

INTELLECTUAL PROPERTY RIGHTS AND ECONOMIC GROWTH

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ABSTRACT. I study the role of intellectual property rights (IPRs) in a model of endogenous growth with sequential innovation. The model mimics the innovation process in hi-tech sectors where many of the techniques and products used in R&D are recent discoveries themselves. Depending on the structure of the patent system, some of these discoveries might still be protected by a patent. In this context IPRs affect both the revenues and the cost of the innovation. I use a dynamic general equilibrium model to study the efficient patent length and its properties.

1. INTRODUCTION

Technological progress, especially in hi-tech sectors, is widely recognized as one of the main forces behind economic growth. In these sectors innovation is sequential, meaning that new products build upon existing ones.

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This is evident when the new good is an improvement over existing products. It is also true when the discovery of the new good requires the use in R&D of research inputs which are previous discoveries themselves. In hi-tech sectors the number of these research inputs often grows large. Patent pools constitutes considerable evidence of this phenomenon. A patent pool is an agreement between two or more patent holders to license one or more of their patents together as a package. The first patent pool was formed in 1856 around intellectual property conflicts in the sewing machine industry. Other important patent pools are those on movie projectors in 1908, on aircrafts in 1917, on radio in 1924, and many more until the most recent one on Blue-ray Disc promoted by MPEG LA in 2007. Some of these patent pools are composed by an incredibly high number of patents: the MPEG2 pool contains more than 600 patents by 25 patent holders. In the absence of a patent pool, the innovator has to purchase a high number of research inputs from different patent holders. This was the case of Golden Rice, whose discovery required the use of around forty patented products and processes (Graff, Cullen, Bradford, Zilberman, and Bennett 2003). Or the development of a malaria vaccine based on the MSP1 protein, that would infringe upon 39 patent families (Commission on Intellectual Property Rights 2002).

These examples show that Intellectual Property Rights (IPRs) affect both the cost and the revenues of the innovator. Therefore the widely believed view that stronger IPRs increase the incentives to innovate no longer holds. This intuition is at the basis of the literature on sequential innovation initiated by Scotchmer (1991). This literature, with the contributions of Green and Scotchmer (1995), Chang (1995),

Scotchmer (1996), and many others, uses two stage innovation models where the first innovation builds the foundations for the second one. It then studies how the optimal patent policy should be designed in order to redistribute profits between the two innovators, so that all the socially valuable innovations are performed. Through static partial equilibrium models, this literature analyzes carefully the effects of IPRs on the profitability of innovation.

The literature on endogenous growth, on the other hand, focuses on the dynamic effects of R&D and innovation on growth. The contribution of this literature has been to make technological progress the result of agents' rational economic decisions. The papers of Romer (1990), Aghion and Howitt (1992), Grossman and Helpman (1991), just to cite a few, have brought fundamental insights on the relationship between R&D and economic growth. This literature also recognizes the cumulative nature of innovation, with each innovation building upon the previous one. In these models knowledge is freely available to each subsequent innovator, so that the IPRs policy does not affect the cost of innovation. It does however affect the revenues of the innovator. In this context the incentives to innovate are monotonically increasing in patent length, hence the adoption of an infinite patent life policy.

Therefore there are two separate streams of literature: the one on sequential innovation studies in depth the effects of IPRs on innovation by means of static partial equilibrium models. The one on endogenous growth uses dynamic general equilibrium models to study the effects of innovation on economic growth, but it is not concerned with the

response of innovation to different IPRs policy. In order to fully understand how patent policy can stimulate economic growth, there is a need to build a bridge between these two literatures.

Surprisingly, no attempt has been done yet in this sense. There is a growing literature on optimal patent length, but this literature is mostly concerned with a different issue. These models assume that the incentive to innovate is strictly increasing in patent length, and then study the trade-off between the growth enhancing effect and the static inefficiency of the monopoly provided by the patent. Judd (1985) finds that, under certain conditions, infinite patents achieve the efficient rate of growth. This is because when all goods are patented and priced the same, the marginal rate of substitution still equals the marginal rate of transformation and the inefficiency disappear. The same conclusion does not hold true for finite patent length: in this case goods whose patent is expired are priced at marginal cost, while patent protected goods are sold at a mark-up over the marginal cost. Futagami and Iwaisako (2007) reverse this result and find that the optimal patent length is finite. This is because they add labor in the production function of the final good. Therefore, even if all intermediate goods are priced the same in the infinite patent case, there is an inefficiency in the choice between labor and intermediate goods. Acemoglu and Akcigit (2007) expand the analysis to include state-dependant IPRs. They find that patent policy should depend on the technological gap between the leading firm and the followers. O'Donoghue and Zweimuller (2004) assume infinite patent length and focus on policy instruments other from patent length. In particular they examine in depth the patentability requirement and the leading breadth. All these papers, while clarifying

many aspects of the role of patent policy in growth, do not take into account the sequential nature of innovation.

To the best of my knowledge this is the first paper studying sequential innovation in a dynamic general equilibrium model where IPRs affect both the cost and the revenues of the innovation. In this paper I develop a model of endogenous growth with cumulative innovation, where the innovator has to use a generic number n of research inputs in R&D in order to come up with a new product. The n research inputs are previous innovations, so that the model is a dynamic general equilibrium version of Llanes and Trento (2007). The R&D success rate follows a stochastic poisson process, meaning that the time of innovation is uncertain. In this context patent length plays a central role: by the time R&D takes place, patents on some of the research inputs might have expired. Thus, depending on patent length, a fraction of the research inputs might be sold at a competitive price. The bigger this fraction, the lower - *ceteris paribus* - the cost of R&D. But reducing patent length also reduces expected revenues from the innovation. This trade off is the central theme of the paper.

I find that the infinite patent length always provides a less than optimal research effort in the steady-state equilibrium. This result is new with respect to the existing literature on quality based growth. This literature finds that infinite patents might lead to an equilibrium rate of innovation that can be slower, equal or faster than the optimal one. The new result of this paper stems from the fact that the model takes into account the effect of IPRs on the cost of the innovation. The R&D effort is hump-shaped as a function of patent length and there exists an optimal length, which is finite; a finite patent length increases the

equilibrium level of innovation relative to an infinite patent. Also in this case, though, the steady state level of innovation is always below the one a benevolent planner would choose. In other words, a varying patent length is not a good policy instrument to achieve the socially optimal rate of innovation. I finally perform some simulations to study the characteristics of the optimal patent length. I find that it is decreasing in the size of the market for the new good. This result is in line with the static analysis of Boldrin and Levine (2005) and it is relevant in the context of the recent discussions about introducing an IPRs system in developing countries. It means that, if the current patent length in advanced countries is optimal, enforcing IPRs in developing countries should be accompanied by a reduction of patent length in developed countries.

The rest of the paper has the following structure: in section 2 I set up the model to be used throughout the paper. Section 3 studies the efficient R&D effort as the solution to the social planner problem. Section 4 analyzes the decentralized equilibrium with infinite patent length, and compares it to the efficient one. In section 5 I study the decentralized equilibrium with finite patent life, and I analyze the optimal patent length and its characteristics. Then in section 6 I draw some conclusion and policy implications of this model.

2. THE MODEL

The structure of the model is a classical endogenous growth model, in line with Grossman and Helpman (1991). In this framework I introduce a technology for R&D that mimics the innovation process of hi-tech sectors, where many R&D inputs are previous innovations themselves. R&D effort depends on the access to these previous discoveries, which

in turns depends on the IPRs policy. This structure allows me to study the optimal patent length in a decentralized equilibrium.

Environment. Time is continuous and denoted by τ . There is one final good supplied in j different qualities, where j is an integer number and it is expanding with innovation. Each innovation increases the quality of the good by a factor $\gamma > 1$. I denote quality j by q_j , therefore $q_j = \gamma^s q_{j-s}$.

There is a mass L of identical households, each provided with one unit of labor that he supplies inelastically in exchange for a wage w . Households only derive utility from the consumption of the final good.

Preferences. Households' utility is increasing in both the quality and the quantity of the good consumed. Let d_j be total consumption of quality j good, then lifetime utility of the representative household is:

$$(1) \quad U = \int_0^\infty e^{-\beta\tau} \log \left[\sum_j \gamma^j d_j(\tau) \right] d\tau$$

where β is the subjective discount rate.

Technology. Labor is the only input in the production of goods on any quality. The production function is linear, with one unit of good requiring one unit of labor to be produced, independently of its quality:

$$(2) \quad x_j = l_j$$

R&D. Quality improvement, as the source of economic growth, is determined endogenously by R&D. I build the R&D production function taking into account two intrinsic features of hi-tech sectors: (i) innovation is sequential, meaning that new goods are built upon existing goods; (ii) a large number of recently discovered techniques and

products are used in research. In order to catch these features I assume that innovation requires the use in R&D of n lower quality goods. In particular I assume that the discovery of quality $j + 1$ requires the use in R&D of qualities $j - 1, \dots, j - n$. The R&D effort for the $j + 1$ innovation is measured as follows:

$$(3) \quad y_{j+1} = A \left(\sum_{i=j-n}^{j-1} x_i^\rho \right)^{\frac{1}{\rho}}$$

where A is a scale parameter, x_i is the amount of quality i good used as input in R&D, and $\rho \in [0, 1]$ is a parameter related to the substitutability between inputs. In what follow I set $A = n^{(\rho-1)/\rho}$ in order to eliminate increasing returns from specialization. Therefore y only depends on the total amount of inputs used ¹.

All firms share the same R&D technology (3). Innovation is characterized by uncertainty: the time of the discovery of a new product follows a Poisson process. The bigger the research effort, the higher the instantaneous probability of discovering a new good, λy . Because of this, the expected time between the discovery of quality $j - 1$ and quality j is $\frac{1}{\lambda y_j}$. Notice that because (3) is a constant returns to scale technology, and because the sum of N poisson processes with parameter λ is a poisson process with parameter $N \lambda$, the number of firms engaging in R&D is irrelevant.

3. SOCIAL PLANNER

The social planner has to allocate resources efficiently between the production of goods for consumption and goods to be used in R&D. The total amount of resources (labor) is L .

¹In most models of monopolistic competition $A = 1$. This implies that, for a fixed amount of total input $X = nx$, output is increasing in n , see (Romer 1987)

The planner maximizes (1), subject to the production technology (2), the R&D technology (3), and the resource constraint $L = \sum_{i=0}^j l_i$

Since all goods share the same technology, the planner will choose to produce only the highest quality good (j) for consumption. Also, because R&D is a concave and symmetric technology, he will choose to use the same quantity of each input: $x_{j-1} = \dots = x_{j-n} = x$. Substituting this choice of inputs in (3), the research effort becomes: $y_{j+1} = n x$. Also, since qualities $j - n - 1, \dots, 0$ are not used in consumption nor in R&D, it is efficient not to produce them at all.

The resource constraint becomes $L = l_j + y_{j+1}$, where l_j and y_{j+1} are the amount of resources used in the production of the consumption good and in R&D respectively. It follows that total quantity produced of the consumption good is $d_j = L - y_j + 1$. Therefore the optimization problem for the planner is:

$$(4) \quad \max_{y_{j+1}} E(U) = E \int_0^\infty e^{-\beta\tau} \log [q_j(\tau) (L - y_{j+1}(\tau))] d\tau$$

Expected utility is separable in quality $q_j(\tau)$ and quantity $(L - y_{j+1}(\tau))$, and it is easier to work with them separately. Expected utility in quality is $E(U_q) = E \int_0^\infty e^{-\beta\tau} \log [q_j(\tau)] d\tau$. Given that innovation follows a Poisson process with arrival rate λy ,

$$\begin{aligned} E \int_0^\infty e^{-\beta\tau} \log [q_j(\tau)] d\tau &= \frac{q_0}{\beta + \lambda y} + \frac{\lambda y \log(\gamma q_0)}{(\beta + \lambda y)^2} + \frac{(\lambda y)^2 \log(\gamma^2 q_0)}{(\beta + \lambda y)^3} + \dots \\ &= \frac{\log(q_0)}{\beta + \lambda y} \sum_{i=0}^\infty \left(\frac{\lambda y}{\beta + \lambda y} \right)^i + \frac{\log(\gamma)}{\beta + \lambda y} \sum_{i=0}^\infty i \left(\frac{\lambda y}{\beta + \lambda y} \right)^i \\ &= \frac{\log(q_0)}{\beta} + \frac{\lambda y \log(\gamma)}{\beta^2} \end{aligned}$$

Expected utility in quantity is $E(U_d) = E \int_0^\infty e^{-\beta\tau} \log [L - y_{j+1}(\tau)] d\tau = \frac{\log(L-y)}{\beta}$. Setting $q_0 = 1$ we have that total expected utility $E(U) =$

$E(U_q) + E(U_d)$ is:

$$(5) \quad E(U) = \frac{1}{\beta} \frac{\lambda y \log(\gamma)}{\beta} + \log(L - y)$$

The planner faces a trade-off between more consumption today and a more valuable consumption tomorrow. Maximizing (5) with respect to the constant research effort y , gives the following first order condition:

$$(6) \quad \frac{\lambda \log(\gamma)}{\beta} = \frac{1}{L - y}$$

This condition states that the marginal benefit of employing resources in R&D (the *lhs*) must equal its marginal cost (the *rhs*). The marginal benefit is increasing in the Poisson parameter λ , which sets the pace of innovation and in γ , the quality step. Not surprisingly it is decreasing in the subjective discount rate. The marginal cost, on the other hand, is decreasing in L and increasing in y . I will comment on these relationships below. Solving (6) for y , the following proposition holds:

Proposition 1 (Planner Allocation). *The optimal amount of research effort in the steady-state equilibrium is $y^* = L - \frac{\beta}{\lambda \log(\gamma)}$. This is increasing in total labor (L), the size of quality step (γ) and the effectiveness of R&D (λ). It is decreasing in the subjective discount rate (β).*

These relationships are all very intuitive. Some comments on expected welfare (5) follow: welfare is decreasing in β which is to be expected, since β is a measure of impatience discounting future utility. Also the positive effect of λ on welfare is not surprising: λ can be considered as a measure of R&D complexity, where lower values of λ are related to more complex R&D processes. Increasing λ will increase the

effectiveness of R&D (will reduce its complexity) reducing economic growth as a consequence. The last comment is on the effect of total labor supply on growth. In this model, which builds on first generation models of economic growth, there is a scale effect just as there is one in its predecessors. In this model the channel is the opportunity cost of employing resources in the R&D process: the opportunity cost is the foregone marginal utility. Marginal utility is decreasing in quantity, therefore increasing L reduces the marginal cost while leaving the marginal revenues unaffected. In order to balance this effect and bring (6) back to the equilibrium, y must increase.

4. DECENTRALIZED EQUILIBRIUM WITH INFINITELY LIVED PATENTS

This section analyzes the decentralized steady-state equilibrium level of the R&D effort. This level is determined by the interaction of consumers, innovators and input producers (previous innovators).

Representative Consumer. The consumer maximizes total lifetime utility

$$(7) \quad U = \int_0^{\infty} e^{-\beta\tau} \log \left[\sum_j q_j d_j(\tau) \right] dt$$

Given preferences, at each point in time the consumer only consumes the good with the lowest p_j/q_j ratio, where p_j is the price of quality j . I assume that, when faced with goods of the same price/quality ratio, the consumer prefers the good with highest quality. Let $E = \sum_j p_j d_j$ be total expenditures, then at each point in time consumer demands $d_j = E/p_j$ of the good with lowest price/quality ratio, and zero of the rest of the goods.

Consumer maximizes (7) subject to the intertemporal budget constraint:

$$\int_0^{\infty} e^{-r\tau} E(\tau) d\tau \leq A(0)$$

where $A(0)$ represents the discounted value of the stream of factor incomes, and r is the instantaneous interest rate. The solution to the consumer maximization problem is:

$$(8) \quad \frac{\dot{E}}{E} = r - \beta$$

so, when the interest rate equals the discount rate, consumption expenditures are constant in time.

Innovator. The innovator chooses the research effort y to maximize expected profits. When engaging in the discovery of the $j + 1$ quality, he solves:

$$(9) \quad \max_{y_{j+1}} \Pi = \lambda y_{j+1} V_{j+1} - c_{j+1}(y_j)$$

where c_{j+1} is the cost of performing y_{j+1} level of R&D, V_{j+1} denotes total profits from selling quality $j + 1$ in case the R&D process is successful, and again λy_{j+1} is the instantaneous probability of innovation. Later on we will make V_{j+1} explicit, as a function of production technology, patent policy, and R&D technology. For now it is only important to know that V_{t+1} is the sum of (i) the profits from being the quality leader in the final good sector, plus (ii) the profits from being the supplier of one of the n inputs used in the R&D process.

The innovator maximizes (9) in two steps: first, given prices of quality i , for $i = j - 1, \dots, j - n$, he derives conditional factor demands for

the corresponding inputs x_i in order to minimize c_{j+1} :

$$(10) \quad \begin{aligned} c_{j+1} &= \min_{x_i} \sum_{i=j-n}^{j-1} p_i x_i \\ \text{s.t.} \quad n^{\frac{\rho-1}{\rho}} \left(\sum_{i=j-n}^{j-1} x_i^\rho \right)^{\frac{1}{\rho}} &\geq y_{j+1} \end{aligned}$$

Solving (10) we get the conditional factor demands, and the cost function of R&D effort:

$$(11) \quad \begin{aligned} x_i &= n^{-\frac{1}{1-\sigma}} p_i^{-\sigma} \left(\sum_{i=j-n}^{j-1} p_i^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} y_{j+1} \\ c_{j+1} &= n^{-\frac{1}{1-\sigma}} \left(\sum_{i=j-n}^{j-1} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} y_{j+1} \end{aligned}$$

where $\sigma = (1 - \rho)^{-1}$ is the elasticity of substitution between inputs.

After deriving conditional factor demands, the innovator maximizes (9). Because of constant returns to scale and free entry, the first order condition equals the zero-profit condition:

$$(12) \quad \frac{\partial c_{j+1}}{\partial y_{j+1}} = \lambda V_{j+1}$$

Note that, because of constant returns in R&D, the number of firms engaging in research is indeterminate.

Balanced Growth Path. Input producers set the usual monopolistic competition price $p_i = \frac{\sigma w}{\sigma-1}$, and since the R&D production function is symmetric they all set the same price $p_i = p$. Since all inputs are priced the same, and because of symmetry and concavity of (3), the innovator will demand the same quantity of each input. In fact, substituting p into the conditional demand (11) we get $x_i = y_{j+1}/n$ for $i = j - n, \dots, j - 1$. Also, substituting p into the cost function of R&D and solving the zero profit condition (12) for V_{j+1} , we obtain the

following expression:

$$(13) \quad V = \frac{\sigma w}{(\sigma - 1)\lambda}$$

which we will use later to find the equilibrium value of the R&D effort in the balanced growth path. Since V depends only on constant parameters and on the numeraire w , I removed the subscript $j + 1$.

In equilibrium, instantaneous profits of the inputs producers are:

$$(14) \quad \Pi_{ip} = (p_i - w) x_i = \frac{w y_{j+1}}{n(\sigma - 1)}$$

The successful innovator is the quality leader in the market for the consumption good. His quality q_{j+1} is γ times bigger than the previous innovation's quality q_j . Since consumers only consume the quality with the lowest p_i/q_i ratio, the innovator maximize profits by setting a price that is γ times higher than the price of his competitors. The constant marginal cost of one unit of any good is w . Therefore the optimal price for the last innovator is γw . At each point in time he makes profits:

$$(15) \quad \Pi_L = (\gamma w - w) \frac{E}{\gamma w} = \left(1 - \frac{1}{\gamma}\right) E$$

After the following innovation takes place, quality $j + 1$ is replaced by quality $j + 2$ in the final good market. At this point it is sold as an input in the R&D market for the next n innovations.

Therefore total expected revenues of engaging in R&D for the discovery of quality $j + 1$ are:

$$(16) \quad V_{j+1} = \int_0^\infty e^{-r\tau} \left[e^{-(\lambda y_{j+2})\tau} \Pi_L + \sum_{i=2}^{n+1} e^{-(\lambda y_{j+i})\tau} \frac{((\lambda y_{j+i})\tau)^{i-1}}{(i-1)!} \Pi_{ip} \right] d\tau$$

This expression represents the expected value of the innovation $j + 1$. It is equal to the discounted expected profits from selling quality $j + 1$ as a quality leader in the consumption good market first, and then as one of the n inputs producers for the next n innovations. The first term inside the square bracket is the probability that nobody has discovered quality $j + 2$ at time τ , times the profits as quality leader in the market for consumption. The second term is the probability that, at time τ , quality $j + 1$ good is still in the R&D process, times the profits as input producer. For quality $j + 1$ to be part of the R&D process we need that: (i) quality $j + 2$ has already been invented; (ii) no more than n innovations have been made after the invention of quality $j + 1$.

Substituting (14), (15) and the resource constraint² into (16) and solving for V in the balanced growth path, we obtain:

$$(17) \quad V^* = \frac{(\gamma - 1) w (L - y)}{r + \lambda y} + \frac{\lambda y^2 w \left(1 - \left(\frac{\lambda y}{r + \lambda y}\right)^n\right)}{n r (r + \lambda y) (\sigma - 1)}$$

The following result holds:

Proposition 2 (Decentralized Equilibrium). *If a decentralized steady state equilibrium exists, it is unique. The research effort y_E is less than optimal. y_E is decreasing in r , and it is increasing in γ, λ and σ .*

Proof. I find the equilibrium by equating the zero profit condition (13) and the resource constraint (17). Because of constant returns to scale, the zero profit condition (13) does not depend on y . The same is not

²Notice that $\frac{E}{\gamma w}$ in (15) is the quantity sold in the market for consumption good. If the highest quality is j , then $d_j = \frac{E}{\gamma w} = L - y_{j+1}$

true for equation (17). In particular:

$$(18) \quad \frac{\partial V^*}{\partial y} = \frac{w \left(\lambda L \left(2r + \lambda y - \left(\frac{\lambda y}{r + \lambda y} \right)^n ((2+n)r + \lambda y) \right) - nr(\sigma - 1)(\gamma - 1)(r + \lambda L) \right)}{nr(r + \lambda y)^2(\sigma - 1)}$$

This expression is negative if H is negative, where H is:

$$(19) \quad H = -n\theta^n + (1 - \theta^n) \left(2 + \frac{\theta}{1 - \theta} \right)$$

with $\theta = \left(\frac{\lambda y}{r + \lambda y} \right) < 1$. H is positive if and only if $n < \left(\frac{1}{\theta^n} - 1 \right) \left(2 + \frac{\theta}{1 - \theta} \right)$.

We have $\frac{\partial \left[\left(\frac{1}{\theta^n} - 1 \right) \left(2 + \frac{\theta}{1 - \theta} \right) \right]}{\partial \theta} < 0$ and, applying de L'Hospital, $\lim_{\theta \rightarrow 1} \left[\left(\frac{1}{\theta^n} - 1 \right) \left(2 + \frac{\theta}{1 - \theta} \right) \right] = n$. Therefore H is negative and so is (18).

Also, for $y = 0$, V^* is the discounted value of the infinite stream of profits as leader in the consumption good: $V^* |_{\{y=0\}} = \frac{(\gamma-1)wL}{r}$. And for $y = L$, $V^* |_{\{y=L\}} = \frac{\lambda L^2 w \left(1 - \left(\frac{\lambda L}{r + \lambda L} \right)^n \right)}{nr(r + \lambda L)(\sigma - 1)} > 0$

Now we know that: (i) (13) is positive and constant in y ; (ii) (17) is positive for all $y \in [0, L]$ and it is decreasing in y . Therefore there exist an equilibrium if and only if $V^* |_{\{y=0\}} < V < V^* |_{\{y=L\}}$. In this case the equilibrium is unique. Notice that, in the end, the existence of the equilibrium depends on the value of L . Therefore an opportune choice of L assures the existence and uniqueness of the equilibrium.

Also substituting the optimal value of R&D, y^* (the one solving the social planner problem), we get (17) < (13). This proves that the decentralized level of research effort is suboptimal and it is lower than y^* . Figure (1) depicts the uniqueness and the sub-optimality of the equilibrium.

The effects of r , γ , λ and σ on y_E follows from straightforward applications of the implicit function theorem. ■

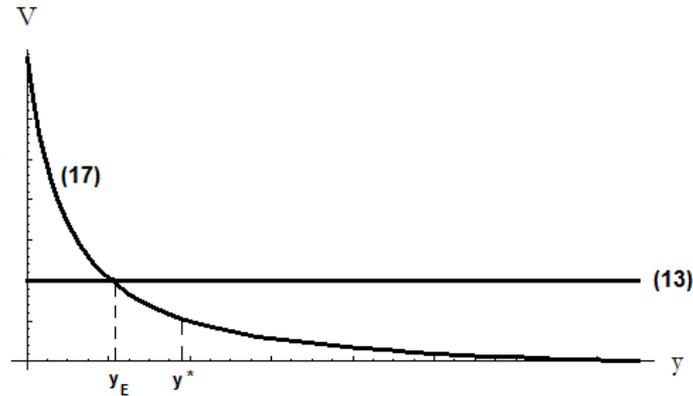


FIGURE 1

That, in the presence of infinite patents, the decentralized equilibrium delivers an insufficient level of R&D is new. The previous literature on quality-ladder based growth, by ignoring the "anti-commons" effect studied here, concluded that, depending on different constellations of parameters, R&D effort could be higher, equal or lower than socially desirable. This paper shows that incorporating a more realistic R&D production function, where IPRs affect both cost and revenues of the innovator, leads to a different conclusion. Patents of infinite length always fail short of delivering the correct incentives to innovate as the monopoly power they attribute to past innovators more than compensate for the extra profits they bestow upon the current one. The next section analyzes whether a finite patent length may achieve the socially desirable innovation effort and, in that case, the properties of the optimal patent length.

5. DECENTRALIZED EQUILIBRIUM WITH FINITE PATENT LENGTH

This section analyzes the decentralized equilibrium when patents have a finite life. Let the patent policy parameter $\phi > 0$ represent patent length. I assume that the innovator does not have any advantage

over imitators so that, after the patent expires, a costless imitation process immediately drives the price down to the marginal cost w .

We have seen in the previous section that when patents are infinitely lived the research effort is lower than optimal. Therefore if a finite patent length has to bring an efficiency gain, it must reduce the cost of R&D more than it reduces the expected revenues. A necessary condition for such a patent policy to exist is that its length is shorter than the expected arrival rate of n innovations. To understand why, one should look separately at the effects of the patent length on expected revenues and expected cost. On the one hand, any finite patent length reduces expected revenues: take for simplicity the case of an innovator who makes profits until next innovation arrives. His expected profits are the integral from zero to infinity of the period profits times the probability that nobody innovates. Now introduce a patent of any length. With a finite patent life, total profits are reduced because now the integration lasts only until the patent expires.

On the other hand, expected cost is reduced only if at least one research input runs out of its patent while it is still being demanded, that is before n , or more, new goods have been invented following it. When a new innovation process begins in the BGP, the lowest quality research input is expected to have been discovered $\frac{n}{\lambda y}$ time earlier. Therefore introducing a finite patent life only reduces the expected cost of R&D if $\phi < \frac{n}{\lambda y}$, where $\frac{n}{\lambda y}$ is the expected arrival rate of n innovations. For these two opposite forces to have a positive net effect the cost reduction must dominate the revenues' reduction.

With finite patent life the representative consumer analysis stays the same: he will only consume the good with the highest quality/price

ratio. Because of the effects on cost and revenues from R&D, the innovator's problem changes. Equation (9) becomes:

$$(20) \quad \max_{y_{j+1}} \Pi_\phi = \lambda y_{\phi,j+1} V_{\phi,j+1} - c_{\phi,j+1}(y_j)$$

The cost of R&D has also changed. Since the research inputs whose patent expired are sold at marginal cost w , the new cost minimization problem for the innovator is:

$$(21) \quad c_{\phi,j+1} = \min_{\{x_i, x'_i\}} \sum_{i=j-n}^{j-1} (p_i x_i I_i + w x'_i (1 - I_i))$$

$$s.t. \quad n^{\frac{\rho-1}{\rho}} \left(\sum_{i=j-n}^{j-1} x_i^\rho I_i + (x'_i)^\rho (1 - I_i) \right)^{\frac{1}{\rho}} \geq y_j$$

where

$$(22) \quad I_i = \begin{cases} 1 & \text{if } \phi < \sum_{k=0}^{j-i-1} \frac{1}{\lambda y_{i+k}} \\ 0 & \text{otherwise} \end{cases}$$

is the indicator function for input i 's patent. It takes the value one if the patent is still valid and zero if it expired. In equation (21), x_i are the inputs still covered by a patent and sold at a non-competitive price p_i , and x'_i are the inputs whose patent has expired, so that they are sold at marginal cost w .

The solution to (21) gives the conditional factor demand for the two types of inputs, x_i and x'_j , and the cost function of R&D y :

$$(23) \quad x_i = n^{-\frac{1}{1-\sigma}} p_i^{-\sigma} \left(\sum_{i=j-n}^{j-1} p_i^{1-\sigma} I_i + w^{1-\sigma} (1 - I_i) \right)^{\frac{\sigma}{1-\sigma}} y_j$$

$$(24) \quad x'_j = n^{-\frac{1}{1-\sigma}} w^{-\sigma} \left(\sum_{i=j-n}^{j-1} p_i^{1-\sigma} I_i + w^{1-\sigma} (1 - I_i) \right)^{\frac{\sigma}{1-\sigma}} y_j$$

$$c_{\phi,j} = n^{-\frac{1}{1-\sigma}} \left(\sum_{i=j-n}^{j-1} p_i^{1-\sigma} I_i + w^{1-\sigma} (1 - I_i) \right)^{\frac{1}{1-\sigma}} y_j$$

where $\sigma = (1 - \rho)^{-1}$ is the elasticity of substitution between the inputs. The first order condition of (20) is:

$$(25) \quad \frac{\partial c_{\phi,j+1}}{\partial y_{j+1}} = \lambda V_{\phi,j+1}$$

and again, because of constant returns to scale and free entrance, this is the zero profit condition.

Balanced Growth Path. In order to find the steady-state equilibrium I follow the same steps as in the previous section: I first find the equilibrium prices and demands, and then use the zero profits conditions and the resource constraint to find the equilibrium in the balanced growth path.

Because of the symmetry of the R&D production function, all inputs covered by a patent have the same price, which is the usual mark up over the marginal cost $p = \frac{\sigma w}{\sigma - 1}$. Substituting this equilibrium price into (23) we obtain the inputs demand. Notice that, because inputs are priced differently depending on being patent protected or not, the demands will no longer be symmetric. Also, substituting the optimal p in the cost function and solving the zero profit condition (25) in the

balanced growth path for $V_{\phi,j+1}$, we obtain:

$$(26) \quad V_{\phi} = (\sigma - 1)^{-2} s^{-s} n^{\frac{1}{\sigma-1}} w \mu \left(n(\sigma - 1)^2 \sigma^s + (\sigma - 2) \nu \lambda y \phi \right)$$

where $\mu = \left(n - \lambda y \phi + \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \lambda y \phi \right)^{\sigma/(\sigma-1)}$, and $\nu = (\sigma^{\sigma}(\sigma - 1) - \sigma(\sigma - 1)^s)$.

To obtain this condition we use the fact that, in the balanced growth path, the fraction of inputs whose patent is still valid is $\frac{\phi}{n/\lambda y}$.

Instantaneous profits for the input producer in the balanced growth path are:

$$(27) \quad \Pi_{\phi,ip} = (p_i - w)x_i = n^{1/(\sigma-1)}(\sigma - 1)^{\sigma-1} \sigma^{\sigma} w y \mu$$

where μ is as in (26).

As in the infinite patents case, the last innovator finds it profitable to set the price γw . Instantaneous profits for the innovator as the quality leader in the consumption good market is:

$$(28) \quad \Pi_{\phi,L} = (\gamma w - w) \frac{E}{\gamma w} = \left(1 - \frac{1}{\gamma} \right) E$$

Total expected profits for the successful innovator are:

$$(29) \quad V_{\phi,j+1} = \int_0^{\phi} e^{-r\tau} \left[e^{-(\lambda y_{j+2})\tau} \Pi_{\phi,L} + \sum_{i=2}^{n+1} e^{-(\lambda y_{j+i})\tau} \frac{((\lambda y_{j+i})\tau)^{i-1}}{(i-1)!} \Pi_{\phi,ip} \right] d\tau$$

The solution to this integral along the balanced growth path, given the resource constraint, is:

$$(30) \quad V_{\phi}^* = \frac{(1 - e^{-(\lambda y + r)\phi}) (\gamma - 1) w (L - y)}{r + \lambda y} + B \left(n^{1/(\sigma-1)} (\sigma - 1)^{\sigma-1} \sigma^s w y \mu \right)$$

where μ is as in (26) and

$$B = \sum_{i=0}^n \left[\left(\frac{\lambda y}{\lambda y + r} \right)^{i+1} \left(1 - e^{-(\lambda y + r)\phi} \sum_{j=0}^{n-i} \frac{(\lambda y \phi)^j}{j!} \right) \right] - \frac{e^{-(\lambda y + r)\phi}}{\lambda y + r} \left(\frac{(\lambda y \phi)^n}{n!} + \frac{(\lambda y)^2 \phi}{\lambda y + r} \right) \sum_{i=2}^n \frac{(\lambda y \phi)^i}{i!}$$

Equations (26) and (30) determine the steady-state equilibrium level of R&D y_ϕ . (26) is strictly increasing in y , while (30) is decreasing in y for $y < \frac{n}{\phi\lambda}$, which is the only interesting case, since we have seen before that the patent length must be shorter than the expected arrival rate of n innovations. As in the case of the infinite patent length, a large enough value of L assures that the two lines cross. Figure (2) shows the steady state equilibrium where the two curves cross.

Proof. Proof in the Appendix ■

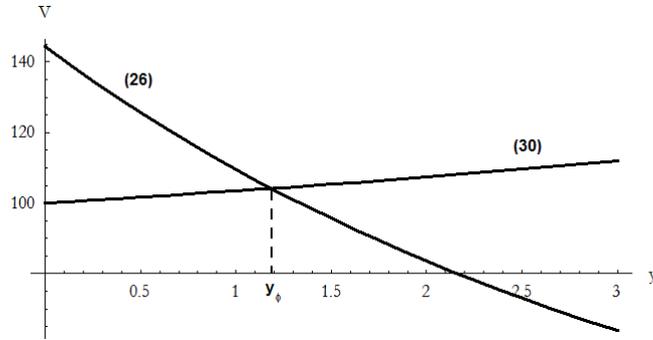


FIGURE 2

In what follows I present the results of some simulations that aim at analyzing the characteristics of the optimal patent policy. The parameters have been chosen as follows: for the annual discount rate, equal to the interest rate, I take values $r \in [0.03, 0.05]$. For γ and λ I used a reasoning similar to Stokey (1995): the average annual rate of economic growth due to technological progress is between 0.5 and 1 per cent. In

the model the expected rate of economic growth is the probability of innovation times the quality improvement $\lambda y(\gamma - 1)$. Since y is endogenous to the model it is complicated to calibrate the model to exactly match the data on economic growth. There exist many values of γ and λ that can match the data, therefore I let both parameters vary within a reasonable range. I take $\lambda \in [0.01, 0.02]$ and $\gamma \in [1.2, 1.65]$. The wage rate does not affect the equilibrium, since it is just the numeraire in the model. I take the elasticity of substitution between inputs $\sigma = 4$, but then I run robustness check for $\sigma \in [2, 5]$. The number of inputs entering the R&D process in the baseline case is 5, but I allow it to increase up to 25. I take total labor $L = 10$ in order to get a stable equilibrium as I run robustness checks for the parameters. I also let L vary to study how this affects the optimal patent policy.

Although the quantitative results obviously change for different values of the parameters, the qualitative results are very robust. The main result is the following:

Proposition 3. *the level of R&D effort with finite patents (y_ϕ) is hump shaped in ϕ . No finite patent length reaches the optimal level of R&D y^* , therefore the optimal patent policy (ϕ^*) is the one that maximizes y .*

Figure (3) depicts the efficient level of R&D y^* , the research effort under infinite patent policy y_E , and the research effort under finite patent policy y_ϕ , as a function of patent length.

A simple way to reach the efficient level of R&D in this model would be to eliminate patent protection and give a fixed subsidy equal to y (the cost of R&D effort) to all firms trying to innovate.

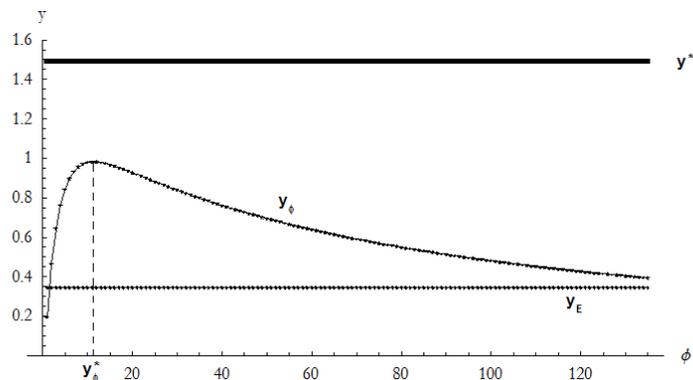


FIGURE 3

A second interesting result is that the optimal patent length is decreasing in L . This is because a higher value of L implies a larger market for the new innovation and more profits in the final good market. This result is in line with the static model of Boldrin and Levine (2005) who show that an increase in the market size for innovation should be accompanied by a decrease of patent strength. Figure (4) shows the negative relationship between L and ϕ^* .

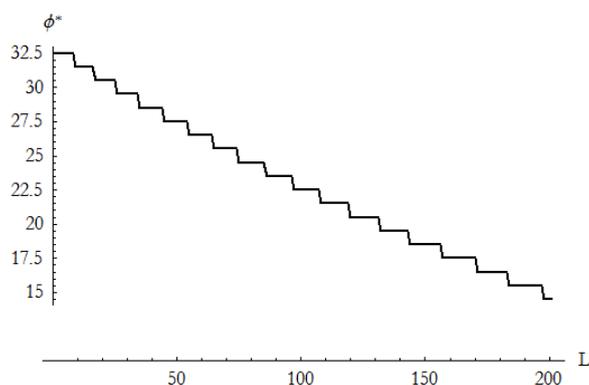


FIGURE 4

6. CONCLUSIONS AND POLICY IMPLICATIONS

In this paper I investigate the effects of intellectual property rights on economic growth when the latter is due to sequential innovations

feeding on earlier ones. Previous models of endogenous innovation abstracts from the fact that earlier innovations are inputs in the R&D process for new ones, thereby assuming an increasing relationship between the length of the patent and the incentives to innovate. The literature on static sequential innovation has made clear that, when innovation is sequential, patents affect both the revenue from and the cost of innovations. This is especially true in high tech sectors, where many of the products and processes used in R&D are previous innovation themselves, and therefore likely to be patent protected. In this context, stronger patent protection does not necessarily increase the incentives to innovate and it may well reduce it. We show that this is the case in general, and especially so for a realistically calibrated version of our model.

This is the first time in which this dual effect of IPRs on innovation is considered in a dynamic general equilibrium setting, and the results are novel. In particular, I find that, in the steady-state equilibrium, an IPRs system with infinitely lived patents does not maximize either social welfare or the rate of economic growth: it provides a level of R&D which is too low.

I also analyze the steady-state balanced growth path with finite and optimal patent life. I find that R&D effort is initially increasing and then decreasing in patent length. This inverse-U shape is easily explained by two effects. Because of the, extreme, assumption of costless and timeless imitation, zero patent protection results in no innovation. As patent protection increases, the level of R&D steadily increases, until it reaches its maximum at a level which is higher than its corresponding level with infinite patents. After that it slowly decreases

until it reaches its infinite patent length equilibrium value. [I conjecture that in the case in which imitation is neither costless nor timeless (to be added) the relation between R&D intensity and patent length may well become monotone decreasing at realistic parameter values.]

I also find that, under the maintained assumptions, there exists a unique optimal finite patent length. This optimal length is decreasing in the size of the market for the innovation. These findings are important as they imply that, if the present patent length is optimal, enforcing IPRs in developing countries should be accompanied by a reduction of patent length in developed countries.

Two natural extensions of this model are being completed and will be added in future versions. In the first I introduce specialized labor into the R&D production function, in other words I assume that new goods can be obtained either by means of patented inputs or by using specialized labor to discover around existing patents, or a combination of both. Because of the monopoly distortion created by the presence of patents, the demand for specialized labor is higher than it would be at the efficient allocation. Obviously, this has an effect on the wage of the specialized labor, which is also higher than it would be in the absence of patents. A second extension assumes that imitation is neither costless nor timeless. This provides the innovator with a first mover advantage, yielding competitive rents and affecting the optimal patent policy, which is now of a shorter length than in the baseline case. I study the configurations of parameter values for which the optimal patent length is zero, i.e. that competitive equilibrium yields the social efficient allocation.

REFERENCES

- ACEMOGLU, D., AND U. AKCIGIT (2007): "State-Dependent Intellectual Property Rights Policy," *NBER Working Paper*.
- AGHION, P., AND P. HOWITT (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60(2), 323–351.
- BOLDRIN, M., AND D. LEVINE (2005): "IP and Market Size," *Mimeo*.
- CHANG, H. F. (1995): "Patent Scope, Antitrust Policy, and Cumulative Innovation," *RAND Journal of Economics*, 26(1), 34–57.
- COMMISSION ON INTELLECTUAL PROPERTY RIGHTS (2002): "Integrating Intellectual Property Rights and Development Policy: Report of the Commission on Intellectual Property Rights," London.
- FUTAGAMI, K., AND T. IWAISAKO (2007): "Dynamic analysis of patent policy in an endogenous growth model," *Journal of Economic Theory*, 127(1), 306–334.
- GRAFF, G., S. CULLEN, K. BRADFORD, D. ZILBERMAN, AND A. BENNETT (2003): "The public-private structure of intellectual property ownership in agricultural biotechnology," *Nature Biotechnology*, 21(9), 989–995.
- GREEN, J. R., AND S. SCOTCHMER (1995): "On the Division of Profit in Sequential Innovation," *RAND Journal of Economics*, 26(1), 20–33.
- GROSSMAN, G. M., AND E. HELPMAN (1991): "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, 58(1), 43–61.
- JUDD, K. (1985): "On the Performance of Patents," *Econometrica*, 53(3), 567–586.
- LLANES, G., AND S. TRENTO (2007): "Anticommons and optimal patent policy in a model of sequential innovation," *Economics Working Papers*, Universidad Carlos III.
- O'DONOGHUE, T., AND J. ZWEIMULLER (2004): "Patents in a Model of Endogenous Growth," *Journal of Economic Growth*, 9(1), 81–123.
- ROMER, P. M. (1987): "Growth Based on Increasing Returns Due to Specialization," *The American Economic Review*, 77(2), 56–62.
- (1990): "Endogenous Technological Change," *The Journal of Political Economy*, 98(5), 71–102.

SCOTCHMER, S. (1991): "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law," *Journal of Economic Perspectives*, 5(1), 29–41.

——— (1996): "Protecting Early Innovators: Should Second-Generation Products Be Patentable?," *RAND Journal of Economics*, 27(2), 322–331.

STOKEY, N. (1995): "R&D and Economic Growth," *The Review of Economic Studies*, 62(3), 469–489.