induce acceptance, the royalty l^i cannot exceed the prevailing patent power $\hat{\alpha}^i$, $i \in \{0, 1\}$.

When the developer accepts the offer l^i, he will exert an investment effort $\hat{e}(l^i)$, with an expected project return $\hat{v}(l^i)$. The patent-holder's expected licensing income is $\hat{u}(l^i) = \hat{e}(l^i)l^i v_{11}$. Denote l^* as the optimal offer that maximizes the patent-holder's expected licensing income in the absence of the participation constraint $l^i \leq \hat{\alpha}^i$:

$$l^* = \arg\max_{l} \hat{u}(l) = \hat{e}(l)lv_{11}.$$

Denote $\hat{u}(l^*) = \hat{u}^*$ and $\hat{v}(l^*) = \hat{v}^*$. We will impose the following assumption in this section.[16]

Assumption 2. $\hat{u}(l) = l\hat{e}(l)v_{11}$ is strictly concave in l and the maximizer $l^* = \arg\max_l \hat{u}$ is an (unique) interior solution, $l^* \in (0, 1)$.

Given a refinement effort r and the associated patent power $\hat{\alpha}^1$ and $\hat{\alpha}^0$, with the opportunity of offering an *interim* license, the patent-holder's expected payoff depends

[15] When the patent-holder cannot verify the developer's capacity to adopt the technology and enter into the market, i.e., when the patent-holder cannot be sure that the developer will be a real threat to her monopoly, she may not want to engage in any licensing before the developer has "proven" himself and successfully completed the project.

[16] The first- and second-order conditions can be found in Section 3, $d\hat{u}/d\hat{\alpha}$ and $d^2\hat{u}/d\hat{\alpha}^2$, with $\hat{\alpha}$ replaced by l, and setting $\pi = v_{11}$ as well as $u_{11} = u_{10} = 0$. It is easy to check that the first-order condition is strictly positive at $l = 0$ and strictly negative at $l = 1$ (for $\hat{e}(1) = 0$). Again, the second-order condition is satisfied globally for quadratic c_E.

on whether l^* is feasible under the prevailing patent power. When $\hat{\alpha}^0 \geq l^*$, the patent-holder can offer l^* at both events $a^P = 0$ and 1, with an expected payoff

$$F(\hat{v}^*)\hat{u}^* - c_R.$$

The technology adopter will have a constant payoff $\hat{v}^*$, and given adoption, the patent-holder enjoys a constant licensing income $\hat{u}^*$. When $\hat{\alpha}^0 < l^* \leq \hat{\alpha}^1$, the optimal offer l^* is feasible only when the patent office issues a favorable decision $a^P = 1$; and when $a^P = 0$, the patent-holder can only charge a licensing term $\hat{\alpha}^0$ because of the binding participation constraint. The patent-holder's expected payoff is

$$F\big(q\hat{v}^* + (1-q)\hat{v}(\hat{\alpha}^0)\big)\left[q\hat{u}^* + (1-q)\hat{u}(\hat{\alpha}^0)\right] - c_R = F(V^{in.})u_\tau^{in.} - c_R,$$

where $V^{in.} \equiv [q\hat{v}^* + (1-q)\hat{v}(\hat{\alpha}^0)]$ and $u_\tau^{in.} \equiv [q\hat{u}^* + (1-q)\hat{u}(\hat{\alpha}^0)]$ are the developer's and patent-holder's post-adoption payoffs, respectively. It is easy to see that $V^{in} \geq V$ and $u_\tau^{in.} \geq u_\tau$; and when $\hat{\alpha}^0 < l < \hat{\alpha}^1$, we have strict inequality for both payoffs. For simplicity, we will again assume that the payoff $F(V^{in.})u_\tau^{in.} - c_R$ is strictly concave in r. And when $\hat{\alpha}^1 < l^*$, the participation constraint $l^i \leq \hat{\alpha}^i$ is binding for both $i \in \{0,1\}$. Whatever the patent-holder decision is, *interim* licensing is irrelevant, and the patent-holder gets the same payoff as before, $F(V)u_\tau - c_R$.

We are concerned with how the *interim* licensing opportunity will affect the patent-holder's refinement decision as well as technology adoption incentives. When $r = \underline{r}$, denote $\hat{\alpha}^1(\underline{r})$ as $\underline{\alpha}^1$ and $\hat{\alpha}^0(\underline{r})$ as $\underline{\alpha}^0$. It turns out that introducing this opportunity may eliminate the refinement effort from a patent-holder with high invention quality α, the one is supposed to have strong refinement incentives in the previous case.

Proposition 3. *(Interim licensing). Suppose that the patent-holder can offer a license after the patent office decision and before the developer incurs the specific investment.*

When α is large enough so that $\underline{\alpha}^0 \geq l^*$, the optimal refinement effort is either no refinement or large enough so that $\hat{\alpha}^0 < l^* < \hat{\alpha}^1$. And when

$$F'(\hat{v}^*)\hat{u}^*(1-q)\frac{\partial \hat{v}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\bigg|_{\hat{\alpha}^0 \to l^{*-}} \frac{\partial \hat{\alpha}^0}{\partial r} < c_R', \tag{3}$$

at r such that $\hat{\alpha}^0 \to l^{*-}$, interim licensing will mute any refinement effort by the patent-holder. Given $l^* = 1/2$, this condition is more likely to hold when α is higher.

When α is small enough so that $\underline{\alpha}^1 < l^*$, then interim licensing will not have significant negative impact on the refinement effort. That is, it is impossible to have the case where under interim licensing the optimal refinement is low enough so that

$\hat{\alpha}^1 < l^*$, but without interim licensing the refinement effort is high enough so that $\hat{\alpha}^0 < l^* < \hat{\alpha}^1$.

The analysis suggests that, *interim* licensing may not improve the clarity of the patent rights for a high quality invention (high α). Nevertheless, by alleviating the hold-up problem it can substitute for the patent-holder's refinement activity. To see this, recall that with quadratic investment, given adoption, the average probability that the developer will successfully conduct the investment project is $q\hat{e}^1+(1-q)\hat{e}^0 = (1-\alpha)Kv_{11}$. Fixing α and distribution $F(\cdot)$, the degree of technology is thus determined by the developer's expected payoff from using the new invention, $V(\alpha, r) = (1-\alpha)^2\Psi v_{11}^2 K/2$. By $\partial\Psi/\partial r > 0$, the upper bound of this payoff is thus $(1-\alpha)v_{11}^2 K/2$, when $r = 1$.

Suppose that α is so large that $\underline{\alpha}^0 \geq l^*$ and condition (3) holds, the patent-holder gives up any refinement activity under *interim* licensing. This is the case only if $\alpha > l^* = 1/2$. The optimal royalty $l^* = 1/2$ thus determines the developer's payoff, $\hat{v}^* = v_{11}^2 K/8$. Given adoption, the expected successful probability under *interim* licensing is higher than without it, $\hat{e}(l^*) = Kv_{11}/2 > (1-\alpha)Kv_{11}$, for $\alpha > 1/2$. And the developer's payoff will also be greater if α is large enough: $v_{11}^2 K/8 \geq (1-\alpha)v_{11}^2 K/2$ whenever $\alpha \geq 3/4$. In other words, for α large enough, *interim* licensing may eliminate refinement without hampering technology diffusion. Note that this comparison is made by using the highest refinement $r = 1$ when there is no *interim* licensing. Any lower r will only strengthen the result.

Remark. An interesting result shown in the proof of this proposition is how *interim* licensing may change the adoption *v.s.* revenue trade-off when $\alpha > 1/2$ and $\underline{\alpha}^0 < l^* < \underline{\alpha}^1$. When α falls in the middle range such that $\underline{\alpha}^0 < l^* < \underline{\alpha}^1$, the developer's and patent-holder's post-adoption payoffs are always $V^{in\cdot}$ and $u_\tau^{in\cdot}$, respectively. And when $\alpha > 1/2$, we show that $\partial V^{in\cdot}/\partial r > \partial V/\partial r$ and $\partial u_\tau^{in\cdot}/\partial r > \partial u_\tau/\partial r$, that is, *interim* licensing will increase the responsiveness of adoption to refinement while reducing the negative impact of refinement on the patent-holder's post-adoption payoff. Furthermore, with quadratic investment cost and so $l^* = 1/2$, we show that the latter effect may be so strong that $u_\tau^{in\cdot}$ may be *increasing* in r for r close to 1/2. In other words, there may be no more adoption *v.s.* revenue trade-off. ||

□ *Ex ante* **licensing:** Let us turn to *ex ante* licensing, which takes place before patent office decision. Referring to Figure 2, we can further distinguish between two cases: bargaining before or after the patent-holder incurs the refinement efforts. To focus on the change these licensing opportunities would bring, we do not allow *interim* licensing here.

When *ex ante* licensing still takes place after the patent-holder chooses the refinement level, first notice that, by assuming the adoption cost c_f as the technology adopter's private information, it doesn't matter for the patent-holder whether the license is offered before or after the technology adoption decision is made. Lacking other means to screen this information, at most one license will be accepted along the equilibrium path.[17] In addition, we assume that even if the adopter has taken the license, he can still walk away at no cost. That is, if he decides not to adopt the patent-holder's technology and remains inactive, then there is no way the patent-holder can impose any fine or extract any surplus from the adopter. This can be justified by the assumptions that the technology adopter is protected by limited liability, and that by remaining idle he gets zero payoff.

Let l^{**} be the optimal value of the following program:

$$l^{**} = \arg\max_{l} F\big(\hat{v}(l)\big)\hat{u}(l) \quad \Rightarrow \quad FOC: \; F'\hat{u}\frac{\partial \hat{v}}{\partial l} + F\frac{\partial \hat{u}}{\partial l} = 0.$$

That is, l^{**} is the optimal licensing term that maximizes the patent-holder's expected licensing income by taking into account technology adoption decision. (If the optimizers are not unique, choose the smallest one.) This is the optimal *ex ante* licensing term the patent-holder will offer when facing only the technology adoption issue.[18]

Because $F' > 0$ and $\partial\hat{v}/\partial l < 0$, the *ex ante* optimal royalty $l^{**} < l^{*}$. Bringing in the technology adoption concern will reduce the optimal royalty relative to the case where the adopter has already employed the patent-holder's invention.

Despite the opportunity of *ex ante* licensing, if he decides to use the patent-holder's invention the technology adopter can do so without taking the license and then resolving any patent disputes in court. When *ex ante* license is offered after the patent-holder's refinement decision, at the contracting stage a participation constraint facing the patent-holder is that the technology adopter's expected payoff by taking the *ex ante* license should be no lower than $V(\alpha, r)$, what he would expect by proceeding alone.

[17] If two licenses give different adoption payoffs, the developer will surely choose the one with higher payoff. And if two licenses leads to the same adoption payoff for the developer, and thus the same adoption probability, but different post-adoption revenue for the patent-holder, the patent-holder will again choose the one with higher revenue.

[18] For simplicity, we assume that the patent-holder cannot do better by offering random contracts, i.e., $(l^s)_{s=1,\dots,S}$ such that the royalty l^s is realized with probability $q^s \in (0,1)$, with $\sum_{s=1}^{S} q^s = 1$. By the same reasoning, this can be guaranteed by the same conditions as in Lemma 1 below.

Suppose that $\partial V(\alpha,r)/\partial r > 0$, as is the case under quadratic investment cost c_E. The payoff $V(\alpha,r)$ attains its minimal value at $r = \underline{r}$. It follows that, for α high enough such that $\underline{\alpha}^0 \geq l^{**}$, the optimal royalty l^{**} is implementable in the absence of *any* refinement effort, $\hat{v}(l^{**}) \geq \hat{v}(\underline{\alpha}^0) > V(\alpha,\underline{r})$. The patent-holder will have no incentives to make any refinement efforts at all. And this is true as long as $\hat{v}(l^{**}) \geq V(\alpha,\underline{r})$.

Now suppose that $\hat{v}(l^{**}) < V(\alpha,\underline{r})$, and so for all $r \in [\underline{r},1]$, $V(\alpha,r) > \hat{v}(l^{**})$, the adopter will not accept the royalty l^{**}. Intuitively, in this case more refinement effort will only increase the adopter's reservation value $V(\alpha,r)$ and reduce the feasible royalty terms. We should expect no refinement either.

To show this result, note that the patent-holder's *ex ante* license offering may contain two royalties, l^0 and l^1, contingent on the patent office decision $a^P \in \{0,1\}$. Given the refinement decision, the patent-holder can offer $(l^0,l^1) = (\hat{\alpha}^0,\hat{\alpha}^1)$ that fully replicates the outcome of no *ex ante* licensing. There is no loss of generality in assuming that the technology adopter, once decided to use the patent-holder's invention, will also take the *ex ante* license.

The following lemma nevertheless finds a sufficient condition under which it is optimal for the patent-holder to offer a fixed royalty $l^0 = l^1$. (Its proof is in Appendix A.) It requires that the program $\max_l \hat{u}(l)$ is well-behaved and the elasticity of investment effort $\hat{e}$ with respect to the royalty l,

$$\xi \equiv \frac{l}{\hat{e}}\frac{\partial \hat{e}}{\partial l} = -\frac{l}{\hat{e}}\frac{v}{c_E''(\hat{e})},$$

is strictly monotonic in l for $l \leq l^*$. Because

$$\frac{\partial \xi}{\partial l} = -\frac{v}{\hat{e}(c_E'')^2}\left[\hat{e}c_E'' - l\frac{\partial \hat{e}}{\partial l}(c_E'' + \hat{e}c_E''')\right],$$

the requirement on elasticity holds as long as the investment cost function satisfies $c_E''' \geq 0$, which is certainly true for the quadratic function.

Lemma 1. (Optimal ex ante license). Suppose that Assumption 2 holds. When the elasticity ξ is strictly monotonic in l for $l \leq l^*$, the optimal contract (l^0,l^1) that solves the program: $\forall q$,

$$\begin{aligned} \max_{(l^0,l^1)} \quad & q\hat{u}(l^1) + (1-q)\hat{u}(l^0) \\ s.t. \quad & q\hat{v}(l^1) + (1-q)\hat{v}(l^0) = \bar{v}, \end{aligned}$$

has the property that $l^0 = l^1$.

By this lemma, if the optimal license leaves the developer an expected adoption payoff $\bar{v}$, then the patent-holder will optimally construct the license such that the royalty is invariant in patent office decision, $l^0 = l^1$. Let $\tilde{l}$ be the optimal licensing offer, which must satisfy $\hat{v}(\tilde{l}) \geq V(\alpha, r)$, where r is the refinement effort chosen by the patent-holder at time 1. The patent-holder's expected payoff, when making the refinement decision, is[19]

$$F\big(\hat{v}(\tilde{l})\big)\hat{u}(\tilde{l}) - c_R, \quad s.t. \;\; \hat{v}(\tilde{l}) \geq V(\alpha, r).$$

Suppose that the refinement effort r and the subsequent *ex ante* license are both chosen at the optimal level. When the patent-holder exerts some refinement effort, $r \in (\underline{r}, 1)$, the developer's participation constraint must be binding, $\hat{v}(\tilde{l}) = V(\alpha, r)$. If not, $\hat{v}(\tilde{l}) > V(\alpha, r)$, then the patent-holder can marginally reduce the refinement effort, saving the refinement cost without jeopardizing her ability to offer the same $\tilde{l}$ and receive the same expected licensing income $F(\hat{v}(\tilde{l}))\hat{u}(\tilde{l})$.

Obviously, when the developer has a preference toward the clarity of the patent rights, $\partial V/\partial r > 0$, the patent-holder will optimally choose *not* to exert any refinement effort, the optimal $r = \underline{r}$. A higher r requires a larger cost c_R and will raise the developer's reservation value at the contracting stage, $V(\alpha, r)$, and thus reduce the feasible set of *ex ante* contracts. For the patent-holder, there is no gain in engaging in patent refinement.

Lastly, consider an *ex ante* license offered before the patent-holder makes the refinement effort. If at the contracting stage the patent-holder can commit to the refinement effort she'll exert later, then we go back to the previous case. A higher refinement effort will raise the adopter's reservation value $V(\alpha, r)$ at the contracting stage. Again there will be no refinement at all. Suppose that the patent-holder lacks this commitment power.

If the technology adopter refuses the patent-holder's licensing offer, then the game continues as if there is only *ex post* licensing. Denote $\hat{r}$ as the patent-holder's optimal refinement effort in that case. The technology adopter is willing to accept an *ex ante* license when his payoff is higher than $V(\alpha, \hat{r})$. An *ex ante* license that can fully mimic the outcome without *ex ante* licensing, and thus will be accepted by the developer is: a royalty $l^0 = \hat{\alpha}^0$ ($l^1 = \hat{\alpha}^1$) is imposed when the patent office issues a decision $a^P = 0$

[19] Notice that given r, and thus given q, a feasible contract that would leave the patent-holder the same payoff is $l^0 = \hat{\alpha}^0$ and $l^1 = \hat{\alpha}^1$. By Lemma 1, the patent-holder thus is strictly better off at the optimal *ex ante* contract.

($a^P = 1$, respectively). (And by Lemma 1, to keep the developer's adoption payoff at $V(\alpha, \hat{r})$, the patent-holder can do strictly better by offering a constant royalty.) It follows that *ex post* licensing will become an off-path event, and the patent-holder and developer will agree on an *ex ante* license. After the agreement, the patent-holder will no longer exert any refinement effort.

Again, no refinement effort from the patent-holder doesn't mean that the overall technology adoption rate will suffer. Indeed, in the case where the *ex ant* licensing takes place before the refinement decision,[20] the fact that the optimal *ex ante* license $\tilde{l}$ has to satisfy the participation constraint $\hat{v}(\tilde{l}) \geq V(\alpha, \hat{r})$ implies that the technology adoption probability will not be damaged by this earlier bargaining opportunity. Furthermore, in the case of quadratic investment cost, without *ex ant* licensing the probability that the developer will successfully conduct the investment project is $(1-\alpha)Kv_{11}$. And by the participation constraint: $\forall r > 1/2$,

$$\hat{v}(\tilde{l}) = (1-\tilde{l})^2 v_{11}^2 \frac{K}{2} \geq V(\alpha, \hat{r}) = (1-\alpha)^2 \Psi v_{11}^2 \frac{K}{2} \quad \Rightarrow \quad (1-\tilde{l})^2 \geq (1-\alpha)^2 \Psi > (1-\alpha)^2,$$

for $\partial \Psi / \partial r > 0$ whenever $r > 1/2$ and $\Psi = 1$ at $r = 1/2$. The successful probability, $\hat{e}(\tilde{l}) = (1-\tilde{l})v_{11}K$, thus, is strictly larger than that without *ex ante* licensing.

Proposition 4. (Ex ante licensing). Suppose that the paten-holder can offer a license before the patent office decision. When Assumption 2 holds and the elasticity ξ is strictly monotonic in l for $l \leq l^*$, regardless of the exact timing of ex ante licensing the inventor no incentives to exert any refinement effort.

When ex ante licensing takes place before the refinement decision, the overall probability that the developer will successfully conduct the downstream investment with the new invention is higher than that without ex ante licensing.

5 Multiple Patent-Holders

In this section we consider a simple symmetric multiple patent-holders case. Suppose that there are two patent-holders, P_1 and P_2, with identical *ex ante* invention quality α. The developer can decide whether to use both inventions or only one of it, and then exerts an unobservable investment effort $e \in [0,1]$ with a quadratic cost $c_E =$

[20] When *ex ante* licensing takes place after the refinement decision, the optimal royalty is either l^{**} (when $\hat{v}(l^{**}) \geq V(\alpha, \underline{r})$) or $\tilde{l}$ such that $\hat{v}(\tilde{l}) = V(\alpha, \underline{r})$ (when $\hat{v}(l^{**}) < V(\alpha, \underline{r})$). However, the optimal refinement without *ex ante* licensing, $\hat{r}$, maybe lead to $\hat{v}(l^{**}) < V(\alpha, \hat{r})$. We need further information about $\hat{r}$ to compare the effect on technology adoption.

$e^2/2K$. To incorporate P_i's invention, the developer has to incur an adoption cost c_i, $i \in \{1,2\}$. As in the basic model, c_i is the developer's private information, and *i.i.d.* over $[0,\infty)$ with CDF $F(\cdot)$. A successfully completed project (with probability e) generates a value $\bar{v}$ when both inventions are incorporated into the project, and $\rho\bar{v}$ when only one invention is adopted. Assume that $\rho \in [0,1]$. This parameter captures the substitutability between the two technologies. When $\rho = 0$, they are perfect complements; the developer needs to combine both into the downstream product. And when $\rho = 1$, they are perfectly substitutes; either of them generates the same revenue stream (if successfully developed), and there is no additional benefit to incorporate both technologies.

For simplicity, assume that both patent-holders are pure licensors and have no other income, and ignore *interim* and *ex ante* licensing opportunities. Denote $r_i \in [\underline{r}, 1]$ as the refinement effort exerted by P_i, with a common refinement cost c_R for both $i \in \{1,2\}$. The patent-holders make refinement decisions simultaneously, and the chosen r_1 and r_2 are observable to other players. Assume that the patent office makes independent decisions about the two patent applications. Let $q_i \equiv \alpha r_i + (1-\alpha)(1-r_i)$ be the probability that the patent office issues a favorable decision to P_i, and $(\hat{\alpha}_i^1, \hat{\alpha}_i^0)$ the pair of patent power, $i \in \{1,2\}$. Also assume that the court makes infringement judgement independently.

If the developer only adopts P_i's technology, $i \in \{1,2\}$, then everything goes as in the case of pure licensor in Section 3, with $v_{11} = \rho\bar{v}$. Given quadratic investment cost, let

$$\Psi_i \equiv \frac{(1-r_i)^2}{q_i} + \frac{r_i^2}{1-q_i} \quad \text{and} \quad \Lambda_i \equiv \frac{r_i(1-r_i)}{q_i(1-q_i)}, \quad i \in \{1,2\}.$$

The post-adoption payoffs are

$$V_i = \Psi_i(1-\alpha)^2\rho^2\bar{v}^2\frac{K}{2}$$

for the developer, and

$$u_\tau^i = \Lambda_i\alpha(1-\alpha)\rho^2\bar{v}^2K$$

for the patent-holder P_i whose invention is used into the development project. The patent-holder P_{-i} whose invention is not used receives no return.

If the developer incorporates both technologies, then he is subject to both patent-holders' patent rights. Given the two patentees' patent power $\hat{\alpha}_1$ and $\hat{\alpha}_2$, the developer chooses the investment e to maximize

$$\hat{\bar{v}}(\hat{\alpha}_1, \hat{\alpha}_2) \equiv \max_e (1-\hat{\alpha}_1)(1-\hat{\alpha}_2)e\bar{v} - c_E.$$

The post-adoption payoff is

$$\bar{V} \equiv q_1 q_2 \hat{\bar{v}}(\hat{\alpha}_1^1, \hat{\alpha}_2^1) + q_1(1-q_2)\hat{\bar{v}}(\hat{\alpha}_1^1, \hat{\alpha}_2^0) + (1-q_1)q_2\hat{\bar{v}}(\hat{\alpha}_1^0, \hat{\alpha}_2^1) + (1-q_1)(1-q_2)\hat{\bar{v}}(\hat{\alpha}_1^0, \hat{\alpha}_2^0).$$

With quadratic investment cost, $\hat{e} = (1-\hat{\alpha}_1)(1-\hat{\alpha}_2)K\bar{v}$ and $\hat{\bar{v}} = (1-\hat{\alpha}_1)^2(1-\hat{\alpha}_2)^2\bar{v}^2K/2$, and, after some computation,

$$\begin{aligned} \bar{V} &= [q_1(1-\hat{\alpha}_1^1)^2 + (1-q_1)(1-\hat{\alpha}_1^0)^2][q_2(1-\hat{\alpha}_2^1)^2 + (1-q_2)(1-\hat{\alpha}_2^0)^2]\frac{K}{2}\bar{v}^2 \\ &= \Psi_1\Psi_2(1-\alpha)^4\frac{K}{2}\bar{v}^2. \end{aligned}$$

To derive the patent-holders' post-adoption payoffs, suppose that if the developer infringes on both patents, then the surplus $\bar{v}$ is equally shared by both patentees. Given a pair of patent power $(\hat{\alpha}_1, \hat{\alpha}_2)$ and the corresponding probability of success $\hat{e}(\hat{\alpha}_1, \hat{\alpha}_2)$, the patent-holder P_i expects an *ex post* licensing payment

$$\hat{e}(\hat{\alpha}_i, \hat{\alpha}_{-i})\left[\hat{\alpha}_i\hat{\alpha}_{-i}\frac{\bar{v}}{2} + \hat{\alpha}_i(1-\hat{\alpha}_{-i})\bar{v}\right] = \hat{e}(\hat{\alpha}_i, \hat{\alpha}_{-i})\bar{v}\hat{\alpha}_i\left(1-\frac{\hat{\alpha}_{-i}}{2}\right),\ i,-i \in \{1,2\}, i \neq -i.$$

P_i's post-adoption payoff is

$$\begin{aligned} \bar{u}_\tau^i &= \bar{v}\Bigg\{q_i\hat{\alpha}_i^1\left[q_{-i}(1-\frac{\hat{\alpha}_{-i}^1}{2})\hat{e}(\hat{\alpha}_i^1, \hat{\alpha}_{-i}^1) + (1-q_{-i})(1-\frac{\hat{\alpha}_{-i}^0}{2})\hat{e}(\hat{\alpha}_i^1, \hat{\alpha}_{-i}^0)\right] \\ &\quad +(1-q_i)\hat{\alpha}_i^0\left[q_{-i}(1-\frac{\hat{\alpha}_{-i}^1}{2})\hat{e}(\hat{\alpha}_i^0, \hat{\alpha}_{-i}^1) + (1-q_{-i})(1-\frac{\hat{\alpha}_{-i}^0}{2})\hat{e}(\hat{\alpha}_i^0, \hat{\alpha}_{-i}^0)\right]\Bigg\} \\ &= \alpha(1-\alpha)^2\bar{v}^2K\Lambda_i\left(1-\frac{\alpha}{2}\Lambda_{-i}\right), \end{aligned}$$

with quadratic investment cost. With these payoffs, let us consider in turn the developer's technology choice and patent-holders' refinement decisions.

Given adoption cost c_1 and c_2, the developer will incorporate both technologies into the development project if[21]

$$\bar{V} - (c_1 + c_2) > \max\{V_1 - c_1, V_2 - c_2, 0\},$$

which requires that $\bar{V} - V_1 > c_2$ and $\bar{V} - V_2 > c_1$. And he will either choose only one technology, or none at all when $\bar{V} - (c_1 + c_2) \leq \max\{V_1 - c_1, V_2 - c_2, 0\}$.

[21]We maintain the assumption that the developer will adoption a technology only when the payoff is strictly higher from doing so.

Replacing with the payoffs derived above, adopting both technologies requires

$$\Psi_1(1-\alpha)^2\frac{K}{2}\bar{v}^2[\Psi_2(1-\alpha)^2-\rho^2] > c_2 \text{ and } \Psi_2(1-\alpha)^2\frac{K}{2}\bar{v}^2[\Psi_1(1-\alpha)^2-\rho^2] > c_1.$$

And a necessary condition is

$$\min\{\Psi_1, \Psi_2\} > \left(\frac{\rho}{1-\alpha}\right)^2. \tag{4}$$

This condition is more likely to fail whenρ is higher, i.e., there is less complementary between the two technologies, or when the *ex ante* invention quality or patent protection is higher, for

$$\frac{\partial}{\partial\alpha}(1-\alpha)^2\Psi_i < 0, \quad i \in \{1,2\},$$

by the sam reason as $\partial V/\partial\alpha < 0$ in Section 3. More importantly, refinement efforts also have an impact. Condition (4) is less likely to hold when patent-holders exert less refinement efforts, for $\partial\Psi_i/\partial r_i > 0$, $i \in \{1,2\}$. Denote $\underline{\psi} > 1$ as the lowest possible value of Ψ_i, which happens when $r_i = \underline{r}$, then $\Psi_i \in [\underline{\psi}, 1/1-\alpha]$, $i \in \{1,2\}$. When

$$\underline{\psi} < \left(\frac{\rho}{1-\alpha}\right)^2 < \frac{1}{1-\alpha}, \tag{5}$$

the developer will not use both technologies without sufficient refinement efforts from both patent-holders. Even when there is significant synergy between the two technologies (ρ is low), as long as each invention has some stand-alone value (and so $\rho \neq 0$), refinement is required for efficient technology exploitation. Furthermore, by

$$\begin{aligned}\frac{\partial(\bar{V}-V_i)}{\partial r_i} &= (1-\alpha)^2\frac{K}{2}\bar{v}^2[\Psi_{-i}(1-\alpha)^2-\rho^2]\frac{\partial\Psi_i}{\partial r_i}, \quad \text{and}\\ \frac{\partial(\bar{V}-V_i)}{\partial r_{-i}} &= (1-\alpha)^4\frac{K}{2}\bar{v}^2\Psi_i\frac{\partial\Psi_{-i}}{\partial r_{-i}}, \quad i,-i \in \{1,2\}, i \neq -i,\end{aligned}$$

when condition (4) holds, higher refinement efforts from either party will increase both $\bar{V}-V_1$ and $\bar{V}-V_2$ and thus the chance that the developer will adopt both technologies.

Proposition 5. *(Technological synergy and refinement). Consider the case of symmetric multiple patent-holders with quadratic investment cost, and suppose that patentees only get returns from ex post licensing. When each technology has stand-alone value ($\rho \neq 0$) and the invention quality α is not too low, such that condition (5) holds, technological synergy ($\rho < 1$) by itself cannot induce the developer to adopt both technologies. The developer will use both technologies only when refinement efforts r_1 and r_2 are high enough.*

Next let us consider the patent-holders' refinement decisions. Given the developer's technology choice, a patent-holder P_i's expected payoff from refinement is

$$\begin{aligned} U_i &= \Pr\left(\bar{V} - (c_1 + c_2) > \max\{V_1 - c_1, V_2 - c_2, 0\}\right) \bar{u}^i_\tau \\ &\quad + \Pr\left(V_i - c_i \geq \max\{\bar{V} - (c_1 + c_2), V_{-i} - c_{-i}, 0\}\right) u^i_\tau - c_R(r_i). \end{aligned}$$

That is, when $\bar{V} - (c_1 + c_2) > \max\{V_1 - c_1, V_2 - c_2, 0\}$, the developer will use both technologies and P_i gets a post-adoption payoff $\bar{u}^i_\tau$; and when $V_i - c_i \geq \max\{\bar{V} - c_1 - c_2, V_{-i} - c_{-i}, 0\}$, the developer chooses only her technology and she gets a payoff u^i_τ. Otherwise either the developer chooses only the other patentee's technology or none at all, and in both events P_i receives no return. To save the space, define adoption probabilities by:

$$\begin{aligned} &\bar{p} \equiv \Pr\left(\bar{V} - (c_1 + c_2) > \max\{V_1 - c_1, V_2 - c_2, 0\}\right), \quad \text{and} \\ &p_i \equiv \Pr\left(V_i - c_i \geq \max\{\bar{V} - (c_1 + c_2), V_{-i} - c_{-i}, 0\}\right), \quad i, -i \in \{1, 2\}, i \neq -i. \end{aligned}$$

Our interests here is the strategic relationship between the two patent-holders' refinement decisions. But instead of a full-fledge analysis, for simplicity let use consider two special cases: perfect complements ($\rho = 0$) and perfect substitutes ($\rho = 1$). We also assume uniform distribution for the technology adoption cost over the support $[0, 1/\nu]$, with ν small enough. The CDF is thus $F(c) = \nu c$ for $c \in [0, 1/\nu]$.

When $\rho = 1$ and the two technologies are perfect substitutes, by $\Psi_i \leq 1/1 - \alpha$, condition (4) is violated for any level of refinement effort and so the developer will never consider using both technologies.[22] The two patent-holders compete against each other for the developer's adoption of their own technology. The patentee P_i's payoff from refinement is $U_i = p_i u^i_\tau - c_R(r_i)$. On the other hand, when $\rho = 0$ and the two technologies are perfect complements, the developer will consider either using both inventions or note. The patent-holder P_i's expected payoff from refinement is $U_i = \bar{p}\bar{u}^i_\tau - c_R(r_i)$. The following proposition shows that refinement decisions tend to be strategic complements in both cases. (See Appendix A for proof.) Again, for simplicity, we assume that c''_R is large enough so that the patent-holder's expected payoff is strictly concave in refinement effort.

Proposition 6. *(Strategic dependence of refinement decisions). Consider the case of two symmetric patent-holders with only ex post licensing. Assume that investment cost takes a quadratic form, and adoption cost has a uniform distribution.*

[22] In fact, by condition (5) as long as $\rho^2 \geq 1 - \alpha$ the option of adopting both technologies is irrelevant.

Suppose that $\rho = 1$ and so the two technologies are perfect substitutes. When $r_i \geq r_{-i}$ and so $V_i \geq V_{-i}$, the patent-holder P_i's optimal refinement r_i is increasing in r_{-i}, i, $-i \in \{1,2\}$, and $i \neq -i$. And when

$$\frac{\alpha}{(1-q_i)(1-r_i)^2 + q_i r_i^2} < \frac{1-\alpha}{r_i(1-r_i)}, \quad \forall r_i \in [\underline{r}, 1],$$

then refinement decisions are strategic complements to each other. This condition is more likely to hold when α is lower, and will be satisfied when $\alpha \leq 1/2$.

Suppose that $\rho = 0$ and so the two technologies are perfect complements. When the patent-holders have incentives to exert some refinement effort, the optimal r_1 and $r_2 > \underline{r}$, then the two patent-holders refinement decisions are strategic complements.

As shown in Section 4, with multiple patent-holders one might expect the patent-holders to use *interim* or *ex ante* licensing to mitigate the hold-up problem. However, to the extent that earlier licensing is absent and so hold-up threat is severe, Proposition 6 has some interesting policy implication. When refinement decisions are strategic complements, inducing any patent applicant to exercise more refinement effort would have the snowballing effect of increasing the total refinement efforts and thus significantly reduce the overall uncertainty. It would be worthwhile to introduce an incentive scheme to encourage private refinement. And even if the introduced policy only directly affects some, not all, patent-holders, through strategic complementarity the indirect impact could be sizable. (On the other hand, in the case of strategic substitutes, one applicant's higher refinement effort would reduce that of other applicants. Such an incentive scheme has to be carefully designed to take into account this negative effect.)

6 Concluding Remarks

In this paper, we investigated a patent-holder's incentives to reduce the uncertainty surrounding her patent rights. We showed that, under some mild conditions, the patent-holder's refinement effort is motivated by technology diffusion concerns. But even when the patent-holder acts as a pure licensor, and thus has no other income than licensing payment, we found that private refinement effort may vanish when the patent-holder has a sufficiently low quality invention (low α). The last result clearly echoes the general perception in software or e-business industry, where it is claimed that patents have been issued to technologies already in the public domain and the patent office examination fails to clear the enforceability of these patents in court. Earlier licensing, in particular at the *ex ante* stage, could alleviate the hold-up problem and fully substitute the patent-holder's refinement effort without jeopardizing technology diffusion. This result implies that if we rely on early licensing to mitigate hold-up, then fuzzy patent rights is one thing we have to live with.

For future research, the strong assumptions imposed in our model provide quite a few natural extensions. First, the treatment of the multiple patent-holder case in Section 5 is far from complete. Besides deriving the strategic relationship of refinement decisions for other cases of ρ, i.e., the degree of substitutability between the two inventions, it'd be important to let patent-holders offer *interim* or even *ex ante* licenses, jointly or separately. Allowing licensing would contribute to the literature of standard-setting organization and patent pools and deepen our understanding of how these collective rights organizations, or IPR clearinghouses, would affect the overall performance of the patent system. For instance, adding refinement decisions before the formation of a patent pool might help pin down whether the demand or competition margin will bind, and thus whether a patent pool is pro-competitive (Lerner and Tirole, 2004).

Second, we may introduce asymmetric information into the model, for instance, by considering a case where the patent-holder has private information about the invention quality α. The observable refinement effort then serves a signaling function. Because the technology adopter's adoption payoff is decreasing in the invention quality, $\partial V/\partial\alpha < 0$, to encourage adoption a patent-holder might refrain from refinement with an attempt to convince the adopter that she has a very low quality invention, i.e., her α is so low that condition (2) fails. On the other hand, when refinement cannot fully reveal the patent-holder's private information (i.e., when the prevailing equilibrium is not a separating equilibrium), asymmetric information persists at the subsequent licensing bargaining. At that stage, the patent-holder might have an opposite interest and want the developer to believe that she has a higher α in order to extract a more favorable licensing term. It would then be interesting to see how these two forces shape the refinement incentives and licensing outcomes.

Lastly, despite the positive results obtained in Section 4, it might be too optimistic to assume that early (*ex ante*) licensing is always available.[23] And it may well happen

[23]For instance, in the U.S. early publication is not mandatory as long as there are (or will be) no foreign equivalent applications filed in countries where early publication will take place. In this case, a potential developer will have difficulty searching for relevant patent applications and seeking a license before patents are granted. Even under early publication requirement (e.g. European Union and Japan), it typically happens eighteen months after the inventor files the patent application. Because the twenty-year patent

that not all potential developers have the same access to *ex ante* licensing. When it is not available, our results suggest a policy response to let public decision-makers (the patent office or court) be the primary source to provide certainty of the patent system.

To introduce the patent office examiner into the model, we may also want to relax the assumption that the inventor's refinement effort has unbiased effect, $r = \Pr.(a^P = a^C)$ for both $a^C \in \{0, 1\}$. It would be interesting to consider a patent applicant's biased incentives to improve the patent power $\hat{\alpha}^1$ without jeopardizing $\hat{\alpha}^0$, when the patent office issues an unfavorable decision (Meurer and Bessen, 2006). If this is true, and if the patent examination remains an *ex parte* procedure, then patent examiners should be provided with adequate incentives to counter this bias and raise the informativeness of issued patents in the other side. That is, we might consider introducing some "advocacy" elements into patent examination, with the applicant and examiner searching for information to support opposite causes (Dewatripont and Tirole, 1999).

Concerning the role of the court, similar extension could be introduced at the litigation stage. That is, we could consider a situation where both a^P and a^C are signals about a "true" patent boundary, which has a true value 1 (i.e., the patent-holder should get the exclusive power of the technology) with probability α. The probability distribution of a^P is affected by the applicant's as well as the examiner's efforts, while that of a^C is affected by the litigation inputs of patent-holder and potential infringer or patent challenger in court. We can then use this framework to re-examine the "rational ignorance" hypothesis and consider the optimal division of labor between the patent office and private challengers to improve the performance of the patent system (Lemley, 2001; Chiou, 2008).

Appendix

A Proofs

□ Proposition 3

Proof. Let us consider three cases according to the level of α. When $\underline{\alpha}^0 \geq l^*$, i.e., the invention quality α is high enough so that at $r = \underline{r}$, the patent power is strong enough to support the optimal royalty l^* even when the patent office issues an unfavorable

term begins with the filing date, a patent-holder may not want to wait until after publication to start the examination process.

decision $a^P = 0$. (A necessary condition is $\alpha \geq l^*$.) The patent-holder's expected payoff from refinement is[24]

$$U^{in.} = \begin{cases} F(\hat{v}^*)\hat{u}^* - c_R, & \text{if } \; l^* \leq \hat{\alpha}^0 < \hat{\alpha}^1, \\ F(V^{in.})u_\tau^{in.} - c_R, & \text{if } \; \hat{\alpha}^0 < l^* < \hat{\alpha}^1. \end{cases}$$

When the refinement effort is so low that $l^* \leq \hat{\alpha}^0 \leq \underline{\alpha}^0$, the patent-holder can charge l^* regardless of the patent office decision. And if the patent-holder chooses r large enough such that $\hat{\alpha}^0 < l^* < \hat{\alpha}^1$, then l^* is feasible only when the patent office issues a favorable decision to the patent-holder, $a^P = 1$.

To find the optimal r, first notice that when $l^* \leq \hat{\alpha}^0$, the revenue is not affected by r, and so the optimal $r = \underline{r}$ over this range. It follows that if $\partial U^{in.}/\partial r \leq 0$ as $\hat{\alpha}^0 \to l^{*-}$:

$$\begin{aligned} \left.\frac{\partial U^{in.}}{\partial r}\right|_{\hat{\alpha}^0 \to l^{*-}} &= F'\big[q\hat{u}^* + (1-q)\hat{u}(\hat{\alpha}^0)\big]\left[(\hat{v}^* - \hat{v}(\hat{\alpha}^0))\frac{\partial q}{\partial r} + (1-q)\frac{\partial \hat{v}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\frac{\partial \hat{\alpha}^0}{\partial r}\right] \\ &\quad + F\left[(\hat{u}^* - \hat{u}(\hat{\alpha}^0))\frac{\partial q}{\partial r} + (1-q)\frac{\partial \hat{u}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\frac{\partial \hat{\alpha}^0}{\partial r}\right] - c_R' \\ &= F'(\hat{v}^*)\hat{u}^*(1-q)\left.\frac{\partial \hat{v}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\right|_{\hat{\alpha}^0 \to l^{*-}} \frac{\partial \hat{\alpha}^0}{\partial r} - c_R' \leq 0, \end{aligned}$$

then under *interim* licensing the patent-holder will not exert any refinement effort, the optimal $r = \underline{r}$.

Condition (3) is a sufficient, but not necessary, condition to reach no refinement from the patent-holder. If it fails, we need to compare

$$F(\hat{v}^*)\hat{u}^* \gtrless \max_r F(V^{in.})u_\tau^{in.} - c_R$$

to determine the optimal r. The optimal r then exhibits a "bang-bang" property: It is either $\underline{r}$ or "jumps" to some level large enough such that $\hat{\alpha}^0 < l^* < \hat{\alpha}^1$.

Consider when condition (3) is more likely to hold. A higher α will require a larger r to reach the same level of $\hat{\alpha}^0$, i.e.,

$$\frac{\partial \hat{\alpha}^0}{\partial \alpha}d\alpha + \frac{\partial \hat{\alpha}^0}{\partial r}dr = 0 \quad \Rightarrow \quad \frac{dr}{d\alpha} = -\frac{\partial \hat{\alpha}^0/\partial \alpha}{\partial \hat{\alpha}^0/\partial r} = \frac{r(1-r)}{\alpha(1-\alpha)} > 0.$$

On the revenue side, when evaluating at $\hat{\alpha}^0 = l^*$,

$$-(1-q)\frac{\partial \hat{\alpha}^0}{\partial r} = \frac{\alpha(1-\alpha)}{1-q} = \hat{\alpha}^0\frac{1-\alpha}{1-r} = l^*\frac{1-\alpha}{1-r},$$

[24] Because at $\underline{r} > 1/2$, $\underline{\alpha}^1 > \underline{\alpha}^0 \geq l^*$, further increase r will cause $\hat{\alpha}^1 > l^*$.

thus to keep $\hat{\alpha}^0 = l^*$,

$$\frac{\partial}{\partial \alpha}(\frac{1-\alpha}{1-r})d\alpha + \frac{\partial}{\partial r}(\frac{1-\alpha}{1-r})dr = \frac{-1}{1-r}\left(1 + \frac{1-\alpha}{1-r}\frac{\partial \hat{\alpha}^0/\partial \alpha}{\partial \hat{\alpha}^0/\partial r}\right)d\alpha = \frac{-d\alpha}{1-r}\left(1 - \frac{r}{\alpha}\right).$$

And from $\hat{\alpha}^0 = l^*$, we can find $\alpha = rl^*/[rl^* + (1-r)(1-l^*)]$, and therefore $\alpha \geq r$ if and only if $l^* \geq 1/2$ (with quadratic investment cost, $l^* = 1/2$). We can conclude that when $l^* \geq 1/2$, condition (3) is more likely to hold for high values of α.

Next, when α is in an intermediate range such that $\underline{\alpha}^0 < l^* \leq \underline{\alpha}^1$, then for all $r \geq \underline{r}$, the patent-holder's payoff is

$$U^{in.} = F(V^{in.})u_\tau^{in.} - c_R,$$

for a higher r will still cause l^* to stay in the open interval $(\hat{\alpha}^0, \hat{\alpha}^1)$. The first-order conditio to determine the optimal refinement effort is

$$\begin{aligned}\frac{\partial U^{in.}}{\partial r} &= F' \cdot \left[q\hat{u}^* + (1-q)\hat{u}(\hat{\alpha}^0)\right]\left[(\hat{v}^* - \hat{v}(\hat{\alpha}^0))\frac{\partial q}{\partial r} + (1-q)\frac{\partial \hat{v}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\frac{\partial \hat{\alpha}^0}{\partial r}\right] \\ &\quad + F \cdot \left[(\hat{u}^* - \hat{u}(\hat{\alpha}^0))\frac{\partial q}{\partial r} + (1-q)\frac{\partial \hat{u}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\frac{\partial \hat{\alpha}^0}{\partial r}\right] - c_R'.\end{aligned}$$

Comparing with the case where only *ex post* licensing is allowed, the opportunity of *interim* licensing raises both the adopter's and patent-holder's expected payoffs upon adoption. It also changes how these payoffs will respond to the refinement effort,

$$\begin{aligned}\frac{\partial V^{in}}{\partial r} - \frac{\partial V}{\partial r} &= \left[\hat{v}^* - \hat{v}(\hat{\alpha}^1)\right]\frac{\partial q}{\partial r} - q\frac{\partial \hat{v}(\hat{\alpha}^1)}{\partial \hat{\alpha}^1}\frac{\partial \hat{\alpha}^1}{\partial r}, \\ \frac{\partial u_\tau^{in}}{\partial r} - \frac{\partial u_\tau}{\partial r} &= \left[\hat{u}^* - \hat{u}(\hat{\alpha}^1)\right]\frac{\partial q}{\partial r} - q\frac{\partial \hat{u}(\hat{\alpha}^1)}{\partial \hat{\alpha}^1}\frac{\partial \hat{\alpha}^1}{\partial r}.\end{aligned}$$

When $\alpha \geq 1/2$ and so $\partial q/\partial r \geq 0$, *interim* licensing will make the developer's payoff more responsive to the refinement effort, $\partial V^{in}/\partial r > \partial V/\partial r$, and reduce the negative impact of refinement effort on the patent-holder's licensing income, $\partial u_\tau^{in.}/\partial r > \partial u_\tau/\partial r$. In fact, it may even happen that the refinement effort will *increase* licensing income, $\partial u_\tau^{in.}/\partial r > 0$. To see this, consider the case of quadratic investment cost, under which the optimal royalty is $l^* = 1/2$ and so $\hat{u} = Kv_{11}^2/4$. The expected licensing payment is

$$u_\tau^{in.} = \left[\frac{q}{4} + (1-q)\hat{\alpha}^0(1-\hat{\alpha}^0)\right]Kv_{11}^2 = \left[\frac{q}{4} + \alpha(1-\alpha)\frac{r(1-r)}{1-q}\right]Kv_{11}^2.$$

The impact of r on u_τ^{in} is proportional to

$$\frac{\partial}{\partial r}\left[\frac{q}{4} + \alpha(1-\alpha)\frac{r(1-r)}{1-q}\right] = \frac{\alpha - (1-\alpha)}{4} + \frac{\alpha(1-\alpha)}{(1-q)^2}[(1-r-r)(1-q) + r(1-r)(\alpha - (1-\alpha))],$$

which is positive when $\alpha > 1/2$ and $r \to 1/2$ (assuming $\underline{r}$ is sufficiently close to 1/2). In this case, the adoption *v.s.* revenue tradeoff will disappear, and the patent-holder will have higher incentives to engage in refinement.

Lastly, when α is small enough so that $\underline{\alpha}^1 < l^*$, the patent-holder's expected payoff from refinement is

$$U^{in.} = \begin{cases} F(V)u_\tau - c_R, & \text{if} \quad l^* > \hat{\alpha}^1 > \hat{\alpha}^0, \\ F(V^{in.})u_\tau^{in.} - c_R, & \text{if} \quad \hat{\alpha}^0 < l^* \leq \hat{\alpha}^1. \end{cases}$$

With low refinement *interim* licensing is irrelevant regardless of the patent office decision. And only r is large enough so that $\hat{\alpha}^0 < l^* \leq \hat{\alpha}^1$ could the patent-holder offer l^* after a decision $a^P = 1$.

The same as in the high α case, we can check the first-order condition at the boundary between the two regimes to gain some information about the optimal refinement effort. Here it should be evaluated at $\hat{\alpha}^1 \to l^{*+}$:

$$\begin{aligned} \left.\frac{\partial U^{in.}}{\partial r}\right|_{\hat{\alpha}^1 \to l^{*+}} &= F' \cdot \left[q\hat{u}^* + (1-q)\hat{u}(\hat{\alpha}^0)\right]\left[(\hat{v}^* - \hat{v}(\hat{\alpha}^0))\frac{\partial q}{\partial r} + (1-q)\frac{\partial \hat{v}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\frac{\partial \hat{\alpha}^0}{\partial r}\right] \\ &\quad + F \cdot \left[(\hat{u}^* - \hat{u}(\hat{\alpha}^0))\frac{\partial q}{\partial r} + (1-q)\frac{\partial \hat{u}(\hat{\alpha}^0)}{\partial \hat{\alpha}^0}\frac{\partial \hat{\alpha}^0}{\partial r}\right] - c_R'. \end{aligned} \tag{6}$$

If this term is (weakly) negative, then the optimal refinement effort falls into the lower end. Furthermore, comparing the first-order conditions with and without *interim* licensing at $\hat{\alpha}^1 \to l^{*+}$ shows that

$$\left.\frac{\partial U^{in.}}{\partial r}\right|_{\hat{\alpha} \to l^{*+}} - \left.\frac{\partial U}{\partial r}\right|_{\hat{\alpha} = l^*} = -F' \cdot \left[q\hat{u}^* + (1-q)\hat{u}(\hat{\alpha}^0)\right] q \left.\frac{\partial \hat{v}(\hat{\alpha}^1)}{\partial \hat{\alpha}^1}\right|_{\hat{\alpha} = l^*} \frac{\partial \hat{\alpha}^1}{\partial r} > 0.$$

The first-order condition jumps upward at the point $\hat{\alpha}^1 = l^*$. We can conclude that in this case *interim* licensing will not discourage too much the refinement effort, if ever. That is, if without *interim* licensing the optimal $\hat{r}$ is large enough so that $\hat{\alpha}^0 < l^* < \hat{\alpha}^1$, then *interim* licensing will maintain a refinement effort such that the relation $\hat{\alpha}^0 < l^* < \hat{\alpha}^1$ still holds. Conversely, when expression (6) is (weakly) negative so that the optimal refinement effort under *interim* licensing falls into the low range such that $l^* > \hat{\alpha}^1 > \hat{\alpha}^0$, then it would already be the case without *interim* licensing. *Q.E.D.*

□ Lemma 1

Proof. First notice that the optimal $l^i \leq l^*$, $i \in \{0, 1\}$. For if not, some $l^i > l^*$ and so $\partial \hat{u}/\partial l^i < 0$ (by the global concavity of $\hat{u}$ in l), then a small reduction in l^i will increase both the patent-holder's and technology adopter's payoffs.

Now, suppose that $l^0 \neq l^1$. Consider a small change of royalties, (dl^0, dl^1), that keeps the constraint satisfied:

$$q\frac{\partial \hat{v}}{\partial l^1}dl^1 + (1-q)\frac{\partial \hat{v}}{\partial l^0}dl^0 = 0 \quad \Rightarrow \quad dl^0 = -\frac{q(\partial \hat{v}/\partial l^1)}{(1-q)(\partial \hat{v}/\partial l^0)}dl^1.$$

The impact of these changes on the objective function is:

$$q\frac{\partial \hat{u}}{\partial l^1}dl^1 + (1-q)\frac{\partial \hat{u}}{\partial l^0}dl^0 = qdl^1\left(\frac{\partial \hat{u}}{\partial l^1} - \frac{\partial \hat{u}}{\partial l^0}\frac{\partial \hat{v}/\partial l^1}{\partial \hat{v}/\partial l^0}\right) = qdl^1\frac{\partial \hat{u}}{\partial l^0}\left(\frac{\partial \hat{u}/\partial l^1}{\partial \hat{u}/\partial l^0} - \frac{\partial \hat{v}/\partial l^1}{\partial \hat{v}/\partial l^0}\right),$$

where

$$\frac{\partial \hat{u}}{\partial l} = (\hat{e} + l\frac{\partial \hat{e}}{\partial l})v \text{ and } \frac{\partial \hat{v}}{\partial l} = -\hat{e}v.$$

When both l^0 and l^1 are in the interval $(0, l^*] \subset (0,1)$ but $l^0 \neq l^1$, say, $l^0 < l^1$, then

$$\begin{aligned} & \frac{\partial \hat{u}(l^1)/\partial l^1}{\partial \hat{u}(l^0)/\partial l^0} - \frac{\partial \hat{v}(l^1)/\partial l^1}{\partial \hat{v}(l^0)/\partial l^0} = \frac{\hat{e}(l^1) + l^1\frac{\partial \hat{e}(l^1)}{\partial l^1}}{\hat{e}(l^0) + l^0\frac{\partial \hat{e}(l^0)}{\partial l^0}} - \frac{\hat{e}(l^1)}{\hat{e}(l^0)} \\ = \quad & \frac{1}{\hat{e}(l^0)(\partial \hat{u}/\partial l^0)}\left[\hat{e}(l^0)l^1\frac{\partial \hat{e}(l^1)}{\partial l^1} - \hat{e}(l^1)l^0\frac{\partial \hat{e}(l^0)}{\partial l^0}\right] = \frac{\hat{e}(l^1)}{\partial \hat{u}/\partial l^0}[\xi(l^1) - \xi(l^0)]. \end{aligned}$$

Therefore when $\xi(l)$ is strictly monotonic in l in this range, whenever $l^0 \neq l^1$, there is a pair of feasible changes (dl^0, dl^1) that can increase the value of the objective function. It can't be optimal.

If some element, say, l^0 hits the boundary point 0, i.e., $l^0 = 0 < l^1 \leq l^*$, then

$$\hat{e}(l^0)l^1\frac{\partial \hat{e}(l^1)}{\partial l^1} - \hat{e}(l^1)l^0\frac{\partial \hat{e}(l^0)}{\partial l^0} = \hat{e}(l^0)l^1\frac{\partial \hat{e}(l^1)}{\partial l^1} < 0,$$

thus a feasible pair of changes that will increase the patent-holder's payoff is $dl^1 < 0$ and dl^0 such that the constraint still holds. *Q.E.D.*

□ Proposition 6

Proof. First consider the case of $\rho = 1$, when technologies are perfect substitutes. The patentee P_i's payoff is $U_i = p_i u_\tau^i - c_R$, with first-order condition:

$$\frac{\partial p_i}{\partial r_i}u_\tau^i + p_i\frac{\partial u_\tau^i}{\partial r_i} = c_R'(r_i).$$

By the assumption of large enough c_R'' and so the global concavity of the objective function, and because u_τ^i is not affected by r_{-i}, the sign of dr_i/dr_{-i} is the same as

$$\frac{\partial^2 p_i}{\partial r_{-i}\partial r_i}u_\tau^i + \frac{\partial p_i}{\partial r_{-i}}\frac{\partial u_\tau^i}{\partial r_i}, \tag{7}$$

where $\partial u^i_\tau / \partial r_i < 0$. Intuitively, a higher refinement by the other patentee will reduce the probability that the developer will use P_i's technology, $\partial p_i / \partial r_{-i} < 0$. And so if the second term dominates, the optimal r_i is increasing in r_{-i}. We consider two cases according to whether $r_i \geq r_{-i}$ or $r_i < r_{-i}$.

When $r_i \geq r_{-i}$ and so $V_i \geq V_{-i}$, the probability that P_i's technology will be used is

$$\begin{aligned}
p_i &= \Pr(c_i < V_i, c_i - c_{-i} \leq V_i - V_{-i}) = \int_0^{V_i} \int_{\max\{0, c_i - (V_i - V_{-i})\}}^{1/\nu} dF(c_{-i}) dF(c_i) \\
&= \int_0^{V_i - V_{-i}} \int_0^{1/\nu} dF(c_{-i}) dF(c_i) + \int_{V_i - V_{-i}}^{V_i} \int_{c_i - (V_i - V_{-i})}^{1/\nu} dF(c_{-i}) dF(c_i) \\
&= F(V_i) - \int_{V_i - V_{-i}}^{V_i} F(c_i - V_i + V_{-i}) dF(c_i).
\end{aligned}$$

In this case,

$$\begin{aligned}
\frac{\partial p_i}{\partial r_{-i}} &= -\left[F(V_i - V_{-i} - V_i + V_{-i}) F'(V_i - V_{-i}) + \int_{V_i - V_{-i}}^{V_i} F'(c_i - V_i + V_{-i}) dF(c_i) \right] \frac{\partial V_{-i}}{\partial r_{-i}} \\
&= -\left[\int_{V_i - V_{-i}}^{V_i} F'(c_i - V_i + V_{-i}) dF(c_i) \right] \frac{\partial V_{-i}}{\partial r_{-i}} < 0, \quad \text{and} \\
\frac{\partial^2 p_i}{\partial r_{-i} \partial r_i} &= \frac{\partial^2 p_i}{\partial r_i \partial r_{-i}} = -\Big[F'(V_i - V_i + V_{-i}) F'(V_i) \\
&\quad - F'(V_i - V_{-i} - V_i + V_{-i}) F'(V_i - V_{-i}) - \int_{V_i - V_{-i}}^{V_i} F'' dF(c_i) \Big] \frac{\partial V_i}{\partial r_i} \frac{\partial V_{-i}}{\partial r_{-i}} \\
&= -\left[\nu^2 - \nu^2 - \int 0 dF(c_i) \right] \frac{\partial V_i}{\partial r_i} \frac{\partial V_{-i}}{\partial r_{-i}} = 0,
\end{aligned}$$

by uniform distribution. The expression (7) is strictly positive in this case.

When $r_i < r_{-i}$ and so $V_i < V_{-i}$, the adoption probability is

$$\begin{aligned}
p_i &= \int_0^{V_i} \int_{c_i - V_i + V_{-i}}^{1/\nu} dF(c_{-i}) dF(c_i) = \int_0^{V_i} \left[1 - F(c_i - V_i + V_{-i}) \right] dF(c_i) \\
&= F(V_i) - \int_0^{V_i} F(c_i - V_i + V_{-i}) dF(c_i).
\end{aligned}$$

The impacts of refinement efforts are

$$\begin{aligned}
\frac{\partial p_i}{\partial r_{-i}} &= -\left[\int_0^{V_i} F'(c_i - V_i + V_{-i}) dF(c_i) \right] \frac{\partial V_{-i}}{\partial r_{-i}} < 0, \quad \text{and} \\
\frac{\partial^2 p_i}{\partial r_{-i} \partial r_i} &= -\left[F'(V_i - V_i + V_{-i}) F'(V_i) - \int_0^{V_i} F'' dF(c_i) \right] \frac{\partial V_{-i}}{\partial r_{-i}} \frac{\partial V_i}{\partial r_i} = -\nu^2 \frac{\partial V_{-i}}{\partial r_{-i}} \frac{\partial V_i}{\partial r_i} < 0.
\end{aligned}$$

To pin down the sign of the expression (7), we need to consider both terms. With these terms in hand,

$$\frac{\partial^2 p_i}{\partial r_{-i}\partial r_i}u_\tau^i + \frac{\partial p_i}{\partial r_{-i}}\frac{\partial u_\tau^i}{\partial r_i} = -\nu^2\frac{\partial V_{-i}}{\partial r_{-i}}\frac{\partial V_i}{\partial r_i}u_\tau^i - \left(\int_0^{V_i}\nu^2 dc_i\right)\frac{\partial V_{-i}}{\partial r_{-i}}\frac{\partial u_\tau^i}{\partial r_i}$$

$$= -\nu^2\frac{\partial V_{-i}}{\partial r_{-i}}\left(\frac{\partial V_i}{\partial r_i}u_\tau^i + V_i\frac{\partial u_\tau^i}{\partial r_i}\right) = -\nu^2 u_\tau^i V_i\frac{\partial V_{-i}}{\partial r_{-i}}\left(\frac{\partial V_i/\partial r_i}{V_i} + \frac{\partial u_\tau^i/\partial r_i}{u_\tau^i}\right).$$

By quadratic investment cost, $V_i = \Psi_i(1-\alpha)^2\bar{v}^2K/2$ and $u_\tau^i = \Lambda_i\alpha(1-\alpha)\bar{v}^2K$, and so

$$\frac{\partial V_i/\partial r_i}{V_i} + \frac{\partial u_\tau^i/\partial r_i}{u_\tau^i} = \frac{\partial\Psi_i/\partial r_i}{\Psi_i} + \frac{\partial\Lambda_i/r_i}{\Lambda_i}$$

$$= \frac{\alpha^2[r_i-(1-r_i)]/q_i^2(1-q_i)^2}{[(1-q_i)(1-r_i)^2+q_ir_i^2]/q_i(1-q_i)} - \frac{\alpha(1-\alpha)[r_i-(1-r_i)]/q_i^2(1-q_i)^2}{r_i(1-r_i)/q_i(1-q_i)}.$$

It follows that, when

$$\frac{\alpha}{(1-q_i)(1-r_i)^2+q_ir_i^2} < \frac{1-\alpha}{r_i(1-r_i)},$$

the expression (7) is strictly positive in the case of $r_i < r_{-i}$ as well. This condition is thus sufficient for refinement decisions to be strategic complements. After some computation, the condition is

$$[\alpha-(1-\alpha)^2]r_i(1-r_i) < \alpha(1-\alpha)[r_i^3+(1-r_i)^3].$$

For $r_i \geq \underline{r} > 1/2$, $r_i(1-r_i)$ is decreasing and $r_i^3+(1-r_i)^3$ increasing in r_i, and at $r_i = 1/2$, the two terms are equal to 1/4. Therefore, when

$$\alpha-(1-\alpha)^2 \leq \alpha(1-\alpha),$$

or, equivalently, when $\alpha \leq 1/2$, the condition is guaranteed. And when $\alpha > 1/2$, because $\alpha-(1-\alpha)^2$ is increasing and $\alpha(1-\alpha)$ decreasing in α, given any r_i this condition is more likely to hold when α is lower.

Next, suppose that $\rho = 0$ and the two technologies are perfect complements. The patentee P_i's payoff is $U_i = \bar{p}\bar{u}_\tau^i - c_R$, where

$$\bar{u}_\tau^i = \alpha(1-\alpha)^2\bar{v}^2K\Lambda_i\left(1-\frac{\alpha}{2}\Lambda_{-i}\right),$$

and by uniform distribution,

$$\bar{p} = \Pr(\bar{V} > c_1+c_2) = \int_0^{\bar{V}}\int_0^{\bar{V}-c_1} dF(c_2)dF(c_1) = \nu^2\frac{\bar{V}^2}{2},$$

with $\bar{V} = (1-\alpha)^4 \bar{v}^2 \Psi_1 \Psi_2 K/2$. The payoff is thus

$$U_i = \frac{\nu^2}{2}\bar{V}^2 \bar{u}_\tau^i - c_R = \frac{\nu^2}{2}\alpha(1-\alpha)^{10}\frac{K^3}{4}\bar{v}^6 \Psi_i^2 \Psi_{-i}^2 \Lambda_i \left(1 - \frac{\alpha}{2}\Lambda_{-i}\right) - c_R.$$

By the same token, the strategic property is determined by the sign of $\partial^2 U_i / \partial r_{-i} \partial r_i$. The relevant part is $\Psi_i^2 \Psi_{-i}^2 \Lambda_i (1 - \frac{\alpha}{2}\Lambda_{-i})$. Note that for all r_{-i} and α, $1 > \frac{\alpha}{2}\Lambda_{-i}$.

$$\frac{\partial U_i}{\partial r_i} \propto \Psi_{-i}^2 \left(1 - \frac{\alpha}{2}\Lambda_{-i}\right)\left(2\Psi_i \Lambda_i \frac{\partial \Psi_i}{\partial r_i} + \Psi_i^2 \frac{\partial \Lambda_i}{\partial r_i}\right).$$

To have the optimal $r_i > \underline{r}$, it must be that

$$2\Psi_i \Lambda_i \frac{\partial \Psi_i}{\partial r_i} + \Psi_i^2 \frac{\partial \Lambda_i}{\partial r_i} > 0,$$

which in turn requires that $2\alpha\Lambda_i > (1-\alpha)\Psi_i$, and so α cannot be too small. Suppose this is true. The strategic relationship then hinges on

$$\frac{\partial^2 U_i}{\partial r_{-i}\partial r_i} \propto 2\Psi_{-i}\left(1 - \frac{\alpha}{2}\Lambda_{-i}\right)\frac{\partial \Psi_{-i}}{\partial r_{-i}} - \frac{\alpha}{2}\Psi_{-i}^2 \frac{\partial \Lambda_{-i}}{\partial r_{-i}} > 0.$$

Refinement decisions are thus strategic complements. *Q.E.D.*

B Later Adoption

In this appendix, we consider an alternative timing of technology adoption, i.e., at time 2.5, after learning the patent office decision, and check whether the results in Section 3 still hold in this late adoption scenario. For simplicity, we assume quadratic investment cost and a uniform distribution for the adoption cost, and do not consider *interim* licensing. We consider (i) the effect of a higher refinement effort on the overall probability that the developer will adopt the technology and successfully introduce the final product; and (ii) the patent-holder's incentive to engage in refinement.

After learning the patent office decision, and so the patent power $\hat{\alpha}$, the developer will incur c_f if and only if $\hat{v}(\hat{\alpha}) > c_f$. The probability that the developer will adopt the technology and successfully enter the downstream market is $F(\hat{v}(\hat{\alpha}))\hat{e}(\alpha)$. Given r, the overall probability that the developer will successfully develop a product is

$$qF(\hat{v}(\hat{\alpha}^1))\hat{e}^1 + (1-q)F(\hat{v}(\hat{\alpha}^0))\hat{e}^1,$$

which, under quadratic investment cost and uniform distribution, is

$$\begin{aligned}
& F(\hat{v}(\hat{\alpha}^1))q(1-\hat{\alpha}^1)Kv_{11} + F(\hat{v}(\hat{\alpha}^0))(1-q)(1-\hat{\alpha}^0)Kv_{11} \\
= \;& (1-\alpha)Kv_{11}\left[(1-r)F(\hat{v}(\hat{\alpha}^1)) + (1-r)F(\hat{v}(\hat{\alpha}^0))\right] \\
= \;& (1-\alpha)\nu\frac{K^2}{2}v_{11}^3\left[(1-r)(1-\hat{\alpha}^1)^2 + r(1-\hat{\alpha}^0)^2\right] = (1-\alpha)^3\nu\frac{K^2}{2}v_{11}^3\left[\frac{(1-r)^3}{q^2} + \frac{r^3}{(1-q)^2}\right].
\end{aligned}$$

The effect of refinement is determined by

$$\frac{\partial}{\partial r}\left[\frac{(1-r)^3}{q^2}+\frac{r^3}{(1-q)^2}\right]=\frac{r^2}{(1-q)^2}-\frac{(1-r)^2}{q^2}+2\alpha\left[\frac{r^2}{(1-q)^3}-\frac{(1-r)^2}{q}^3\right].$$

After some calculation, it can be verified that this term is strictly positive for all $r>1/2$. A higher refinement effort again increases the likelihood that the developer will successfully introduce the final product.

For the patent-holder, given patent power $\hat{\alpha}$, her expected payoff is

$$[1-F(\hat{v})+F(\hat{v})(1-\hat{e})]\,u_{10}+F(\hat{v})\hat{e}\,[\hat{\alpha}\pi+(\hat{\alpha})u_{11}]=u_{10}+F(\hat{v})\hat{e}\,[u_{11}-u_{10}+\hat{\alpha}(\pi-u_{11})]\,.$$

Let us consider the two special cases: $\pi=u_{10}$, and $\pi=v_{11}$ and $u_{11}=u_{10}=0$.

When $\pi=u_{10}$, the patent-holder prefers to be the only player in the downstream market. Her expected payoff given the patent office decision, and thus patent power $\hat{\alpha}$, is $u_{10}-(u_{10}-u_{11})(1-\hat{\alpha})F(\hat{v})\hat{e}$. When determining the optimal refinement effort, her payoff is

$$u_{10}-(u_{10}-u_{11})\left[q(1-\hat{\alpha}^1)F(\hat{v}(\hat{\alpha}^1))\hat{e}^1+(1-q)(1-\hat{\alpha}^0)F(\hat{v}(\hat{\alpha}^0))\hat{e}^0\right]-c_R.$$

Similar to the previous case, the patent-holder will have no incentives to make any refinement effort if the term $q(1-\hat{\alpha}^1)F(\hat{v}(\hat{\alpha}^1))\hat{e}^1+(1-q)(1-\hat{\alpha}^0)F(\hat{v}(\hat{\alpha}^0))\hat{e}^0$ is increasing in r. Using quadratic investment cost and uniform distribution,

$$\begin{aligned} & q(1-\hat{\alpha}^1)F(\hat{v}(\hat{\alpha}^1))\hat{e}^1+(1-q)(1-\hat{\alpha}^0)F(\hat{v}(\hat{\alpha}^0))\hat{e}^0=(1-\alpha)\left[(1-r)F(\hat{v}(\hat{\alpha}^1))\hat{e}^1+rF(\hat{v}(\hat{\alpha}^0))\hat{e}^0\right] \\ = \; & (1-\alpha)\nu\frac{K^2}{2}v_{11}^3\left[(1-r)(1-\hat{\alpha}^1)^3+r(1-\hat{\alpha}^0)^3\right]=(1-\alpha)^4\nu\frac{K^2}{2}v_{11}^3\left[\frac{(1-r)^4}{q^3}+\frac{r^4}{(1-q)^3}\right].\end{aligned}$$

The impact of r is summarized in $(1-r)^4/q^3+r^4/(1-q)^3$, and

$$\begin{aligned} & \frac{\partial}{\partial r}\left[\frac{(1-r)^4}{q^3}+\frac{r^4}{(1-q)^3}\right]=\frac{r^3}{(1-q)^4}(1-q+3\alpha)-\frac{(1-r)^3}{q^4}(q+3\alpha) \\ = \; & \frac{1}{q^4(1-q)^4}\left[r^3q^4(1-q+3\alpha)-(1-r)^3(1-q)^4(q+3\alpha)\right]>0,\end{aligned}$$

whenever $r>1/2$. Therefore in later adoption the patent-holder also has no incentives to make any refinement effort in this case.

Next, consider if the patent-holder is a pure licensor, i.e., $\pi=v_{11}$ and $u_{11}=u_{10}=0$. When deciding the optimal refinement effort, the patent-holder's expected payoff is

$$v_{11}\left[qF(\hat{v}(\hat{\alpha}^1))\hat{\alpha}^1\hat{e}^1+(1-q)F(\hat{v}(\hat{\alpha}^0))\hat{\alpha}^0\hat{e}^0\right]=\alpha v_{11}\left[rF(\hat{v}(\hat{\alpha}^1))\hat{e}^1+(1-r)F(\hat{v}(\hat{\alpha}^0))\hat{e}^0\right].$$

With a quadratic investment cost function and uniform distribution,

$$
\begin{aligned}
& rF(\hat{v}(\hat{\alpha}^1))\hat{e}^1 + (1-r)F(\hat{v}(\hat{\alpha}^0))\hat{e}^0 = \nu\frac{K^2}{2}v_{11}^3\left[r(1-\hat{\alpha}^1)^3 + (1-r)(1-\hat{\alpha}^0)^3\right] \\
= \;& (1-\alpha)^3\nu\frac{K^2}{2}v_{11}^3\left[\frac{r(1-r)^3}{q^3} + \frac{(1-r)r^3}{(1-q)^3}\right].
\end{aligned}
$$

The impact of the refinement effort on the expected payoff is

$$
\begin{aligned}
& \frac{\partial}{\partial r}\left[\frac{r(1-r)^3}{q^3} + \frac{r^3(1-r)}{(1-q)^3}\right] = \frac{(1-r)^2}{q^4}\left[(1-r)q - 3\alpha r\right] + \frac{r^2}{(1-q)^4}\left[-r(1-q) + 3\alpha(1-r)\right] \\
= \;& \frac{1}{q^4(1-q)^4}\left\{3\alpha r(1-r)\left[rq^4 - (1-r)(1-q)^4\right] - q(1-q)\left[r^3q^3 - (1-r)^3(1-q)^3\right]\right\}. \qquad (8)
\end{aligned}
$$

Consider some special cases: When $\alpha \to 1$, $q \to r$ and $1-q \to 1-r$, the numerator in expression (8) approaches to

$$
3r(1-r)[r^5 - (1-r)^5] - r(1-r)[r^6 - (1-r)^6] > 0, \quad \text{for all } r \in (\tfrac{1}{2}, 1);
$$

when $\alpha \to 1/2$, both q and $1-q \to 1/2$, the numerator approaches to

$$
\begin{aligned}
& \frac{3}{2}r(1-r)(\frac{r}{16} - \frac{1-r}{16}) - \frac{1}{4}\left[\frac{r^3}{8} - \frac{(1-r)^3}{8}\right] \\
= \;& \frac{1}{32}\left[3r(1-r)[r-(1-r)] - [r^3 - (1-r)^3]\right] = -\frac{(2r-1)^3}{32} < 0;
\end{aligned}
$$

and when $\alpha \to 0$, $q \to 1-r$ and $1-q \to r$, the numerator approaches to zero. Furthermore, differentiating the numerator with respect to α, when evaluating at $\alpha = 0$, leads to

$$
3r^2(1-r)^2\left\{(1-r)^3 - r^3 - [r-(1-r)]r(1-r)\right\} < 0,
$$

which suggests that expression (8) is negative and so the patent-holder has no incentives to engage in refinement when α is around zero. Although less clear-cut, we still obtain similar qualitative results that the patent-holder would want to exert some refinement effort when α is large enough, but not when α is close to zero or 1/2.[25]

[25] To consider the general case, define $\Delta r^x = r^x - (1-r)^x$, where x is a positive integer. The numerator of expression (8), in terms of α and r, is

$$
\begin{aligned}
& \alpha^5 r(1-r)[2\Delta r^5 + r(1-r)\Delta r^4] + \alpha^4(1-\alpha)[8r^2(1-r)^2\Delta r^3 + 4r^3(1-r)^3\Delta r - \Delta r^8] \\
- \;& 4\alpha^3(1-\alpha)^2 r(1-r)\Delta r^1[r^3 + (1-r)^3 - 2r^2(1-r)^2] - 6\alpha^2(1-\alpha)^3 r^2(1-r)^2[r-(1-r)]^3 \\
- \;& 3\alpha(1-\alpha)^4 r^2(1-r)^2[\Delta r^3 + \Delta r^1].
\end{aligned}
$$

When α is large enough, the term α^5 dominates, and so the patent-holder would have incentives to engage in refinement.

References

Ayres, I. and P. Klemperer (1999), “Limiting Patentees’ Market Power without Reducing Innovation Incentives: The Perverse Benefits of Uncertainty and Non-Injunctive Remedies,” *Michigan Law Review*, 97: 985-1033.

Caillaud, B. and A. Duchên, (2005), “Patent Office in Innovation Policy: Nobody’s Perfect,” working paper.

Chiou, J.-Y., (2008), “The Patent Quality Control Process: Can We Afford An (Rationally) Ignorant Patent Office?” working paper.

Dewatripont, M. and J. Tirole, (1999), “Advocates,” *Journal of Political Economy*, 107 (1): 1-39.

Gans, J., D. Hsu, and S. Stern, (2007), “The Impact of Uncertain Intellectual Property Rights on the Market for Ideas: Evidence from Patent Grant Delays,” working paper.

Langinier, C. and P. Marcoul (2003), “Patents, Search of Prior Art and Revelation of Information,” working paper.

Lemley, M., (2001), “Rational Ignorance at the Patent Office,” *Northwestern University Law Review*, 95(4): 1495-1529.

Lemley, M. and C. Shapiro, (2005), “Probabilistic Patents,” *Journal of Economic Perspectives*, 19 (2): 75-98.

Jaffe, A. and J. Lerner, (2004), *Innovation and its Discontents: How Our Broken Patent System is Endangering Innovation and Progress, and What to Do About it,* Princeton University Press.

Lerner, J. and J. Tirole, (2004), “Efficient Patent Pools,” *American Economic Review*, 94 (3): 691-711.

Lichtman, D., (2005), “Substitutes for the Doctrine of Equivalents: A Response to Meurer and Nard,” working paper.

Meurer, M. and J. Bessen, (2006), “Patent Litigation with Endogenous Disputes,” *American Economic Review: Papers and Proceedings*, 96 (2): 77-81.

Meurer, M. and J. Bessen, (2008), *Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk*, Princeton University Press.

Meurer. M and C. Nard, (2005), "Invention, Refinement, and Patent Claim Scope: A New Perspective on the Doctrine of Equivalents,' '*Georgetown Law Journal*,

Prady, D. (2008), " "Is That a Gun in Your Pocket...?" Applications for Patents and Costly Verification," working paper.

Risch, M., (2007), "The Failure of Public Notice in Patent Prosecution," *Harvard Journal of Law and Technology*, 21 (1): 180-232.

Shapiro, C. (2001), "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard-Setting," *Innovation Policy and the Economy*, Volume I, A. Jaffe, J. Lerner, and S. Stern, eds., MIT Press.

Shuett, F. (2008), "Inventors and Impostors: An Economic Analysis of Patent Examination," working paper.