Liquidity Constraints of the Middle Class

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Abstract

This paper combines impatience with large recurring expenditures to replicate the observation that middle-class U.S. households consume much more out of transitory income than permanent income theory predicts. In the present model, households make a large recurring expenditure of exogenous timing and endogenous size; hence, in spite of their impatience, households save in anticipation of this expenditure. When it occurs, a borrowing constraint taking the form of equity requirements on collateralizable durable goods limits household's debt. Although the standard Euler equation usually holds good, the household is always *liquidity constrained*, in the sense that they value assets that provide liquidity more than their fundamental value. These constraints are strongest when wealth is highest. We contrast a calibrated version of the model with evidence from the 2001 tax rebate.

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1 Introduction

Are liquidity constraints common among middle class U.S. households? Evidence surveyed below suggests that liquidity constraints are not concentrated among the poor, and that there is a positive association between ownership of financial assets and spending out of temporary income. To address this issue we develop a model of households with middle to high-income characteristics, and contrast the quantitative results from the model to existing evidence on households' spending reactions to income changes.

Households in the model are home owners and hold financial assets. They are impatient, i.e., their rate of time preference is higher than the interest rate. Homes are treated as part of the durable goods stock, and all debt should be collateralized by durable goods. The borrowing constraint takes the form of equity requirements (down payment and a given rate of repayment) on those goods.

Asset accumulation in the model is generated by large expenditures at exogenous intervals, and there is no uncertainty in the model. These expenditures generate saving in spite of impatience. The role of assets in this model can be interpreted as an extreme and simplified version of precautionary savings. This combination of impatience and a motive for asset accumulation is similar as in Carroll (2001). Carroll stresses precaution as the motive for asset accumulation, balancing impatience which causes asset decumulation.

In the empirical literature on consumption and consumption response to tax changes, ownership of financial assets is considered an indication that the household is not liquidity constrained. Assets are typically seen as the outcome of lucky past realizations of income shocks. Hence, the amount of assets is taken as measures of liquidity availability. In the present model, this channel for asset accumulation is shut down, and the level of assets signals the opposite: It indicates a need for funds and hence the extent of a liquidity *shortage*.

A basic feature of the model is that, at a given point in time, most households appear unconstrained in the sense that their Euler equation currently holds. Nevertheless, they are liquidity constrained in an intertemporal sense. A binding borrowing constraint in the future affects current behavior—Zeldes (1989) mentions this situation as implying a "global" constraint. Specifically, we separate the question whether the *borrowing constraint* binds, from whether the *liquidity constraint* binds.

Let us define precisely these two concepts here, which usually are not separated. As in the standard characterization, a borrowing constraint binds when the household cannot freely borrow to increase current consumption—or the Euler equation does not currently hold.

Liquidity is defined as usual: Liquidity is the characteristic of an asset which can be transformed into consumption goods at no cost when so desired. These liquidity services are not valued in a standard deterministic framework. To value liquidity, there should be either uncertainty or a future binding borrowing constraint. We follow the latter route, and define a binding liquidity constraint as a generalization of the borrowing constraint: The liquidity constraint binds if the borrowing constraint binds either in the present or in the future.

This definition formalizes a basic intuition about liquidity: A household is liquidity constrained when, even without uncertainty, the utility value of \$1 on hand is higher than the present utility value of $\$(1 + interest\ rate)^j$ for some j periods ahead, using the discount factor $\beta^j u'(c_{t+j})/u'(c_t)$—expressed in standard notation. The degree to which a household is liquidity constrained depends on the proximity of the binding borrowing constraint and on it's strength—the size of it's Lagrange multiplier.

The focus of this paper is on the quantitative comparison of the model's implications to the empirical results about households' additional spending in the face of income tax changes. We focus on the results in Shapiro and Slemrod (2003a,b) and Johnson, Parker and Souleles (2006) about the 2001 tax rebate, and in Parker (1999), who uses changes in Social Security tax withholding.

Interestingly, these papers present evidence, not unanimous, that high-income and high-assets households tend to respond more rather than less to after-tax income changes. This stands in contrast to the presumption that these households are less likely to be liquidity constrained. Shapiro and Slemrod (2003a) find that spending responses tend to increase with both income and ownership of stocks, and Parker (1999) finds that these responses are stronger for high-assets households than for low-assets households. These results challenge the view that assets indicate lower likelihood of liquidity constraints. Our model rationalizes the stronger response of high-assets households because these assets indicate a forthcoming shortage of resources. Hence, according to the definition above, households with higher assets may be more rather than less liquidity constrained. In other words, the asset level indicates an effective shortening of the planning horizon, which in turn generates a stronger response to temporary income.

2 Evidence on Consumption Reaction to Income Changes

In 2001, most U.S. households received tax rebates of \$300 or \$600, as part of a 10 years tax cut. We review here results about the consumption effects of this rebate in Shapiro and Slemrod (2003a) and Johnson, Parker and Souleles (2006). We focus on the results in these two studies about the following questions:

1. What percentage of households spent most of the rebate? or, what percentage of the rebate was spent? Shapiro and Slemrod address the first specification; Johnson, Parker and Souleles the second.

2. How persistent the rebate was expected to be? This is key for interpreting the results on the first question: Higher expected persistence should increase the fraction spent.

3. Did households respond in a Ricardian fashion? If they did, liquidity constraints, which are the focus of this paper, did not play an important

role. The survey conducted by Shapiro and Slemrod contains questions aimed at providing information about this and the previous question.

4. Were the rebates expected as of 2000 (or previously)? This issue is important for our quantitative analysis, where we use annual intervals. If the debate was expected, then the effect in 2001 should be smaller.

5. What is the relationship between the household's asset levels and the fraction spent? Typically, low assets households are considered liquidity constrained, and thus are expected to spend more. In this paper we expect the opposite. Assets' levels should be higher for households closer to a big expenditure, and thus will be expected to be more liquidity constrained.

From the evidence in the studies above, the answers to these questions are the following:

1. Johnson, Parker and Souleles found that nondurable consumption increased by about 2/3 of the rebate during a six month period (close to 40 percent during the three-month period of the rebate, and the remaining in the following three months). Shapiro and Slemrod found that 22 percent would spend most of the rebate, while the rest would save most of it. Note that this percentage is not an expenditure fraction of the rebate—or marginal propensity to consume. Using plausible distributions of the marginal propensities to consume across those who would "mostly spend" and "mostly save", Shapiro and Slemrod (2003b), calculate an average marginal propensity to consume of about 1/3. The fraction is significantly lower than Johnson, Parker and Souleles, also because it refers to total spending, and not only to nondurable consumption.

2. Shapiro and Slemrod's survey includes questions about expected size of future rebates and about future government spending (Table 5). Regarding future government spending, 26 percent responded that they expect higher spending, 55 percent expect no change, and 19 percent expect smaller spending. Hence, 81 percent expect the same or higher government expenditures. If households recognize the link between spending and taxes—or if they have "Ricardian beliefs"—they should also expect a short-lived tax cut and a future tax increase. Indeed, to the answer whether they expect smaller, bigger, or the same size tax cuts in the future, 37 percent believe they will be smaller, 47 percent replied the same, and only 16 percent responded higher. Hence, these answers are tilted towards lower tax cuts. Both answers, to future government spending and tax cuts, suggest that the rebates were considered temporary.

3. Slemrod and Shapiro (Table 5) checked whether households who expect higher government spending would correspondingly spend less, and found no regularity in this respect. Similarly, there is no correlation between expectations of smaller rebates in the future and larger spending fraction. Hence, their survey finds no evidence of Ricardian behavior.

4. As Johnson, Parker and Souleles stress, the fact that the Tax Act was passed by Congress in May 2001, while the rebates were sent during July-September, indicates that the rebate was fully expected several months in advance. They also point out that during the 2000 election campaign, George W. Bush included the tax cut in his platform. However, it is doubtful that as of year 2000 on average, households started to count on a rebate. We adopt the view that during 2000 households did not expect the rebate.

5. The evidence on the consumption response to the rebate by assets level is particularly interesting in the present context. Dividing the sample into three brackets by liquid assets, Johnson, Parker and Souleles (Table 5) find that the low assets group spent much more than the middle range group, which is consistent with that group being liquidity constrained. However, also the top assets group spent more than the middle range group, although the difference is not statistically significant. They find the same pattern by income brackets, which can proxy for other financial assets. Shapiro and Slemrod (Table 2) perform similar tests using the value of stocks. They divide the sample into six groups: The first with no stocks, and other five groups corresponding to stock ownership brackets from $1-$15,000 to $250,000 and above. The spending fraction of the no-stock group is 19.5, and the spending fraction of the other five groups are 13.1, 18.1, 26.7, 33.6 and 22.9. Remarkably, the spending fraction *increases* with stock ownership, with exceptions when moving to the lowest stock bracket, and to the top bracket—which we address later. They find the same pattern using income levels.

Additional evidence on the marginal propensity to consume by assets level is presented by Parker (1999), who studies the consumption response to predictable changes in Social Security Tax withholding. He finds a nondurable consumption elasticity of 1/2 to these income changes. This elasticity corresponds to a marginal propensity of 0.2 of nondurable consumption. This response differs across two groups by financial asset holdings: A high-asset group, with assets enough for 6 months of nondurable consumption or more, and a low-asset group, with assets less than enough for one month of nondurable consumption. Parker finds (Table 5, panel A) that the consumption elasticity of the high-asset households is higher, 0.8, and statistically significant, while for the low-asset group the elasticity is lower, 0.5, and statistically insignificant.

Shapiro and Slemrod, and Parker, discuss different explanations for the larger marginal propensities to consume of households with high assets levels. Parker focuses on bounded rationality: Households with high income may not gain much by smoothing consumption to the withholding changes in Social Security taxes. Shapiro and Slemrod's explanation is closer in spirit to the present paper. Those with low stocks, tend to spend less than those without stocks at all, and less than those with higher stock levels because they are in the process of building their optimal level of assets. We'll comment later on the implications of this mechanism and the one in the present paper.

3 The Model

The model addresses a household's problem combining impatience with a motive for asset accumulation. Impatience pushes the household against a borrowing constraint, while a large periodic expenditure works in the opposite direction, generating asset accumulation. This expenditure is thought of as capturing college expenditures, weddings, etcetera. The model is solved under certainty, but the forthcoming periodic expenditure generates saving in a similar fashion as a precautionary motive does. Hence, this model has similar features as those stressed by Carroll (2001) in a stochastic environment and precautionary saving.

The borrowing constraint is modelled as equity requirements on the durable goods stock—thought of as including housing. There are two advantages in using this particular borrowing constraint, rather than a standard constraint on assets. One advantage is realism: The vast majority of household debt in the U.S., takes this form. According to the 2001 Survey of Consumer Finances, 90 percent of all household debt is collateralized by homes and vehicles. The calibration in Section 4.1 and Campbell and Hercowitz (2009) provide more details. We expect this to be relevant for the quantitative evaluation of the calibrated model. The second advantage is theoretical. The borrowing constraint depends on the collateral value of the durable goods stock, which is endogenous—rather than exogenous, as the standard lower limit on assets. In the present setup, a new durable good has value as providing liquidity by expanding the collateralizable assets. Even if the household is not currently borrowing constrained, this value, which is the Lagrange multiplier on the collateral accumulation equation, reflects the proximity of a binding borrowing constraint. Hence, this multiplier will provide a measure of how valuable liquidity is.

3.1 The Household's Problem

The household's preferences are

$$\sum_{t=0}^{\infty}\beta^t \left\{(1-\theta)\ln C_t + \theta \ln S_t + \mu_t \ln M_t\right\}, \quad 0<\beta<1, \tag{1}$$

where C_t is ordinary non-durable consumption, S_t is the stock of durable goods and M_t is extraordinary non-durable consumption, which yields utility periodically according to the indicator μ_t. This indicator follows a τ periods cyclical pattern with $\mu_t = \mu > 0$ every τ periods and $\mu_t = 0$ in the other periods. This specification generates a periodic expenditure of exogenous timing but endogenous size. This feature resembles the models of asset demand by Baumol (1952) and Tobin (1956).

We assume that the household is impatient, i.e., the discount factor satisfies $\beta R < 1$, where R is the gross interest rate, assumed to be constant.

The household is endowed with one unit of labor which is supplied inelastically at the wage rate W_t. Denoting net financial assets at the beginning of period t with A_t, the household's budget constraint is

$$C_t = W_t + RA_t + (1-\delta) S_t - A_{t+1} - S_{t+1} - M_t, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate of durable goods.

Debt is constrained by collateralizable durable goods and an exogenous equity requirement. This mimics a typical feature of most loan contracts in the United States: An equity share that starts with a down payment and increases over time as debt amortizes more rapidly than the goods depreciate. The parameters capturing this feature are the initial equity share $0 \leq \pi < 1$, and the rate of repayment $\phi \geq \delta$ which determines equity accumulation. It can be shown (Campbell and Hercowitz (2009)), that these equity requirements imply that the collateral value of the durable stock evolves as

$$V_{t+1} = (1-\phi)V_t + \frac{(1-\delta)(1-\pi)}{R}(S_{t+1} - (1-\delta) S_t). \tag{3}$$

In (3), the first term reflects the depreciation rate ϕ of the collateral value, which translates into a required amortization of the debt. The second term represents the contribution of new durable goods purchases, $S_{t+1} - (1-\delta) S_t$, to the collateral value of the stock. This contribution depends negatively on the initial equity share π, as well as on δ and R given that the goods depreciate, and debt accrues interest, till repayment in period $t+1$. The expression $(1-\delta)(1-\pi)/R$ corresponds, therefore, to a maximum loan-to-value ratio allowed.

The borrowing constraint is then

$$A_{t+1} \geq -V_{t+1}. \tag{4}$$

An additional constraint imposed is[1]

$$M_t \geq 0. \tag{5}$$

Given V_0, A_0 and S_0, the household chooses sequences of C_t, S_{t+1}, M_t and A_{t+1} to maximize the utility function in (1) subject to the sequences of constraints in (2), (3), (4) and (5). Expressing the Lagrange multipliers on the four constraints with Ψ_t, Ξ_t, Γ_t and Λ_t, the optimality conditions are

$$\Psi_t = \frac{(1-\theta)}{C_t}, \tag{6}$$

$$\Psi_t - \Xi_t \frac{(1-\delta)(1-\pi)}{R} = \beta \frac{\theta}{S_{t+1}} + \beta (1-\delta) \Psi_{t+1} - \beta (1-\delta) \Xi_{t+1} \frac{(1-\delta)(1-\pi)}{R}, \tag{7}$$

$$\Psi_t - \Lambda_t = \frac{\mu_t}{M_t}, \tag{8}$$

$$\Psi_t = \Gamma_t + \beta R \Psi_{t+1}, \tag{9}$$

[1] When this constraint binds, the utility function includes the product zero times $log(0)$, and the first-order condition (8) will include the division of zero by zero. We interpret these two expressions as zero. An alternative would be to express (5) as $M_t \geq \underline{M}$, and let $\underline{M}$ approach zero after the derivation of the first-order conditions.

$$\Xi_t = \Gamma_t + \beta(1-\phi)\Xi_{t+1}, \tag{10}$$

$$\Psi_t \left[W_t + RA_t + (1-\delta) S_t - A_{t+1} - C_t - S - M_t\right] = 0, \tag{11}$$

$$\Xi_t \left[(1-\phi)V_t + \frac{(1-\delta)(1-\pi)}{R}\left(S_{t+1} - (1-\delta) S_t\right) - V_{t+1}\right] = 0, \tag{12}$$

$$\Gamma_t \left(V_{t+1} + A_{t+1}\right) = 0, \tag{13}$$

$$\Lambda_t M_t = 0, \tag{14}$$

$$\Psi_t, \Xi_t, \Gamma_t, \Lambda_t \geq 0. \tag{15}$$

The multiplier on the budget constraint in (6), Ψ_t, represents the marginal value of current resources. In (9), Γ_t is the deviation from the standard Euler equation, which is positive when the borrowing constraint currently binds.

Iterating (10) forwards yields Ξ_t as a present value of the current and future values of Γ_t. That is, Ξ_t measures how close the next binding constraint is and it's strength—the value of Γ_t. The multiplier Ξ_t is closely related to our definition of a liquidity constraint. If $\Xi_t = 0$, for example, there is no binding constraint in sight. This implies that the utility value of \$1 on hand exactly equals the present utility value of $\$R^j$ for any j periods ahead discounted by $\beta^j \Psi_{t+j}/\Psi_t$—or $(\beta R)^j \, \Psi_{t+j}/\Psi_t = 1$. If, alternatively, $\Gamma_{t+j} > 0$ for some $j \geq 1$, both $\Xi_t > 0$ and $(\beta R)^j \, \Psi_{t+j}/\Psi_t < 1$ hold. Then, as j becomes smaller, Ξ_t increases and $(\beta R)^j \, \Psi_{t+j}/\Psi_t$ declines.

Because a higher value for Ξ_t indicates that the binding constraint is closer, temporary income will increase consumption smoothed over a shorter period—and hence current consumption will react more. We will illustrate the connection between Ξ_t and the closeness of the forthcoming borrowing constraint in Section 3.2.

Equation (7) characterizes optimal durable good purchases. If the borrowing constraint never binds, i.e., $\Xi_t = \Xi_{t+1} = 0$, then this equation equates the purchasing cost of the durable good—the marginal cost of current resources—to it's marginal utility plus it's discounted resale value. When Ξ_t and Ξ_{t+1} are positive, the purchasing cost of the durable good is lower because it provides $(1-\delta)(1-\pi)/R$ of collateral, whose liquidity value is that times Ξ_t. The resale price of the durable good is correspondingly lower as well.

The optimal periodic expenditure in given by (8). Every τ periods $\mu_t > 0$, and then the value of M is set by the equality of its marginal utility to the marginal cost of resources. In these periods, the constraint $M_t \geq 0$ does not bind, and hence $\Lambda_t = 0$. In all other periods $\mu_t = 0$, so $M_t = 0$ and $\Lambda_t = \Psi_t > 0$.

The model solves only for net assets. The household can save towards the periodic expenditure by accumulating equity above requirement on durable goods, and then take a mortgage on that free equity to pay for the expenditure, or by purchasing assets. We resolve this indeterminacy by assuming that paying the mortgage faster and then taking a new one involves a small cost, while saving by purchasing assets is free of cost. In this situation, gross debt is $B_t = V_t$, and gross assets are then $A_t^g = A_t + B_t$. In other words, the debt is repaid at the required pace, and the expenditure is financed by selling assets.

3.2 Steady-State Cycling

When W_t is constant, the model is in a cycling steady state. The steady-state solution consist in vectors of length τ for all variables in the system. Denoting $\overrightarrow{X} = \{X^\kappa\}_{\kappa=1}^{\tau}$, and defining the leading variables by $F\overrightarrow{X} = \{X^2, X^3, ...X^\tau, X^1\}$, where F is the matrix

$$F = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ 0 & 0 & 0 & 1 & ... \\ & & ... & & \\ 1 & 0 & 0 & ... & 0 \end{bmatrix},$$

we can write the steady-state system in following vectorized form:

$$\begin{aligned}
0 &= -\frac{(1-\theta)}{\overrightarrow{C}} + \frac{(1-\delta)(1-\pi)}{R}\overrightarrow{\Xi} + \frac{\beta\theta}{F\overrightarrow{S}} + \frac{\beta(1-\delta)(1-\theta)}{F\overrightarrow{C}} - \frac{\beta(1-\delta)^2(1-\pi)}{R}F\overrightarrow{\Xi}, \\
0 &= -\frac{(1-\theta)}{\overrightarrow{C}} + \overrightarrow{\Gamma} + \beta R\frac{(1-\theta)}{F\overrightarrow{C}}, \\
0 &= \overrightarrow{\Gamma} - \overrightarrow{\Xi} + \beta F\overrightarrow{\Xi}(1-\phi), \\
0 &= W + R\overrightarrow{A} + (1-\delta)\overrightarrow{S} - F\overrightarrow{A} - \overrightarrow{C} - F\overrightarrow{S} - \frac{\overrightarrow{\mu}}{(1-\theta)}\overrightarrow{C}, \\
0 &= (1-\phi)\overrightarrow{V} + \frac{(1-\delta)(1-\pi)}{R}\left(F\overrightarrow{S} - (1-\delta)\overrightarrow{S}\right) - F\overrightarrow{V}, \\
0 &= \overrightarrow{\Gamma}\cdot\left(F\overrightarrow{V} + F\overrightarrow{A}\right),
\end{aligned}$$

where $1/\overrightarrow{X}$ indicates the elementwise inverse of vector $\overrightarrow{X}$, and "$\cdot$" indicates elementwise multiplication. In this system, we solved for the periodic expenditure using (6), (8), (5) to get

$$\overrightarrow{M} = \frac{\overrightarrow{\mu}}{(1-\theta)} \cdot \overrightarrow{C}.$$

The equation for M takes this form because for $\kappa = \tau$, the non-negativity constraint for M does not bind and hence $\Lambda^\tau = 0$, and when it does bind $\mu^\kappa = 0$, $\kappa = 1, 2, ...\tau - 1$. Hence, $M^\kappa = 0$ for $\kappa = 1, 2, ...\tau - 1$, and $M^\tau = uC^\tau/(1-\theta)$. This is a system of $6 \times \tau$ equations in the $6 \times \tau$ unknowns $\overrightarrow{C}$, $\overrightarrow{S}$, $\overrightarrow{A}$, $\overrightarrow{V}$, $\overrightarrow{\Gamma}$ and $\overrightarrow{\Xi}$, which can be easily solved numerically.

4 Quantitative Analysis

We start by calibrating the model. Then, we compute and discuss the steady-state cycling and the model's response to a one-period transitory income increase. Then, this section focuses on the model's responses to a tax reduction like that of 2001, interpreting the evidence surveyed in Section 2.

4.1 Calibration

We adopt a year as the length of a period.

We calibrate the main parameters, τ and u, to micro data on nonlabor income. Using PSID data for 2001, we compute the average ratio of interest and dividend income to labor income for home owners with 65 years of age or less (for the household head). This sample has 3378 households. We exclude the top 2 percentiles of the interest and dividend income distribution, under the presumption that they do not fit the characterization of impatient. The average ratio is 0.019.[2] Using the interest rate of 4 percent, the resulting estimate of actual gross financial assets to labor income ratio is 0.475. Then, we calibrate τ and u by setting the average ratio of A_t^g to the wage in steady-state cycling—assuming that households are uniformly distributed along the expenditure cycle—equal to 0.475. The problem with this procedure is that different combinations of τ and u can deliver the same average ratio of gross assets to labor income. We proceed here by setting $\tau = 10$, resulting in $\mu = 2.016$. In Section 4.4.1, we check the sensitivity of the results to different values of τ.

The equity requirements and other parameters follows Campbell and Hercowitz (2009). These requirements changed substantially after 1982. Given our focus on the 2001 tax rebate, we set this parameters to the period of low requirements. The initial equity requirement, π, is 0.11, and the rate of repayment, ϕ, is 0.06.

The other parameters are: depreciation rate $\delta = 0.04$, durable goods utility parameter $\theta = 0.37$, time-preference factor $\beta = 0.94$ and gross interest rate, $R = 1/0.96$.

4.2 Steady-State

The steady-state system is solved computationally using the following conjectures to facilitate the solution: (1) The constraint $V^{\kappa+1} + A^{\kappa+1} = 0$ should bind for some κ during the expenditure cycle. Otherwise, the household's debt is continuously lower than allowed. Because the household is impatient, it's utility can be increased by increasing it's debt. (2) This implies that the borrowing constraint should bind at $\kappa = \tau$, because this is the time of highest need for funds during the cycle. (3) If the constraint binds also in other periods, these are likely to be right after the expenditure period—as time left for saving towards the next expenditure is then the longest.

The steady-state cycle from the calibrated model is plotted in Figure 1, where the periodic expenditure takes place in the last period.

The main features of Figure 1 are the following:

- The borrowing constraint multiplier, Γ, has the highest value in period τ; it is still positive in the next period—the first of the next cycle—and then it is zero until the next expenditure.

[2] For comparison, the same ratio for renters, i.e., also excluding the top 2 percentiles of the interest and dividend distribution, is 0.005. The total sample of renters has 2091 households.

- Ξ increases over the cycle—except from period 1 to period 2, after the borrowing constraint ceases to bind.
- Gross financial assets are zero in periods 1 and 2, and then increase.
- A^g and Ξ comove positively, except at the beginning of the cycle when Ξ declines while A^g remains constant.
- Consumption is smoothed—with a declining pattern because of impatience—while the Euler equation holds, i.e., while $\Gamma = 0$.

We interpret these features as follows:

As a present value of the present and future values of Γ, Ξ measures the strength of the liquidity constraint. In period 1 the borrowing constraint binds, while in period 2 it does not; hence, $\Xi^1 > \Xi^2$. During the rest of the cycle, the main feature is the increasing proximity of the next expenditure, a time at which the borrowing constraint will bind and the value of Γ will be the highest. This shows in the increasing pattern of Ξ, or the tightening of the liquidity constraint. An implication of the increasing Ξ is the shortening of the consumption smoothing horizon. In Section 4.3, we address the connection of this shortening of the horizon with the spending response to temporary income changes.

In period τ, all financial assets are spent, and hence $A^1 + B^1 = (A^g)^1 = 0$. Hence, in period 1 the household is similar to a typical financially constrained consumer in the literature, as in Zeldes (1989), for whom a past event depleted all financial assets. Given impatience, it is possible that the borrowing constraint binds also in period 1, as in the present case, and then $(A^g)^2 = 0$ as well. The borrowing constraint could bind for more periods, but eventually the proximity of the next expenditure requires to start saving. This happens in Figure 1 in period 2. At this point, the household begins to hold assets in spite of impatience, similarly as in Carroll (2001).

The positive comovement of Ξ and A^g reflects a positive association between the assets level and the strength of the liquidity constraint. In contrast to the usual assumption in the empirical literature, the level of assets reflects here an increasing need for funds, and hence it signals a shortage of liquidity rather than an abundance of it.

4.3 Dynamic Response to a Temporary Income Change

Figure 2 shows the impulse responses of nondurable consumption, including the periodic expenditure, as a ratio to a transitory and unexpected transfer of one percent of labor income. The number on the vertical axis is the impact effect, or "MPC". Each panel plots a different timing of the transfer, from $\kappa = 1$, i.e., one period after the expenditure, to $\kappa = 10$, the expenditure period. Figure 3 portrays the same for durable purchases. There are two aspects of these graphs we wish to stress: The main one is the MPC's, i.e., the current effects shown on the vertical axes, and the other aspect is the pattern of the dynamic responses.

Let us start with nondurable consumption in Figure 2. The highest MPC is 0.71 for $\kappa = 10$, and then 0.29 for the following period, $\kappa = 1$. In these two periods the borrowing constraint binds, which explains the high MPC's. When the transfer is received in periods 2 to 9, i.e., when the borrowing constraint does not bind, the MPC's are smaller and range from 0.07 to 0.14. The interesting feature of these MPC's is that they go up as the next expenditure gets closer. As we stress below, this happens simultaneously with increasing asset levels.

The dynamic pattern in Figure 2 shows consumption smoothing over a short horizon, which depends on the timing of the transfer. For example, for $\kappa = 5$, 8 percent are spent during the same and four additional periods. After the big expenditure five periods later, the impact of the transfers is nil. When the transfer is received in $\kappa > 5$, the effective horizon is shorter, generating higher MPC's: 9 to 14 percent. The opposite holds for $\kappa < 5$, for which the MPC's range from 7 to 8 percent. In other words, a binding borrowing constraint in the non-distant future greatly reduces the planning horizon in spite of the infinite-horizon optimization.

Figure 3 portrays the corresponding impulse responses for durable goods purchases. This spending refers to the down payment only, i.e., 11 percent of the purchases; the rest is assumed to be borrowed.[3] The MPC's have a similar pattern as for nondurable consumption: They are the highest when the borrowing constraint binds—0.4 and 0.29 for $\kappa = 1$ and $\kappa = 10$—and they increase as the household is closer to the following expenditure at the time of the transfer—from 0.05 to 0.11. The dynamic patterns also illustrate the effectively short horizon; following the next periodic expenditure, the response is very small.[4]

4.4 Dynamic Responses to a Tax Rebate

Here we evaluate the effects of the 2001 tax rebate using the calibrated model. Based on the discussion of the evidence on this rebate in Section 2, the simulation is based on the following assumptions about the households' perceptions at the time of the rebate: (a) The rebate is unexpected prior to 2001, (b) The intertemporal budget constraint of the government is balanced, (c) The future path of government spending is not affected by the tax cut, (d) The tax cut is not very persistent.

From (b) and (c) it follows that the simulation should include expected

[3] The assumption that the household uses the maximum borrowing allowed is natural when the transfer is received while the borrowing constraint binds, i.e., for $\kappa = 1, 10$. For the other periods, the household could use accumulated funds to pay for part of the durable goods—and take later a mortgage on the resulting free equity to finance the next periodic expenditure. As we assumed in Section 4.1, we rule out this behavior.

[4] A puzzling feature of the dynamic responses is the *increase* in the fraction spent on durables precisely at the next expenditure time; this happens in period 10 when $\kappa = 1$, in period 9 when $\kappa = 2$, etcetera. This increase is due to the decline in Γ, and hence in Ξ for next expenditure period—as a consequence of the rebate. This decline is due to the saving incurred at the time of the transfer, which makes the shortage of funds at the time of the expenditure less acute.

higher taxes with present value equal to the tax cut. We model the expected tax adjustment as a smooth permanent tax increase immediately after the discountinuation of the tax cut.

If assumption (c) is relaxed, i.e., if it is government spending which adjusts to satisfy the intertemporal budget constraint of the government, the effects of the rebate are identical to those of the transfer in Section 4.3. How different is this scenario from the case where adjustment takes place via tax increases depends on the effects of future taxes, which is the main issue in this section.

Regarding the perceived persistence of the tax cut, we consider three possibilities: one year, three years, and five years.

Figures 4 and 5 plot the impulse responses for a purely temporary rebate for nondurable consumption and durable purchases. Hence, they are the counterparts of Figures 2 and 3 with added balanced intertemporal budget.

Comparing Figure 4 to Figure 2, we can see a similar pattern for nondurable consumption, but, as it can be expected, the MPC's are lower. What is more surprising is the small extent to which spending generated by a tax postponement is lower than for a completely free transfer: For the $\kappa = 1, 10$, the MPC's are 0.28 and 0.70, slightly lower than 0.29 and 0.71 without the tax adjustment. For the other timing, the MPC's are generally one percentage point lower. Additionally, the responses are here negative after the next expenditure, as households who spent almost all the rebate till then, are faced with higher taxes.

Compared to Figure 3, the responses for durable purchases in Figure 5 display similarly lower profiles as nondurable consumption for most stages over the cycle.[5] The spending response after the next periodic expenditure are slightly negative, but this is not visible in the graph.

We now consider persistence of three years. Figures 6 and 7 display this case for nondurable consumption and durable purchases. Comparing Figure 6 to Figure 4, and Figure 7 to Figure 5, shows that the higher persistence increases, as expected, the magnitude of the responses of both nondurable consumption and durable purchases. Given the higher tax increase than when the tax cut lasts one year only, the responses after the next periodic expenditure are here more negative. For durables, however, the lower level is not noticeable in the graph.

The results for five year persistence, not shown, display a still higher MPC's for most of the κ values, although for $\kappa = 9, 10$, i.e., on the expenditure period and on the preceding one, the MPC's change little. This result can be explained by the effectively short horizon of these households. When the rebate is received very close to the binding borrowing constraint, whether the rebate will continue one period or three periods beyond has small quantitative importance.

Table 1 summarizes the results in this and in Section 4.3.

[5] Note, however, that for $\kappa = 1$, i.e., the immediate response is slightly higher, 0.41 instead of 0.40 without tax adjustment. This is followed by a somewhat deeper decline in the next period: -0.39 instead of -0.36. This profile indicates lower overall purchases, but also a slight advancement of them.

Table 1: Financial Assets and Marginal Propensities to Spend

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Financial Assets		Lump-Sum Transfer	Balanced-Budget Tax Cut		
κ	Net	Gross	One Year	One Year	Three Years	Five Years
1	−1.91	0.00	0.69	0.69	0.70	0.72
2	−2.43	0.02	0.12	0.09	0.27	0.47
3	−2.40	0.10	0.13	0.09	0.30	0.52
4	−2.20	0.21	0.14	0.11	0.33	0.58
5	−1.97	0.35	0.15	0.12	0.37	0.65
6	−1.71	0.52	0.16	0.14	0.43	0.74
7	−1.43	0.72	0.18	0.16	0.50	0.71
8	−1.12	0.95	0.21	0.19	0.59	0.63
9	−0.79	1.21	0.24	0.23	0.51	0.51
10	−1.04	0.67	0.87	0.87	0.93	0.93
Avg	−1.70	0.47	0.29	0.27	0.49	0.65

Columns 2 and 3 show the average of beginning-of-period and end-of-period financial assets on steady-state cycling. This averaging achieves more realism for comparison to the data: Survey questions about current ownership of assets should reflect not only the level of assets brought from the previous year but also saving during the current year. The values in these two columns correspond to ratios to annual labor income—calibrated to match household data as discussed in Section 4.1.

The other columns show the marginal propensities to spend both in non-durable goods, including the periodic expenditure, and in down payments for durable goods purchases.

Comparing the impact of a one-year transfer, column 4, to that of a one-year balanced-budget rebate, column 5, shows that the future tax increase has little effect. This reflects the shortening of the planning horizon given the forthcoming binding borrowing constraint. This implies that future tax increase could be replaced by a government spending cut of the same present value without changing the results much. Hence, assumption (c), that government spending is not affected by the tax cut, is not very important for the results.

For unconstrained households, the effects of the one-year transfer in column 4 should be 0.04/1.04, which corresponds to the permanent-income hypothesis given the interest rate. For the liquidity constrained households in this table, the effects are much higher—from 12 to 87 percent—the first corresponding to those with $\kappa = 2$, for whom the number of unconstrained periods is the largest. For the balanced-budget rebate in columns 5, 6 and 7, the Ricardian counterpart is zero—while here they range from 9 to 93 percent.

A particularly interesting feature of Table 1 is the pattern of the marginal propensities for different level of assets. The pattern of marginal propensities by asset level is not monotonic. The highest marginal propensity corresponds to households in the 7th decile of the asset distribution—those with $\kappa = 10$. The next highest marginal propensity is for households with no assets, except for the five-year persistence in column 7. For the latter case, the second largest marginal propensity is for the 6th decile—those with $\kappa = 6$.

Overall, with some imagination, the relationship between the MPC and assets can be seen as a "forward-slanted S". Households with no assets have a high marginal propensity to spend. Households with positive but small assets have a much lower propensity than those with zero assets, and, peculiarly to this model, also lower than those with higher assets. Hence, for the first 7th deciles, there is a strong positive correlation between asset levels and the propensity to spend. These correlation is reversed at the highest asset deciles, although the marginal propensities remain higher than for the first decile.

4.4.1 Sensitivity Analysis

As mentioned in Section 4.1, the calibration of the expenditure cycle is based on the arbitrary choice of $\tau = 10$. Here, we report the results for two alternative values of the cycle length: $\tau = 6$ and $\tau = 14$. The resulting calibrated values of μ are 1.83 and 2.41—while for $\tau = 10$: $\mu = 2.016$.. The steady-state cycle and the MPC's are shown in the appendix. Overall, the results are similar to those presented above, but there are some differences.

The cyclical pattern for the alternative calibration is shown in Figures A1 and A2. When $\tau = 6$, the borrowing constraint does not bind at the beginning of the cycle—it binds only on $\kappa = 6$—while in the basic calibration, the constraint binds also in the first period of the cycle. This follows from the closer next periodic expenditure, which induces to start saving right after the last expenditure. In contrast, when $\kappa = 14$ the borrowing constraint binds at the beginning of the cycle for three periods.

Tables A1 and A2 show the MPC's for these alternative cases. The average values are similar to those in the basic case, specially for the one-year transfer and rebate. For the three-year and five-year rebate, however, extending the cycle lowers the average MPC because the three and five years persistence becomes less important as the cycle is longer, weakening the effect of extending the horizon beyond the next expenditure period.

4.5 Comparison with Survey Evidence

Here we interpret the survey evidence on the 2001 tax rebate in Shapiro and Slemrod (2003a,b) and in Johnson, Parker and Souleles (2006) using the results from this model, as summarized in Table 1. This comparison is subject to the qualification that these studies addressed a representative sample of the whole U.S. population, while the present model characterizes home owners only.

Shapiro and Slemrod (2003a,b) find that 22 percent of the households spent most of the rebate. The present model approximates this outcome when the rebate is perceived as purely temporary, as in column 5: Total spending as a fraction of the rebate is higher than 50 percent for two values of κ: 1 and 10. Hence, if the model's households are uniformly distributed over the spending cycle, 20 percent of them spend most of the rebate. When perceived persistence is higher, as in columns 6 and 7, this fraction goes up to 40-50 percent for 3 years and 90 percent for five years. The average MPC in column 5 is 26.9, somewhat lower than the 1/3 value inferred in Shapiro and Slemrod (2003b).

Johnson, Parker and Souleles (2006) focus directly on the MPC, and on nondurable consumption only. Their main result is that about 2/3 of the rebate was spent within six months. The results for five-year persistence in column (7) indicate an average marginal propensity of 64.9, very close to their findings, but this figure corresponds to total spending, and not to nondurables only. The average fraction spent on nondurable consumption with five-year persistence is 39.2, which is similar to their average fraction spent during the first three months following the rebate. The six-months estimate in Johnson, Parker and Souleles has a wide standard error, unlike their three-months estimate. The model's value 39.2 for nondurable consumption is one-standard deviation from their estimate 2/3, and thus it's well within a standard confidence interval around this estimate.

Our results provide an explanation to Shapiro and Slemrod's findings about increasing marginal propensities to spend by assets levels—which is puzzling when considering asset ownership as positively related to liquidity. They found that the fraction of households that would spend most of the rebate is 19.5 for the zero stocks group, and for other five groups corresponding to stock ownership brackets from \$1-\$15,000 to \$250,000 and above, these fractions are 13.1, 18.1, 26.7, 33.6 and 22.9. This pattern resembles the "forward-slanted S" from Table 1. If there is a positive association in the data between the fraction of households spending most of the rebate in each group, and the average fraction of the rebate spent, the model rationalizes this evidence along the lines described in Section 4.2. Those with zero assets are households similar to the typical consumer in the literature, for whom a past event—the periodic expenditure here—depleted all financial assets and hence they are borrowing constrained. These households desire to borrow to increase current spending but they can't, so they spend most of the rebate when it comes. As soon as assets become positive, the borrowing constraint ceases to bind. This happens at a time when the next periodic expenditure is still far ahead; hence, households smooth consumption over a relatively large interval, implying a relatively low MPC. Accordingly, asset accumulation reflects the shortening of the consumption smoothing horizon, generating an increasing MPC.

According to the model, the reversal at the top of the asset distribution reflects two effects: One is that the highest marginal propensity applies to households right on the expenditure period where the borrowing constraint is the strongest and thus the MPC is the highest. Correspondingly, they start the period with the highest level of assets, but end it with zero. Therefore, the highest MPC corresponds to a moderately high level of assets but not the highest. The other effect applies to persistent rebates as in columns 6 and 7. As the expenditure period gets closer, the expected persistence increasingly overlaps with the next cycle, and this extends the household's planning horizon.

Shapiro and Slemrod (2003a) discuss another but related mechanism that can generate a positive correlation between assets ownership and the tendency to spend most of the rebate for households with positive assets. If they have an optimal asset level, those with little assets should be in the process of building their optimal stock; hence, they are likely to save most of the rebate. As stock ownership increases, the need to save declines, and the tendency to spend goes up. This mechanism and the one in this paper are based on two mechanisms for building asset stocks, and it's difficult to tell them apart in the evidence, except for the monotonicity of the relationship in the optimal stock story, and the non-monotonic pattern in the present story. The evidence Shapiro and Slemrod present in their Table 2 shows a decline in the fraction of spenders at the top stock bracket, which seems more in line with the present mechanism.

5 Concluding Remarks

How much liquidity constrained are U.S. middle-class households? To address this question, we developed a model where households are home owners and hold financial assets, and measured liquidity constraints with the fraction spent out of a temporary tax rebate—compared to an unconstrained response of zero.

In the model, a future binding borrowing constraint effectively shortens the planning horizon of households who are infinite-horizon planners. A basic feature is the conceptual distinction between binding borrowing constraints and binding liquidity constraints. The calibrated version of the model has the following implications:

- At a any point in time most households are not borrowing constrained, i.e., their current Euler equations hold. However, all of them are liquidity constrained to some degree; they value a consumption unit on hand more than future amounts with the same present value.
- The spending responses to a transitory transfer in the model are much higher than for a permanent-income consumer. Furthermore, the responses to a transitory transfer and to a tax cut financed by a future permanent tax hike are not very different. In other words, future tax changes have little effect on current decisions. This implies that the response of these households to a balanced-budget tax rebate differs greatly from those of a Ricardian permanent-income consumer, for whom the response is zero.
- The volume of assets owned reflects a liquidity shortage rather than liquidity availability. This feature generates a non-monotonic relationship between assets and the marginal propensity to consume out of temporary income. This relationship is positive for household with positive assets.

The last point provides a rationalization of the finding in Shapiro and Slemrod (2003a), that the fraction of households who spent most of the 2001 rebate increases with stock ownership—among households with positive amounts of shares. The model also rationalizes their result that households with zero shares are more likely to spend most of the rebate than households with positive but small amount of shares, but less than some with higher stocks of shares.

According to the results in this paper, if the rebate is expected to persist from one year to five years, households spend in the first year between 27 to 65 percent. These figures are realistic given the evidence on the 2001 tax rebate. We interpret these figures as a quite important degree of liquidity constraints for households who are modelled as home owners and asset holders.

In this paper we did not address the macroeconomic implications of these liquidity constraints. This requires closing the model with a production side, adding patient households, and solving for the general equilibrium. We plan to address general equilibrium separately.

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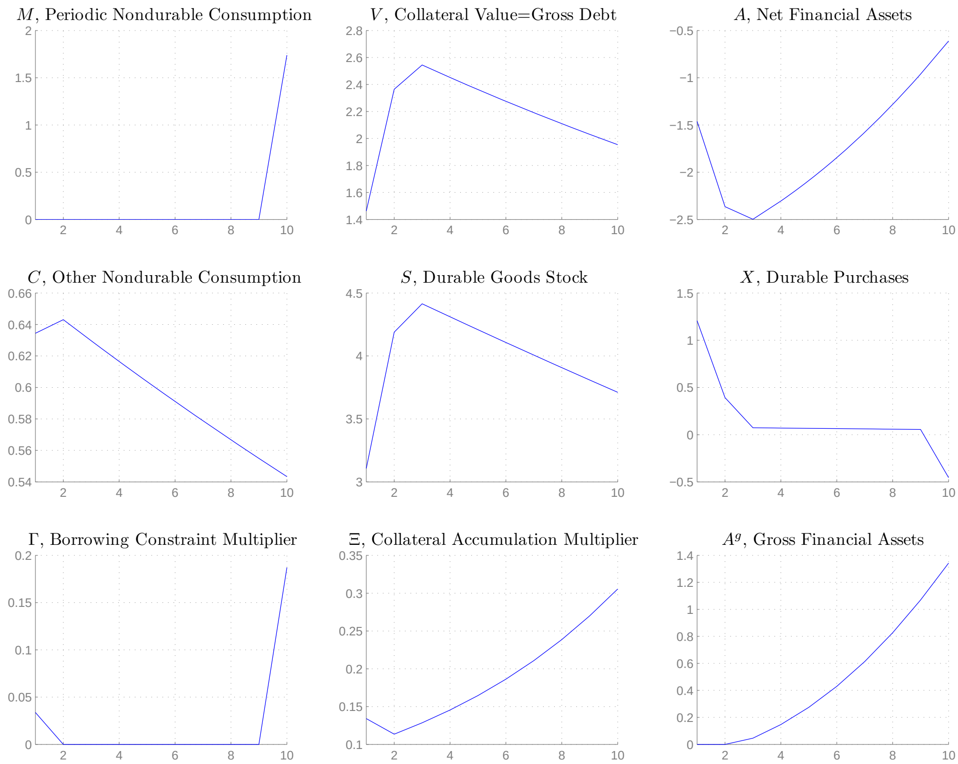

Figure 1: Steady-State Cycle

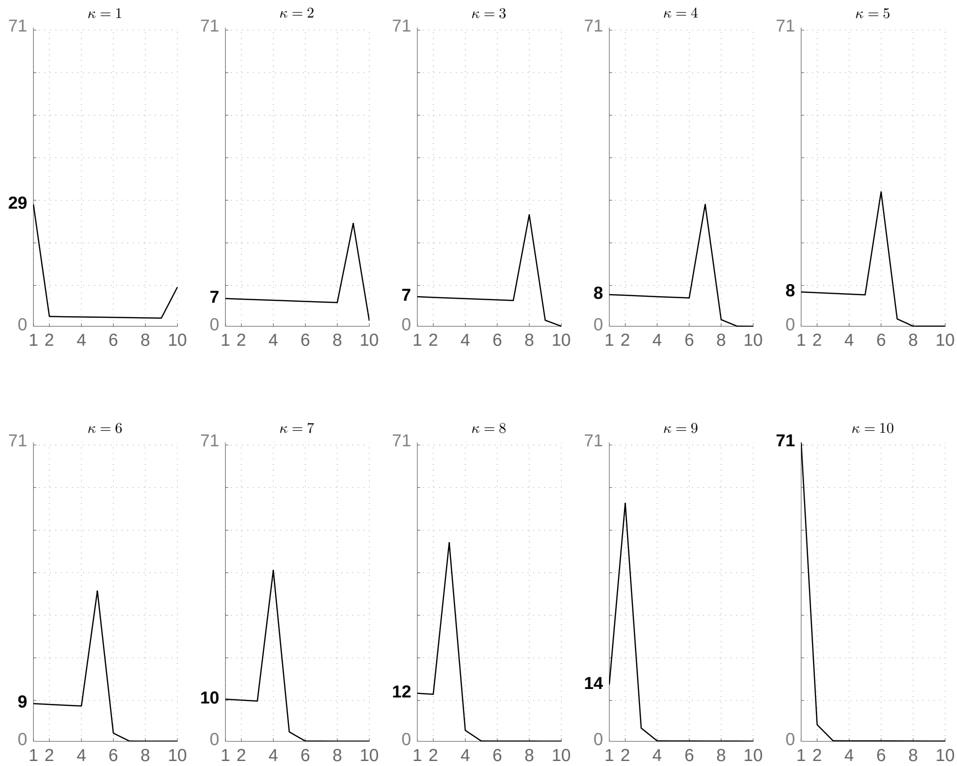

Figure 2: One-Year Transfer – Nondurables

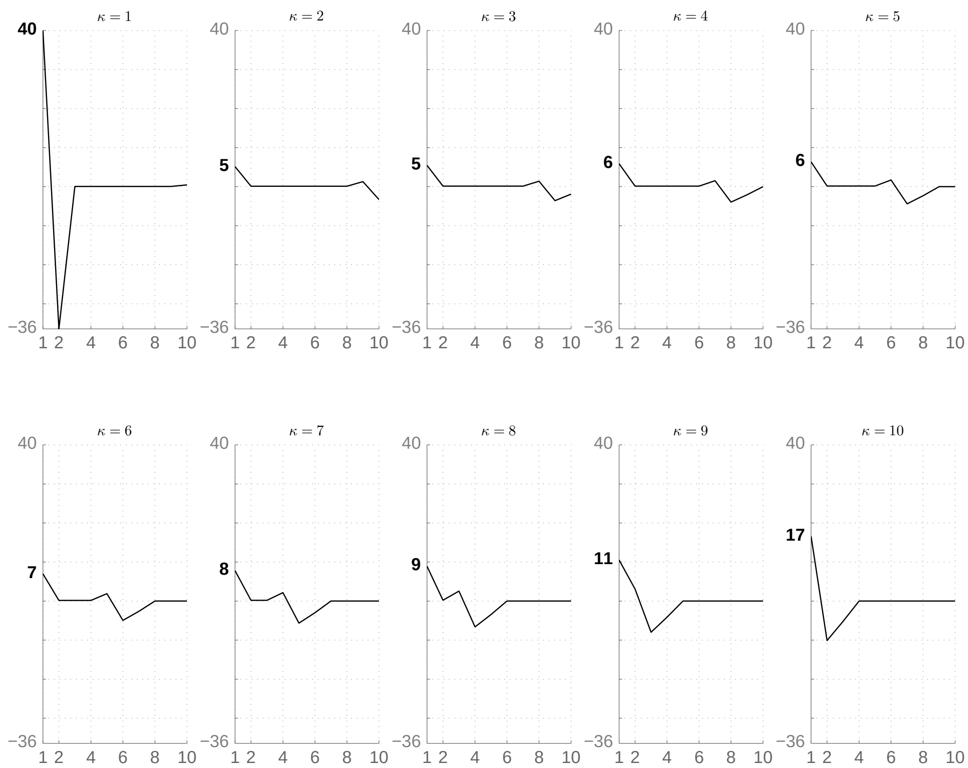

Figure 3: One-Year Transfer – Durable Purchases

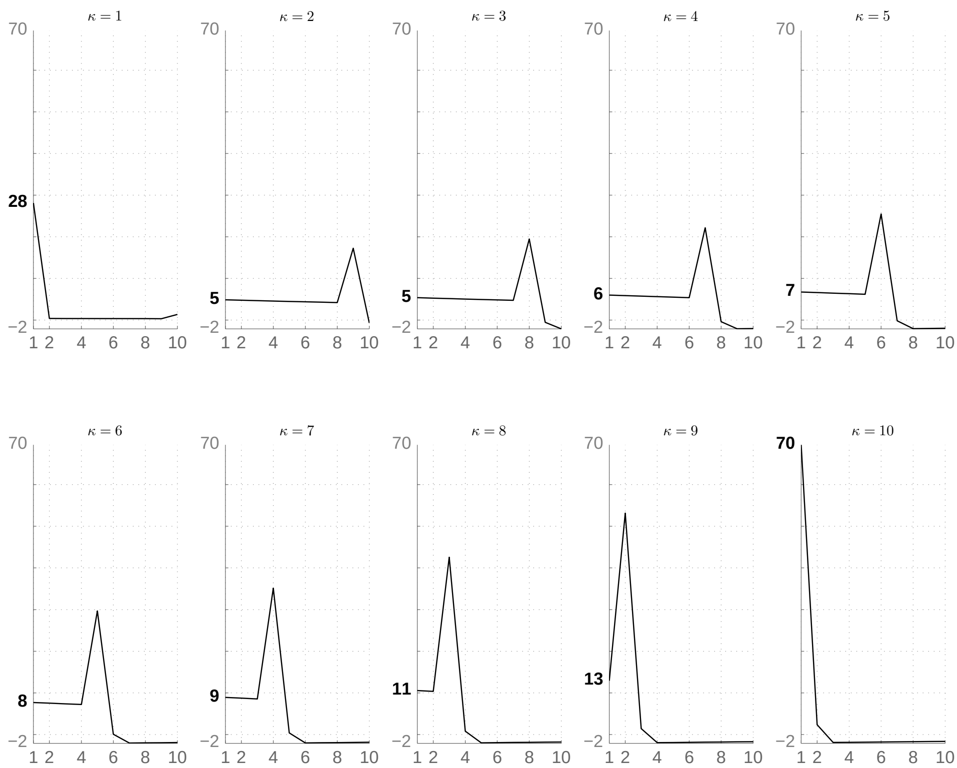

Figure 4: One-Year Tax Rebate – Nondurables

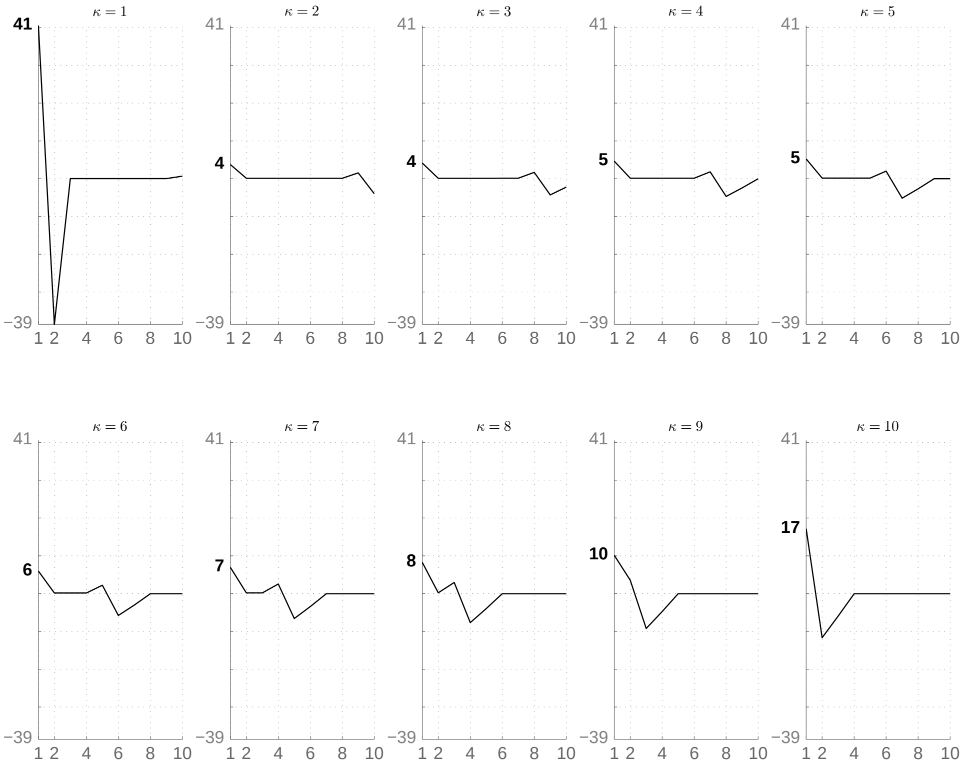

Figure 5: One-Year Tax Rebate – Durable Purchases

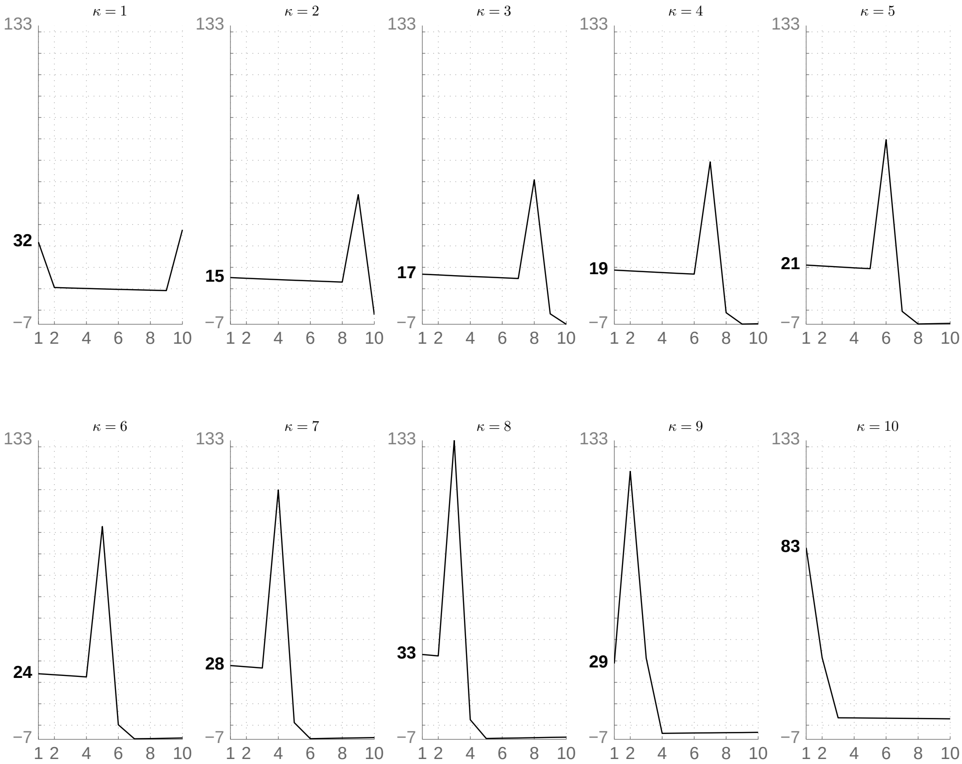

Figure 6: Three-Year Tax Rebate – Nondurables

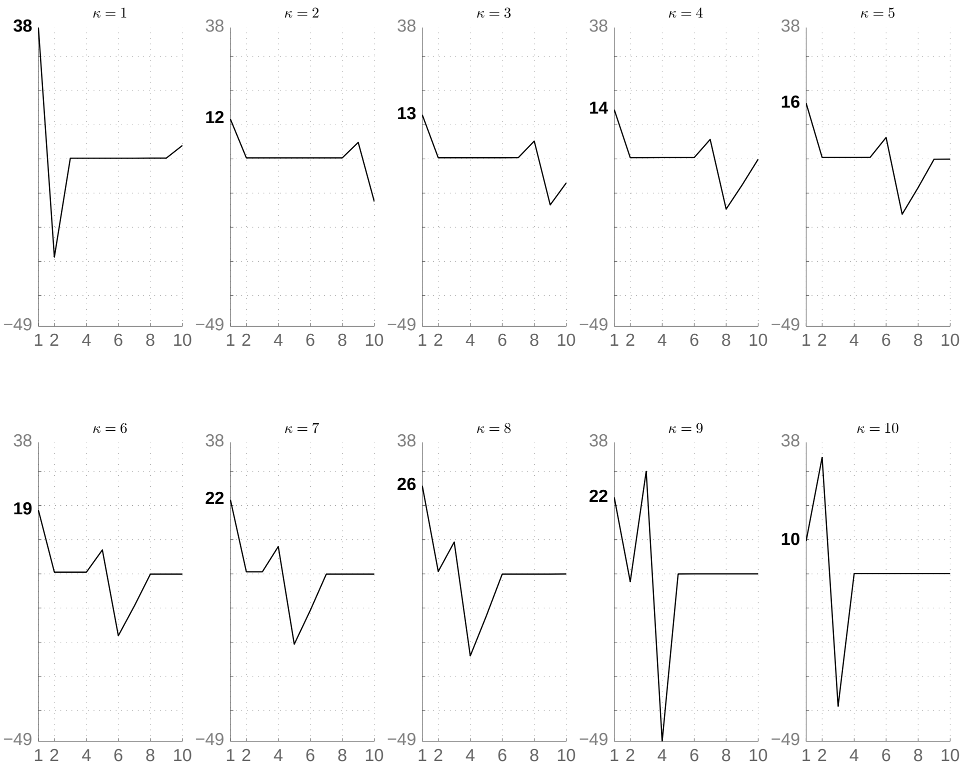

Figure 7: Three-Year Tax Rebate – Durable Purchases

A Appendix: Sensitivity Analysis Results

In this appendix we present the cyclical pattern and MPC's for $\tau = 6$ and $\tau = 14$, discussed in Section 4.4.1.

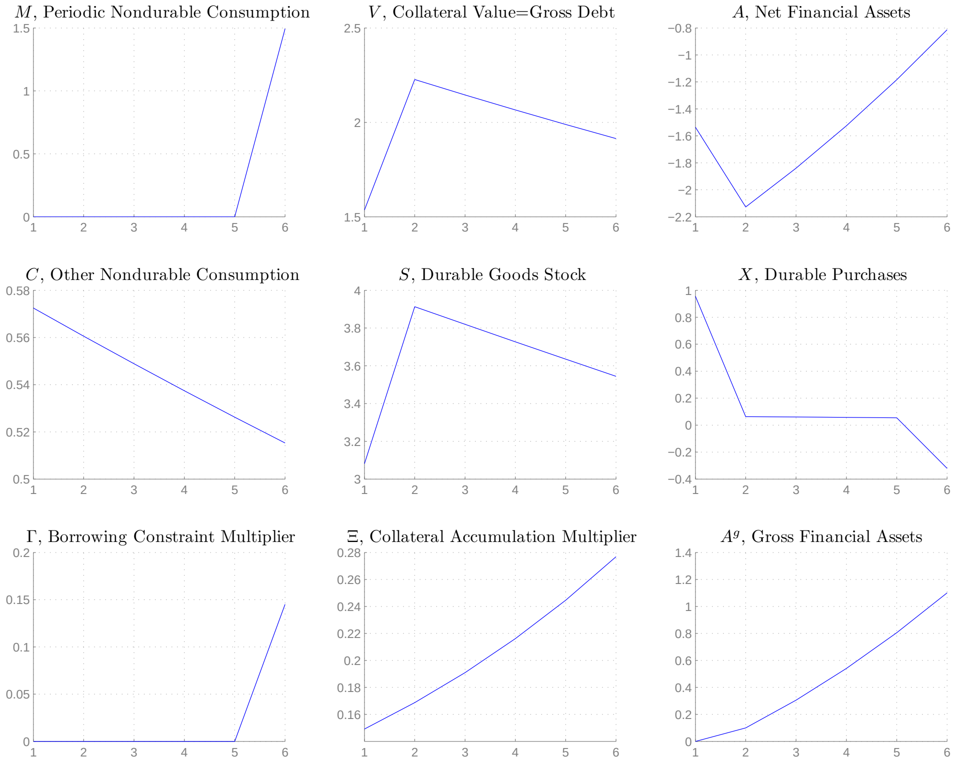

Figure A1: Steady-State Cycle ($\tau = 6$)

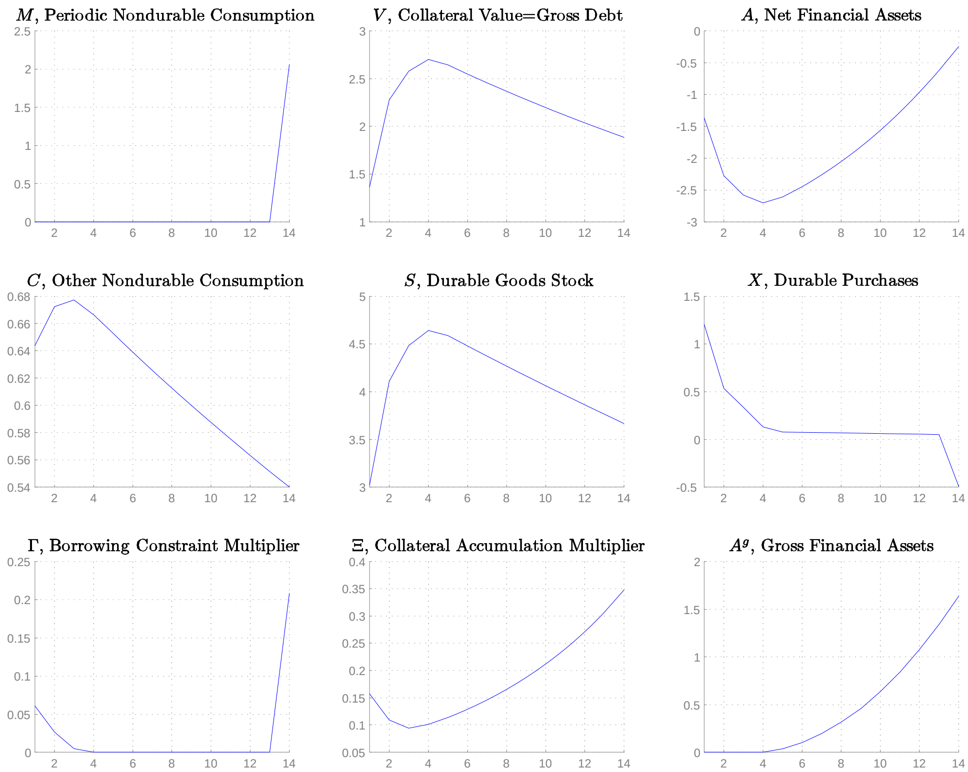

Figure A2: Steady-State Cycle ($\tau = 14$)

Table A1: Financial Assets and Marginal Propensities to Spend ($\tau = 6$)

	Financial Assets		Lump-Sum Transfer	Balanced-Budget Tax Cut		
κ	Net	Gross	One Year	One Year	Three Years	Five Years
1	-1.83	0.05	0.15	0.12	0.37	0.65
2	-1.98	0.20	0.16	0.14	0.43	0.74
3	-1.68	0.42	0.18	0.16	0.50	0.68
4	-1.35	0.67	0.21	0.19	0.59	0.61
5	-1.00	0.95	0.24	0.23	0.48	0.51
6	-1.17	0.55	0.85	0.85	0.88	0.91
Avg	-1.50	0.47	0.30	0.28	0.54	0.68

Table A2: Financial Assets and Marginal Propensities to Spend ($\tau = 14$)

	Financial Assets		Lump-Sum Transfer	Balanced-Budget Tax Cut		
κ	Net	Gross	One Year	for One Year	for Three Years	for Five Years
1	−1.82	0.00	0.71	0.70	0.75	0.76
2	−2.43	0.00	0.70	0.70	0.74	0.74
3	−2.64	0.00	0.68	0.68	0.69	0.71
4	−2.65	0.02	0.11	0.07	0.22	0.38
5	−2.53	0.07	0.11	0.08	0.24	0.42
6	−2.35	0.15	0.12	0.08	0.26	0.46
7	−2.16	0.25	0.12	0.09	0.29	0.50
8	−1.94	0.39	0.13	0.10	0.32	0.56
9	−1.69	0.55	0.14	0.12	0.36	0.63
10	−1.42	0.74	0.16	0.13	0.41	0.71
11	−1.12	0.96	0.17	0.15	0.47	0.67
12	−0.79	1.21	0.19	0.18	0.56	0.60
13	−0.43	1.49	0.23	0.21	0.48	0.49
14	−0.81	0.82	0.89	0.88	0.96	0.97
Avg	−1.77	0.47	0.32	0.30	0.48	0.61