How Democracy Resolves Conflict in Difficult Games

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Abstract. Democracy resolves conflicts in difficult games like Prisoners' Dilemma and Chicken by stabilizing their cooperative outcomes. It does so by transforming these games into games in which voters are presented with a choice between a cooperative outcome and a Pareto-inferior noncooperative outcome. In the transformed game, it is always rational for voters to vote for the cooperative outcome, because cooperation is a weakly dominant strategy independent of the decision rule and the number of voters who choose it. Such games are illustrated by 2-person and n-person public-goods games, in which it is optimal to be a free rider, and a biblical story from the book of Exodus.

How Democracy Resolves Conflict in Difficult Games[1]

1. Introduction

A cornerstone of *democracy* is fair and periodic elections. While there is an ongoing debate about how best to conduct elections (Brams, 2008), here we assume that voters choose between two alternatives, and the alternative with the most votes wins.

We say that voting in a democracy *resolves conflict* if the electorate considers (i) the voting process fair and (ii) the outcome chosen acceptable. There may not be a consensus among the voters that the alternative chosen is the best, but as long as some agreed-upon minimum number of voters (e.g., a majority) support some alternative, this outcome will be implemented.

In this note, we focus on choices that are costly to implement. For example, if voters in a referendum decide to finance a public project, the cost of this project will be reflected in higher taxes they must pay.

Suppose that the project is renovation of a public park, which can benefit everybody—but more so those who use the park frequently than those who don't. In this case, some would argue that those who use the park frequently should pay more for its renovation, such as through the Central Park Conservancy in New York City, which solicits voluntary contributions. But this voluntary approach leads to a *public-goods* or *free-rider problem*, which we model as an n-person Prisoners' Dilemma (PD).

2. Resolution by Voting in a 2-Person PD

To render the subsequent analysis as transparent as possible, we begin with a 2-person PD, wherein one player is a wealthy individual who can make a big contribution

[1] We thank Christian Klamler for valuable comments on an earlier version of this paper.

to the renovation of the park. Suppose his or her contribution is expected to equal the contributions made by the rest of the public, whom we treat as a single player. In the ranking of payoffs to the players below, we assume that the wealthy individual and the rest of the public both prefer partial renovation without contributing (4) to full renovation with contributing (3) to no renovation without contributing (2) to partial renovation with contributing (1), as shown in the payoff matrix below:

Rest of Public ⇒ **Wealthy Individual ⇓**	**Contribute**	**Don't contribute**
Contribute	Full renovation: (3,3)	Partial renovation: (1,4)
Don't contribute	Partial renovation: (4,1)	No renovation: (2,2)

Key: (x, y) = payoff ranking to (wealhy individual, rest of public), where 4 = best, 3 = next best, 2 = next worst, and 1 = worst.
Nash equilibrium underscored.

Each player's strategy of don't contribute *strictly dominates* its strategy of contribute, because it is better whichever strategy the other player chooses. Each player, therefore, has an incentive to be a *free rider*, obtaining the benefit of the public good without contributing to it.

But the choice by both players of don't contribute leads to the next-worst outcome of (2,2), which is the unique *Nash equilibrium*—neither player would have an incentive unilaterally to depart from it lest it do worse (by obtaining 1).[2] The dilemma is that (2,2) is worse for both players than the cooperative outcome of (3,3), wherein both players contribute. But the latter outcome is not a Nash equilibrium—each player would have an

[2] The Nash equilibrium is actually the pair of pure strategies of the players associated with (2,2), not the outcome itself, but for convenience we identify Nash equilibria by the outcomes they produce. Note that there is no possibility of mixed strategies in the game, because payoffs are ordinal rankings, not cardinal utilities.

incentive unilaterally to depart from its strategy associated with it (to obtain 4)—rendering it unstable.

To be sure, (3,3) may be stabilized under certain conditions—for example, in tournament play (Axelrod, 1984), in strategies that evolve over time (Skyrms, 1996; Nowak, 2006), or when players are farsighted (Brams, 1994).[3] Farsighted thinking, which nonhuman animals seem incapable of, is epitomized by Theodore Sorensen's statement about the deliberations of the Executive Committee (ExCom) during the October 1962 Cuban missile crisis:

> We discussed what the Soviet reaction would be to any possible move by the United States, what our reaction with them would have to be to that Soviet reaction, and so on, trying to follow each of those roads to their ultimate conclusions (Holsti, Brody, and North, 1964, p. 188).

Because of such farsighted calculations on both sides, the crisis subsided and war was averted, though some argue that the game played resembled Chicken (game 8 in Figure 1) more than PD.[4]

Farsighted thinking aside, what resolution does democracy, and voting in particular, offer in PD, Chicken, and the other difficult games we present later? Assume that the players in the preceding Prisoners' Dilemma can first vote on whether to contribute or not contribute to financing the renovation of the park. If a majority (i.e.,

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[3] Farsightedness offers a very different resolution of PD than tournament play or evolution. Pinker (2007, p. 71) distinguishes the former from the latter by arguing that "natural selection [in evolution] is like a design engineer in the sense that parts of animals become engineered to accomplish certain things, but it is not like a design engineer in that it doesn't have long-term foresight." Presumably, only humans possess this foresight and can anticipate that if they move from (3,3), it will not necessarily induce their best outcome of (4,1) or (1,4) but, instead, may trigger a countermove by the player receiving 1 to (2,2). Because this outcome is worse for both players than (3,3), (3,3) is a "nonmyopic equilibrium" in PD if the players start at this outcome and think ahead (Brams and Wittman, 1981; Kilgour, 1984; Brams, 1994).

[4] Like PD, the cooperative outcome in Chicken is a nonmyopic, but not a Nash, equilibrium. In fact, the game that best models this crisis, and its resolution, is probably neither PD nor Chicken but a different game (Brams, 1994, pp. 130-138).

both players) must vote to finance the park in order that it be renovated, then their choices and the resulting outcomes are shown in the game below:

Rest of Public ⇒ **Wealthy Individual** ⇓	**Vote to finance**	**Vote not to finance**
Vote to finance	Full renovation: <u>(3,3)</u>	No renovation: (2.2)
Vote not to finance	No renovation: (2,2)	No renovation: (2,2)

Key: (x, y) = payoff ranking to (wealthy individual, rest of public), where 4 = best, 3 = next best, 2 = next worst, and 1 = worst.
Nash equilibrium underscored.

Notice that the option that the park be partially renovated does not appear in the payoff matrix. Instead, the outcomes are starker: The park is either fully renovated or not renovated, which renders the cooperative outcome of full renovation the unique Nash equilibrium; moreover, it is supported by *weakly dominant* strategies of the players.[5] This transformation may be viewed as a mapping of two of the four outcomes in the PD (full renovation and no renovation) into the new game, with voting determining which outcomes these two outcomes replace.

3. Resolution by Voting in an *n*-Person PD

To extend this resolution of a 2-person PD to an n-person public-goods game, assume there are $n \geq 2$ players and two strategies, Cooperate (C) and Defect (D), that each player can choose. If k players cooperate, the payoff to each cooperator is the amount $c(k)$, where $k = 1, 2, \ldots, n$, and the payoff to each defector is the amount $d(k)$,

[5] Why "weakly"? Unlike PD, each player's cooperative strategy associated with (3,3) is not strictly better, whichever strategy the other player chooses: If the other player votes not to finance, either voting to finance or voting not to finance leads to the same outcome of (2,2). Because of this "tie," voting to finance is not *always* better than voting not to finance.

where $k = 0, 1, \ldots, n–1$.[6] An n-person game that satisfies the three properties given below mimics the characteristics of the 2-person PD:

Properties of *n*-Person PD

1. The payoffs $c(k)$ and $d(k)$ are increasing in k. That is, when more players cooperate, all benefit—whether they chose C or D—because more of the public good is provided.
2. For each $k = 1, 2, \ldots, n$, $c(k) < d(k–1)$. That is, comparing the situations in which there are (i) k cooperators and (ii) $k – 1$ cooperators after the defection of a cooperator, each of the defectors in the latter situation receives a greater payoff than each of the cooperators in the former situation, given that the strategies of all other players are fixed.
3. $c(n) > d(0)$. That is, when all players choose D, the resulting outcome is *Pareto-inferior*, or worse for all players, than the outcome in which all cooperate.

Property 2 implies that, for each player, C is a strictly dominated strategy. To see this, fix a player and suppose that $k – 1$ other players choose C and the remaining $n – k$ choose D. Then the focal player will receive $c(k)$ for choosing C and $d(k–1)$ for choosing D. Because this conclusion holds for every value of k, D strictly dominates C for every player.

It follows that the unique Nash equilibrium in the n-person PD is for all players to choose D and receive $d(0)$. Because this strategy profile is supported by strictly

[6] Because $c(k)$ and $d(k)$ are indexed differently, we can compare $c(k)$ and $d(k–1)$ over all k, as we do in property (2) below.

dominant strategies, the resulting all-D Nash equilibrium is especially stable. But by property 3, the nonequilibrium outcome of all-C, at which all players receive $c(n)$, is strictly preferred by all players to $d(0)$. Thus, this n-person PD has a unique strictly dominant strategy of D for each player, but when all players choose it, a strictly Pareto-inferior outcome results.

The resulting n-person PD has all the problems of the 2-person PD and more. When there are only two players, they may well stabilize the cooperative outcome by implementing an enforcement mechanism, such as regular inspections in an arms-control agreement, that transforms the PD into a more benevolent game, with the cooperative outcome as the unique Nash equilibrium.

But if there are many players,[7] this becomes far less feasible—short of transforming the game into a voting game, as we will show next. Whereas the voting game we described in section 2 required that only two players agree to contribute to renovation of the park, we now propose that a decision rule be fixed which determines whether a public good is provided. More specifically, we assume that with the introduction of voting by the players, the n-person PD is played according to the rules given below:

Rules of Transformed *n*-Person PD

1. A decision rule r, satisfying $0 < r \leq n$, is fixed and announced to all players.
2. The players vote, independently and simultaneously, for either C or D.

[7] In the preceding example, we treated the "rest of the public" as a single player, but if the game is among many similar players, then it is properly modeled as an n-person PD. To ameliorate the problem of defections in such a game, wealthy individuals often commit to match the donations of small contributors, thereby enhancing the incentive of these individuals to contribute by guaranteeing that their donations will be increased by some factor.

3. If the number of players that vote for C is $m < r$, then the all-D outcome is implemented, so all players receive $d(0)$. But if $m \geq r$, then the all-C outcome is implemented, so all players receive $c(n)$.

It is easy to check that a player's choice of C or D only affects its payoff when exactly $m - 1$ other players choose C. In this case, the player receives $c(n)$ for choosing C and $d(0)$ for choosing D; by property 3, the player prefers $c(n)$.

Because voting for C sometimes results in a better outcome and never results in a worse outcome, it is a weakly dominant strategy, as it is in the transformed 2-person PD. Thus, the all-C outcome, supported by the players' weakly dominant strategies of voting for C, is the unique Nash equilibrium in the transformed n-person PD.[8]

4. Example of an *n*-Person PD

Suppose there are $n = 10$ players, and the payoff functions to the cooperators and the defectors are $c(k) = 10k - 50$ and $d(k) = 10k$. It is easy to show that the three properties of an n-person PD are satisfied:

1. The payoffs to the players are increasing in k.
2. $c(k) = 10k - 50 < d(k{-}1) = 10(1{-}k)$, which simplifies to $-50 < -10$ and so is satisfied.
3. $c(n) = 100 - 50 > d(0) = 0$, which simplifies to $50 > 0$ and so is satisfied.

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[8] Hardin (1971) shows that all-C is a *Condorcet choice* when pitted against any other strategy combination—that is, a majority of voters would prefer it, except in the case of a tie—but he does not provide a procedure that would implement all-C.

Let $k = 1, 2, . . ., 10$. The payoff for being the k^{th} cooperator, $c(k)$—as opposed to defecting and there being one less cooperator, $d(k–1)$—are shown for representative values of k in the table below:

No. of coop-erators ⇒	$\mathbf{k = 1}$	$\mathbf{k = 2}$	$\mathbf{k = 5}$	$\mathbf{k = 9}$	$\mathbf{k = 10}$
$\mathbf{c(k)}$	–40	–30	0	40	50
$\mathbf{d(k–1)}$	0	10	40	80	90

Notice that $k = 5$ cooperators make the value of cooperation, $c(5) = 0$, equal to the value of defection by everybody, $d(0) = 0$, in the n-person PD. Thus, 5 cooperators is the *breakeven number* at which funding the project has the same value for the cooperators as not funding it.

Whereas all-D at $D(0)$ is the Nash equilibrium in the n-person PD, all-C at $c(10)$, which gives a payoff of 50 to each player, is not an equilibrium. The latter outcome is unstable because if one player defects from all-C, he or she receives a payoff of $d(9) =$ 90. In fact, as we know from the previous analysis, every player has a strictly dominant strategy of defecting in the n-person PD, however many cooperators there are.

Now assume simple-majority rule is used in the transformed n-person PD (i.e., $r =$ 6), so if there are 5 or fewer cooperators, no project is funded. But if there are 6 or more cooperators, everyone, including the defectors, gets a payoff of 50. If we depict the game as a 10-dimensional array in which each of the 10 players can choose between C and D, then C weakly dominates D for each player, whatever the value of r is, but the contingency in which C makes a difference (by raising a player's payoff from 0 to 50) changes when r changes.

Although the value of r does not affect the weak dominance of C, it would be strange indeed if r were not at least a simple majority (6 in our example), because less than a majority of cooperators could implement a project, perhaps against the wishes of a majority. (In the extreme case, it would be a single player, or *dictator*, that would call the shots.) Accordingly, we propose that r be at least a simple majority in the transformed n-person PD.

In fact, a simple majority may be preferable to a qualified majority, because a simple majority is more robust against defectors. Thus in our example, selecting $r = 6$ means that even if up to 4 players choose D (for whatever reasons), the majority would still triumph, whereas this would not be the case for a greater r. In particular, if $r = 10$ (unanimity), one defector can undermine the choice of C by the other 9 players.

5. A Biblical Tale

A story from the Hebrew Bible illustrates how a group, aided by a charismatic leader, may resolve an n-person PD when individuals alone cannot not do so.[9] The story begins after Moses descends from Mount Sinai and discovers that the Israelites, who had grown restive during his absence of forty days and forty nights, had built, with the complicity of Aaron (Moses's brother), a golden calf that they worshiped.

[9] This story is adapted from Brams (2000, 2003, pp. 94-98), but the interpretation of Moses's resolution of an n-person PD via a kind of referendum is new. Passages from the Bible are drawn from *The Torah: The Five Books of Moses* (1962). Schelling (1978, ch. 7) gives several contemporary examples of n-person PDs, such as whether a hockey player should wear a helmet, which was not mandated by the NHL until the 1990s. Prior to 1990, most players refused to wear helmets because it put them at a strategic disadvantage, limiting their peripheral vision, though they were at a substantially greater risk of serious head injury. The dilemma was resolved not by a secret vote of the players, which arguably would have led to the requirement of helmets in the 1970s, but by a public outcry, which put pressure on the NHL. Even so, players who entered the league before the helmet requirement were exempted; the last player to refuse to wear a helmet retired in 1997.

Observing the revelry of the Israelites at the base of the mountain, Moses is enraged and destroys the Ten Commandments. But he must also deal with another problem—the extreme anger of God, who is infuriated by the idolatry of the Israelites and threatens to destroy them:

> "I see this as a stiffnecked people. Now, let Me be, that My anger may blaze forth against them and I may destroy them, and make of you a great nation." But Moses implored the LORD his God, saying, "Let not Your anger, O Lord, blaze forth against Your people, who You delivered from the land of Egypt with great power and a mighty hand. Let not the Egyptians say, 'It was with evil intent that He delivered them, only to kill them off in the mountains and annihilate them from the face of the earth.'" (Exod. 32:9-12)

Moses offers a cogent reason why the Israelites should be spared, asking God to

> turn from Your blazing anger, and renounce the plan to punish Your people. Remember Your servants, Abraham, Isaac, and Jacob, how You swore to them by Your Self and said to them: I will make your offspring as numerous as the stars of heaven, and I will give to your offspring this whole land of which I spoke, to possess forever. And the LORD renounced the punishment He had planned to bring upon His people. (Exod. 32:14-15).

Thus God, realizing the enormous investment he has made in His chosen people, does not brush aside His handiwork out of pique.

Although God relents, Moses must still convince Him that His decision to save His chosen but "stiffnecked" people, who had "acted basely" (Exod. 32:7), is not a foolish one. After wringing a confession out of Aaron for his part in the idolatrous affair, Moses looks with horror on the Israelites, who are "out of control" (Exod. 32:25).

Moses averts catastrophe by seizing the initiative: "Whoever is for the LORD, come here" (Exod. 32:26). Moses's gamble pays off, at least for one tribe:

> And all the Levites rallied to him. He said to them, "Thus says the LORD, the God of Israel: Each of you put sword on thigh, go back and forth from gate to gate throughout the camp, and slay brother, neighbor, and kin." The Levites did as Moses had bidden, and some three thousand of the people fell that day. And Moses said, "Dedicate yourselves to the LORD this day—for each of you has been against son and brother, that He may bestow blessing upon you today." (Exod. 32:26-29).

I interpret Moses's summons to "come here" as less a command than a desperate plea for a sizeable number—if not a majority—of the Israelites to rally to the side of the LORD and renounce their sinful behavior. In effect, Moses asks for the Israelites to vote in a referendum on his leadership.

If only a few Israelites had heeded Moses's plea and supported him, their numbers would not have been sufficient to persuade God that they were willing to turn from their idolatrous ways and worship Him as their rightful God, "who brought you out of the land of Egypt!" (Exod. 22:8). But Moses wants not just a vote of confidence but also seeks the annihilation of all dissidents.

This serves his and God's purpose by wiping out the last vestiges of idolatry among the Israelites. That the faithful are spared reinforces God's message since the time of Adam and Eve—He is stern in punishing sinners—but He is also merciful in protecting those who redeem themselves.

Effectively, Moses's solution to the n-person PD—whereby D is for the Israelites to continue to worship the golden calf and C is for them to return to the God of Israel—is to eliminate the outcome in which some Israelites choose D and some choose C. True, it is nowhere specified that if r Israelites choose C, C will be implemented. To prevent defections from this outcome, Moses deemed it necessary that those who chose D be decimated. This is a gruesome way to achieve consensus, but it is hardly unknown in recent times.

The solution worked, at least for a while. (The Israelites become restive again.) However, we strongly recommend voting, without the sacrifice, as a more civilized way to resolve *n*-person PDs.

6. Other Difficult Games

The hypothetical example we discussed in section 4 illustrates a public-goods or common-pool game (Ostrom, Gardner, and Walker, 1994), in which there is a free-rider problem unless a mechanism like voting is introduced to transform the game into one that encourages cooperation. In the biblical example in section 5, no Israelite alone has an incentive to support Moses—knowing that his or her faith in God will not appease Moses or save the Israelites from the wrath of God—but if Moses can turn the game into a referendum on his leadership and rally a sufficient number to his cause, then he can snuff out idolatry, especially if those that refuse to go along are eliminated.

PD is only one of the 57 distinct 2×2 strict ordinal *games of conflict,* in which there is no mutually best (4,4) outcome. How many of these games can be transformed into more cooperative games through voting?

Define a *cooperative outcome* in a 2×2 strict ordinal game to be one in which each of the two players obtains either its best (4) or its next-best (3) outcome. Call the players' strategies associated with this outcome *cooperative strategies*. Call the other player strategies *noncooperative strategies*, and the outcome associated with these the *noncooperative outcome.* A 2×2 strict ordinal game is *difficult* if it satisfies the following three conditions:

1. There is only one cooperative outcome.
2. The cooperative outcome is not a Nash equilibrium, so at least one player has an incentive to defect from it.
3. The noncooperative outcome is Pareto-inferior to the cooperative outcome, so both players would prefer the cooperative outcome to it.

Obviously, 2-person PD meets these conditions, but so do the ten other games shown in Figure 1.[10] The eleven games, which constitute 19 percent of all the 2×2 conflict games, can be broken down into three classes:

Figure 1 about here

1. The Nash equilibria in four games, including PD, are the Pareto-inferior noncooperative outcomes. Either one or both (in the case of PD) players has a strictly dominant strategy associated with this equilibrium, and neither player has a dominant strategy associated with the cooperative outcome.
2. The Nash equilibia in three games, including Chicken, destabilize the cooperative outcome by inducing the player(s) receiving a payoff of 3 at the cooperative outcome to defect from it.
3. Three games have no Nash equilibria, with one player having an incentive to defect from each outcome, including the cooperative outcome.

[10] Schelling (1978, ch. 7) offers a different classification of PD and non-PD games, using lines and curves on a graph. Different classifications of the 78 2×2 strict ordinal games, which include the 57 games of conflict and 21 games with a mutually best (4,4) outcome, are given in Rapoport and Guyer (1966, 1976) and Brams (1977).

Note that only PD and Chicken are *symmetric games*, in which the payoff ranks along the diagonal are the same and the payoff ranks along the off-diagonal are mirror images of each other.

Clearly, the cooperative outcome in all eleven games has a shaky status because it is not a Nash equilibrium. But when these games are transformed into voting games in the manner we illustrated for PD, the cooperative outcomes take on a new status: Each becomes the unique Nash equilibrium, stabilized by the weakly dominant strategies that support it.

Unlike PD, we will not try to illustrate these games with examples. But it is worth noting that whether all players receive the same payoff of 3 at the cooperative outcome, or one set of players receives 3 and the other set 4 so their benefits differ (think of frequent and infrequent users of a public park), neither set has an incentive to defect from this outcome in the transformed voting game.

If this outcome does not receive at least *r* votes, its failure cannot be attributed to a public-goods or free-rider problem. Rather, it fails because more voters view the provision of the public good as detrimental—that is, they see the cooperative outcome as Pareto-inferior, not Pareto-superior, to the noncooperative outcome. Put another way, it is a public bad, unworthy of their support.

7. Conclusions

Democracy resolves conflict in difficult games like PD and Chicken by stabilizing their cooperative outcomes. It does so by transforming them into games in which voters are presented with a dichotomous choice between a cooperative outcome and a Pareto-inferior noncooperative outcome. In the transformed game, it is always rational for voters to vote for the cooperative outcome, because *C* is a weakly dominant strategy independent of the decision rule *r* and the number of voters who choose *C*.

Why, then, is the cooperative outcome not always selected, given that voters have no incentive to be free riders in the transformed game? The answer is that the public good may not be viewed by enough voters to be worth the cost of providing it. This explanation for the failure of cooperation—that a majority see the public good as, in fact, a public bad—is very different from the claim that free riders undercut the provision of public goods in a democracy. They do so only if enough voters view them as public bads.

What is "enough"? We suggested that simple-majority rule is more robust than qualified majority rule, because it is not so vulnerable to defectors who may, perhaps out of ignorance, fail to recognize what a majority see as a genuine public benefit.

Even charismatic leaders like Moses, whose brilliant defense of the Israelites—despite their serious lapses—persuaded God that they deserved a reprieve, cannot act alone. He succeeded by persuading the Levites, in a kind of referendum, to renounce their idolatry and, less defensibly, slaughter those who did not go along.

In a standard 2-person PD, it would be odd indeed to ask the players to vote on whether to select *C* and, if both do, implement the cooperative outcome. The difficulty of doing so—say, in an arms race—is that there may be no mechanism to enforce cooperation, even when both sides agree to it.

On the other hand, when a government can credibly commit to providing a public good that a majority support, the solution that democracy provides is compelling. In situations in which crime or corruption is rampant, however, voters will need assurances that procedures have been put in place that ensure that a cooperative outcome that a majority supports will actually be implemented. Thus, while the appeal of democracy is considerable in difficult games, questions about how, practically, to resolve conflicts and implement cooperative outcomes must also be answered.

Figure 1

Eleven Difficult Games

Class 1 (4 games)

1 (27)

(3,4)	(1,2)
(4,1)	(2,3)

2 (28)

(3,4)	(1,3)
(4,1)	(2,2)

3 (32)

(3,3)	(1,4)
(4,1)	(2,2)

Prisoners' Dilemma

4 (48)

(3,4)	(1,1)
(4,2)	(2,3)

Class 2 (4 games)

5 (22)

(3,3)	(2,4)
(4,1)	(1,2)

6 (35)

(4,3)	(2,4)
(3,1)	(1,2)

7 (50)

(4,3)	(2,4)
(3,2)	(1,1)

8 (57)

(3,3)	(2,4)
(4,2)	(1,1)

Chicken

Class 3 (3 games)

9 (29)

(4,3)	(1,4)
(3,2)	(2,1)

10 (31)

(4,3)	(1,4)
(2,2)	(3,1)

11 (46)

(3,4)	(2,1)
(4,2)	(1,3)

Key: (x, y) = payoff ranking to (row, column), where 4 = best, 3 = next best, 2 = next worst, and 1 = worst.
Cooperative outcomes in boldface; Nash equilibria underscored.
Numbers of games in parentheses are those given in Brams (1994, pp. 217-219).

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