Hype and Dump Manipulation*

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Abstract

This paper introduces signaling in a standard market microstructure model so as to explore the economic circumstances under which hype and dump manipulation can be an equilibrium outcome. We consider a discrete time, multi-period model with stages of signaling and asset trading. A single informed trader contemplates whether or not to spread a (possibly dishonest) rumor on the asset payoff among uninformed traders. Dishonest rumor-mongering is costly due to regulatory enforcement, and the uninformed traders who access the rumor can be *sophisticated* or *naive*. The sophisticated traders correctly anticipate the relationship between the rumor and the asset payoff, whereas the naive ones take the rumor at its face value as if it truthfully reveals the asset payoff. The presence of sophisticated traders puts the informed trader off from rumor-mongering, because sophisticates fully infer the asset payoff from the rumor, reducing the informational rents enjoyed by the informed trader. Nevertheless we show that it can be optimal for an informed trader to create false hype among uninformed traders provided that there is at least one naive trader in the market and the cost of dishonest rumor-mongering is not too low. The false hype allows the informed trader to sell at an inflated price or buy at a deflated one. Intense regulatory enforcement, which makes dishonest rumor-mongering very costly, may not necessarily curb hype and dump schemes. Market depth and trading volume rise with "hype and dump" while market efficiency decreases.

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1 Introduction

Stock market manipulation is an old yet effective game. One of the most well-known manipulation schemes is the hype and dump manipulation, also referred to as pump and dump. In this scheme, the manipulator artificially inflates the asset price through promotion in order to sell at the inflated price, or deflates the asset price through false hype in order to buy at the deflated price. This practice is illegal under U.S. securities law, yet it is common.[1]

In this paper, we explore the economic circumstances under which the hype and dump manipulation is an equilibrium outcome. Our analysis builds on early work regarding strategic trading à la Kyle (1985). We embed costly signaling into the Kyle model. Specifically, we consider a discrete time, multi-period model in which traders first engage in a signaling game and then a trading game. In the signaling stage, a single informed trader, who knows the risky asset payoff with certainty, contemplates whether or not to send a (possibly false) rumor about the asset to uninformed traders. If she decides to send a rumor, then she also chooses the rumor content. However, there is a cost associated with dishonest rumor-mongering, which captures the risk of prosecution enforced by financial regulatory bodies. Uninformed traders observe the rumor, but not all of them construe it in the same manner. Some of the uninformed traders are sophisticated, and in equilibrium, they correctly anticipate the relationship between the rumor and the asset payoff. The others are naive, and take the rumor at its face value; i.e. they falsely think that the rumor truthfully reveals the asset payoff.[2] In the trading stage, informed and uninformed traders submit market orders to a market maker while taking into account their effect on the price.

Our main result is that hype and dump manipulation is attained in equilibrium if there is at least one naive trader in the market and the cost of dishonest rumor-mongering is high enough. It would not be surprising to see the informed trader spread rumors in order to hype and dump

[1]For more information, see the SEC web site on hype and dump manipulation:
`http://www.sec.gov/investor/pubs/pump.htm`

[2]Note that we abstract from adverse selection problems between informed and uninformed traders in our primitive economic environment: the rumormonger is known to be informed by all traders. The credibility of traders, who send rumors, is definitely an important issue, which is to be studied in future research.

if all the traders were naive. After all, the naive traders believe the rumor to be true, hence they can be easily manipulated by the informed trader to inflate or deflate the price. However, the presence of sophisticated traders discourages the informed trader's rumor-mongering, because sophisticates fully infer the asset payoff from the rumor, hence reduce the informational rents enjoyed by the informed trader. The direction of the trade-off regarding rumor-mongering is therefore a non-trivial matter when sophisticated traders are present in the market.

Our results also show that regulatory enforcement, which makes dishonest rumor-mongering costly, can have unexpected implications on the informed trader's economic incentives. It is straightforward to see that enforcement creates a deterrence mechanism, which discourages informed trader from rumor-mongering. However, enforcement also creates a commitment mechanism for the informed trader: If there were no cost attached to dishonesty, the informed trader's profit would increase in an unbounded manner as her rumor diverged away from the true asset payoff, and as a result, an equilibrium might not be achieved. The cost brought by enforcement allows the informed trader to settle on an equilibrium rumor-mongering strategy; i.e. she can then choose an optimal dishonesty level for the content of her rumor.

Due to this multi-faceted impact of regulatory enforcement on rumor-mongering, the policy implications are delicate. In particular, we show that intense regulatory enforcement may not necessarily curb hype and dump schemes. The intuition: The market maker is willing to provide higher market depth in the presence of intense enforcement, because the level of dishonesty in informed trader's rumor can then be restrained. Higher market depth allows the informed trader to better camouflage her private information against the market maker, therefore her informational rents increase. It turns out that the informed trader's benefit from increasing informational rents can surpass the cost associated with enforcement, hence hype and dump manipulation can be sustained in equilibrium even if the intensity of enforcement is high. However, the fact that regulatory enforcement does not necessarily deter informed traders from hyping and dumping, by no means suggests that employing enforcement measures on rumor-mongering is a bad policy overall. Our analysis does not encompass all possible equilibria that can arise in the model, and so there can be other equilibria in which intense regulatory enforcement prevents manipulation by rumor-mongering.

Numerical examples, confirm this last remark. We show that endogenous rumor-mongering can lead to multiple equilibria. In some of these equilibria, enforcement measures actually deter the informed trader from rumor-mongering.

We also provide welfare and asset pricing implications. On the welfare side, we show that informed trader increases her trading profit by hyping and dumping at the expense of naive traders. On the asset pricing side, we find that market depth and trading volume increase with hype and dump while market efficiency decreases. These implications are broadly consistent with anecdotal and empirical evidence.

From the theoretical perspective, our paper underscores the importance of endogenous information disclosures in trading games: rumor-mongering has significant impact on the trading behavior of market participants, and it also leads to multiple equilibria. Even the decision not to rumor-monger may reveal information in our model since the informed trader makes that decision based on her private information. Also, to the best of our knowledge, this is the first paper to introduce "costly talk"[3] in a trading game so as to analyze manipulation in financial markets.

From the policy perspective, our results warn that policy proposals, which solely focus on the deterrence brought by enforcement and ignore other incentive implications for the market participants involved, can be misleading.

The rest of the paper is organized as follows. Section 1.1 presents the related theoretical literature. Section 1.2 provides anecdotal and empirical evidence on hype and dump manipulation. Section 2 introduces the model. Section 3 presents the equilibrium analysis. Section 4 concludes. All proofs are confined to the Appendix.

1.1 Related literature: theory

Our paper is related to several theoretical literatures. One is the modeling of financial markets in which full rationality is dispensed with (De Long, Shleifer, Summers, and Waldman (1991), Daniel, Hirshleifer, and Subrahmanyam (1998), Gervais and Odean (2001), Kyle and

[3] In costly talk, as the phrase suggests, dishonest signaling is costly.

Wang (1997), and Odean (1998)). Another one is the literature on signaling games: In this literature, Crawford and Sobel (1982) is the seminal paper that introduced cheap talk, however the modeling of signaling stage in our paper is more closely related to a series of recent papers dealing with strategic signaling in a setup where misreporting (i.e. dishonesty) is costly (Blanes (2003), Hong, Scheinkman, and Xiong (2006), Kartik (2005), Kartik, Ottaviani and Squintani (2006)). Also, the literature on principal-agent relationship between financial advisors and investors is relevant (Blanes (2003), Hong, Scheinkman, and Xiong (2006), Ottaviani and Sorensen (2006a, 2006b)). Lastly, but most importantly, our paper is related to a growing literature on market manipulation (Aggarwal and Wu (2003), Allen and Gale (1992), Allen and Gorton (1992), Allen, Litov and Mei (2006), Bagnoli and Lipman (1996), Benabou and Laroque (1992), Chakraborty and Yilmaz (2004, 2004), Fishman and Hagerty (1995), Goldstein and Guembel (2003), Hart (1977), Hart and Kreps (1986), Huddart, Hughes, and Levine (2000), Jarrow (1992, 1994), John and Naranayan (1997), Kumar and Seppi (1992), and van Bommel (2003), Vila (1987, 1989), Vitale (2000)).

Within the theoretical literature on manipulation, Benabou and Laroque (1992; henceforth "BL") and van Bommel (2003; henceforth "vB") are closest to this study. Both investigate environments with informed and uninformed traders, where the informed discloses information to the uninformed in order to manipulate asset prices. Both studies base the signaling periods of their models on Crawford and Sobel (1982). There are, however, a number of important differences between our work and BL and vB.

First and foremost, BL and vB assume that informed traders' price impacts are negligible. Specifically, in BL the market is competitive so that no individual trader has impact on price, and in vB informed traders have wealth constraints so that their impact on price is limited. In our model, all traders exhibit non-price-taking behavior and they all take into account their impacts on price. Especially in the context of hype and dump schemes, it seems more reasonable for the manipulator's trade to have price impact.[4]

[4]For instance, Frieder and Zittrain (2006) reports a well-known case of hype and dump manipulation brought to court by the SEC. In this case, Jonathan Lebed routinely purchased stock accounting for 17% to 46% percent of the stock's market volume for a day, and sent spam e-mail touts on the same day. He then lodged limit orders to sell

Our paper also differs from BL and vB in the assumptions regarding rationality of traders. In BL, bounded rationality appears on the informed traders' side because of the existence of honest traders who always truthfully reveal their private information. vB introduces bounded rationality on both sides: there are honest informed traders who send truthful signals as well as uninformed traders with short term trading horizons. In our model, bounded rationality stems only from uninformed traders: some of the uninformed traders are naive in their interpretation of the rumors. We assume that the informed trader is rational, so she always chooses the disclosure strategy that maximizes her expected profit and faces punishment for dishonest signaling.[5] We believe it makes more sense to attribute bounded rationality to uninformed traders in the context of hype and dump manipulation.

BL and vB do not explore asset pricing implications of informed manipulation, such as the implications for liquidity, trading volume, and market efficiency, whereas we do. On the other hand, BL and vB investigate dynamic trading and rumor-mongering while our analysis is restricted to a static environment. BL also deals with the important issue of credibility of rumor mongers (i.e. whether or not they are indeed informed) while we leave the issue for future research.

Finally, the manipulation scheme analyzed in vB differs from the hype and dump scheme analyzed in our paper. In vB, an informed trader first trades based on her private information to enjoy her informational monopoly rents and then sends an imprecise rumor to uninformed traders in order to trade once again in the case price overshoots the asset payoff after the uninformed traders follow her imprecise yet truthful advice. In the manipulation scheme considered in vB, both informed and uninformed traders increase their profits at the expense of liquidity (noise) traders.

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for the next day's trading session, anticipating a rise in the stock price after the general public received his touts and some acted on them. The trading volume attributed to Lebed in the case makes the assumption of negligible price impact implausible.

[5]Neither BL nor vB explicitly model costly signaling, however they mention punishment as a justification for the existence of honest informed traders who truthfully signal.

1.2 Related literature: anecdotal and empirical evidence

One of the most well-known hype and dump schemes is attributed to Nathan Rothschild. Benabou and Laroque (1992) write the story of his alleged manipulation of British government security prices: "During the battle of Waterloo (1815) the banker Nathan Rothschild, who was known to have superior information from the continent due to a system of carrier pigeons, walked around the city looking dejected, spreading the news that the battle was going badly, and had his agents openly sell British government securities. Meanwhile, he was secretly buying much larger quantities of these securities, taking advantage of the depressed price and of his actual knowledge of an impending victory."

Today the Internet offers unparalleled opportunities to manipulators. There are almost 8000 stocks for which message board activity exists, and these message boards attract the attention of many investors, including manipulators.[6] There have been recent reminders of this attention: In separate incidents, Lucent Technologies, the telecoms network equipment giant, and Emulex, a computer network hardware vendor, saw $7.1bn and $2.6bn wiped off their respective stock market values within hours of bogus press releases appearing on the web.[7] The NEIP, an obscure nearly bankrupt company, saw its stock price rocket up by an impressive 106,600% in a matter of days, thanks to the Internet message boards.[8] Another testament to the effective manipulative rumormongering on the net is the office set up by the SEC just to deal with Internet scammers.[9] "Manipulation on the Internet is where the action is, and appears to be replacing brokerage boiler rooms of the past," said SEC enforcement-division director Richard Walker in 2000.[10]

Recent empirical studies by Frieder and Zittrain (2006) and Hanke and Hauser (2006) show the effectiveness of hype and dump as a manipulation scheme. Frieder and Zittrain (2006) report that approximately 730 million spam e-mails are sent every week, 15% of which tout stocks. They analyze 75,000 unsolicited e-mails, all touting stocks, sent between January 2004 and July

[6] Das and Chen (2006).

[7] Phillips, S., "Cyber scams threaten integrity of the market", Financial Times IT, February 7, 2001.

[8] Leinweber and Madhavan (2001).

[9] Morgenson, G., "Internet's role is implicated in stock fraud", New York Times, December 16, 1999.

[10] Wall Street Journal, November 6, 2000.

2005. The authors find that a spammer who bought shares the day before starting an e-mail campaign and then sold them the day after could make a return on his or her investment of 4.9%. People who buy into the hype and dump scam typically lose 5.25% of their investment in two days. Hanke and Hauser (2006) find that stock spam e-mails have a significant impact on returns, volatility, intra-day spread and trading volume. In particular, they show that trading volume in spammed stocks is significantly higher on and around spam days. Frieder and Zittrain (2006) also have a similar finding: On a day when no spam is detected, the likelihood of a touted stock being the most actively traded stock that day is 8%. However, on days when there is spam and touting, the probability of a touted stock being the single most actively traded stock is 81%.

2 Model

2.1 Setup

We consider an economy with a single traded asset. The economy lasts for three periods, $t = 0, 1, 2$. Trade takes place at $t = 1$ and the final wealth is consumed at $t = 2$. The asset has a risky payoff $\tilde{v}$, which realizes at $t = 2$.[11]

There are four main types of agents in the economy: a single informed trader, N (initially) uninformed traders, liquidity traders, and a competitive market maker.

The informed trader observes the asset payoff v at $t = 0$. After this observation takes place, the informed trader decides whether or not to send a rumor about the asset payoff v to the uninformed traders. If she decides to send a rumor, then she chooses the rumor content $r \in \mathbb{R}$. Any dishonest rumor-mongering ($r \neq v$) comes at a cost to the informed trader in the form of

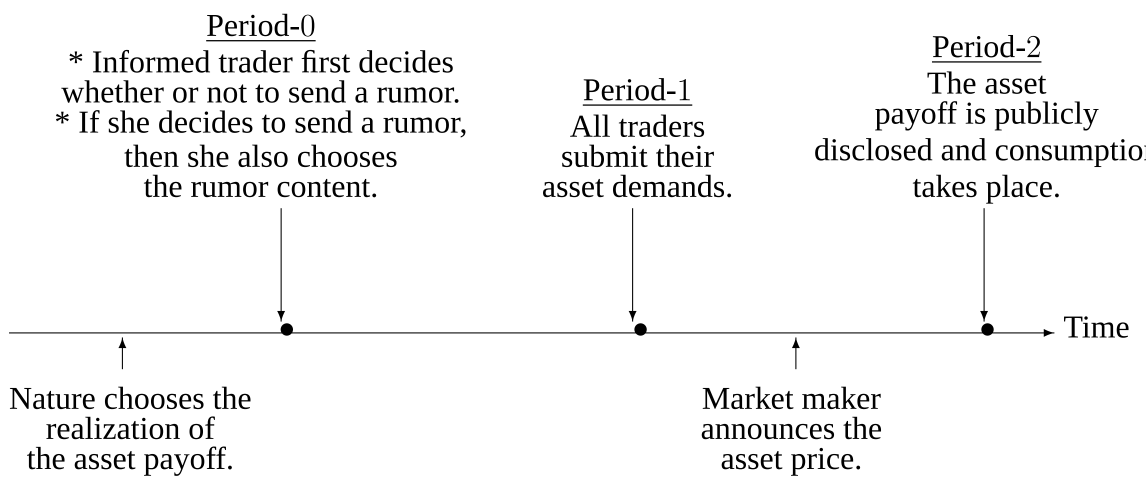
$$\alpha \left(r - v\right)^2,$$

where $\alpha \geq 0$. This cost is meant to capture the prosecution risk associated with dishonest rumor-mongering: the higher the difference between the rumor and the asset payoff, the higher is the

[11]Throughout the text, we use the following convention: random variables are denoted with *tilde* (such as $\tilde{y}$), and the realizations of random variables are denoted without *tilde* (such as y).

expected fine enforced by the financial regulatory bodies, such as the SEC. The parameter α measures the intensity of regulatory enforcement. If the informed trader decides not to send any rumor, she does not incur any cost.

Whether or not the informed trader sends a rumor is observable to all agents in the economy, however the content of the rumor is only observed by the informed trader and the uninformed traders, but not by the market maker. Not all uninformed traders construe the rumor in the same manner. N_s many of them are *sophisticated*: in equilibrium, they correctly anticipate the relationship between the rumor (r) and the asset payoff (v). The remaining $N - N_s$ many of the uninformed traders are *naive*: they take the rumor at its face value, i.e. they consider r equals v.[12]



Both informed and uninformed traders trade in order to maximize their expected profits, and they are risk-neutral. Liquidity traders trade for reasons exogenous to the model: their total demand is determined by the realization L of a random variable $\tilde{L}$. The competitive market maker supplies against the total net demand of informed trader, uninformed traders, and liquidity traders. The market maker neither observes the asset payoff v nor the rumor r, however in equilibrium he correctly perceives the relationship between the rumor r and the asset value v.

[12]There is extensive anecdotal evidence which indicates manipulation by rumor-mongering relies on the presence of naive traders, who take rumors at their face values. Along similar line, Malmendier and Shantikumar (2004) provide empirical evidence on the inability of individual investors to see through the incentives of financial analysts.

The latter assumption guarantees that asset price is set efficiently by the market maker. Therefore the results obtained in this paper cannot be attributed to inefficient pricing.

All trading activities take place at $t = 1$. Trading is order-driven as modelled in Kyle (1985): First, informed, uninformed and liquidity traders submit their asset orders simultaneously to the market maker. Having observed the net total order flow from these traders, the market maker announces the asset price. The market maker makes zero expected profit, hence sets the price equal to the expected value of the asset payoff conditional on the net total order flow.

The asset payoff $\tilde{v}$ and the liquidity demand $\tilde{L}$ are jointly normally distributed with mean $(0, 0)$ and variance-covariance matrix

$$\begin{pmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_L^2 \end{pmatrix}.$$

As mentioned above, in equilibrium, the sophisticated traders and the market maker correctly anticipate the functional form of the rumor, that is the relation between the rumor and the asset payoff. The naive traders are unable to see through the incentives of the informed trader, and they anticipate that the rumor r equals the asset payoff v.The informed trader and all uninformed traders correctly anticipate the market maker's equilibrium pricing strategy, and the market maker correctly anticipates all traders' equilibrium demand strategies.

2.2 Strategies and Payoffs

Strategies. We assume that whether or not a rumor is sent is common knowledge even though the market maker does not observe the rumor content r. The strategies of the informed trader is given by a list of functions $\big(\delta(\cdot), f(\cdot), x^{NR}(\cdot), x^{R}(\cdot\,,\,\cdot)\big)$, where

1. $\delta : \mathbb{R} \rightarrow \{R, NR\}$ is the decision strategy determining whether or not to rumor-monger: $\delta(v) = R$ stands for the decision to "send a rumor", and $\delta(v) = NR$ stands for the decision "not to send any rumor",

2. $f : \mathbb{R} \rightarrow \mathbb{R}$ is the signaling strategy determining the content of the rumor, i.e. $r = f(v)$,

3. $x^{NR} : \mathbb{R} \to \mathbb{R}$ is the demand strategy as a function of v if the informed trader chooses not to send any rumor,

4. $x^{R} : \mathbb{R}^2 \to \mathbb{R}$ is the demand strategy as a function of (v, r) if the informed trader chooses to send the rumor r.

The demand strategies of a sophisticated trader $j \in \{1, ..., N_s\}$ are given by the pair $\{s_j^{NR}, s_j^R(\cdot)\}$, where

1. s_j^{NR} is the demand strategy when $\delta(v) = NR$,
2. $s_j^R : \mathbb{R} \to \mathbb{R}$ is the demand strategy as a function of r when $\delta(v) = R$.

Similarly, the strategies of a naive trader $j \in \{N_s + 1, ..., N\}$ are given by the pair $\{g_j^{NR}, g_j^R(\cdot)\}$, where

1. g_j^{NR} is the demand strategy when $\delta(v) = NR$,
2. $g_j^R : \mathbb{R} \to \mathbb{R}$ is the demand strategy as a function of r when $\delta(v) = R$.

Finally, the pricing strategies of the market maker are given by the pair $\{P^{NR}(\cdot), P^R(\cdot)\}$, where

1. $P^{NR} : \mathbb{R} \to \mathbb{R}$ is the pricing strategy as a function of the total order flow

$$X^{NR}(v, L) = x^{NR}(v) + \sum_{j=1}^{N_s} s_j^{NR} + \sum_{j=N_s+1}^{N} g^{NR} + L$$

when $\delta(v) = NR$,

2. $P^R : \mathbb{R} \to \mathbb{R}$ is the pricing strategy as a function of the total order flow

$$X^R(v, r, L) = x^R(v, r) + \sum_{j=1}^{N_s} s_j^R(r) + \sum_{j=N_s+1}^{N} g^R(r) + L$$

when $\delta(v) = R$.

Payoffs. For each realization v of the asset payoff, r of the rumor, and L of the liquidity trade,

1. the payoff of the informed trader is given by

$$\begin{aligned} \big(v - P^{NR}(X^{NR}(v,L))\big) \cdot x^{NR}(v) \qquad & \text{if } \delta(v) = NR \\ \big(v - P^{R}(X^{R}(v,r,L))\big) \cdot x^{R}(v,r) - \alpha(v-r)^2 \qquad & \text{if } \delta(v) = R, \end{aligned}$$

2. the payoff of the sophisticated trader $j \in \{1, ..., N_s\}$, is given by

$$\begin{aligned} \big(v - P^{NR}(X^{NR}(v,L))\big) \cdot s_j^{NR} \qquad & \text{if } \delta(v) = NR \\ \big(v - P^{R}(X^{R}(v,r,L))\big) \cdot s_j^{R}(r) \qquad & \text{if } \delta(v) = R, \end{aligned}$$

3. the payoff of the naive trader $j \in \{N_s + 1, ..., N\}$, is given by

$$\begin{aligned} \big(v - P^{NR}(X^{NR}(v,L))\big) \cdot g_j^{NR} \qquad & \text{if } \delta(v) = NR \\ \big(v - P^{R}(X^{R}(v,r,L))\big) \cdot g_j^{R}(r) \qquad & \text{if } \delta(v) = R, \end{aligned}$$

4. the payoff of the market maker is given by

$$\begin{aligned} -\big(v - P^{NR}(X^{NR}(v,L))\big) \cdot X^{NR}(v,L) \qquad & \text{if } \delta(v) = NR \\ -\big(v - P^{R}(X^{R}(v,r,L))\big) \cdot X^{R}(v,r,L) \qquad & \text{if } \delta(v) = R. \end{aligned}$$

2.3 Equilibrium

The informed and uninformed traders are expected utility maximizers, and the market maker sets the price so as to make zero expected profit conditional on the total order flow. Our equilibrium concept is similar to perfect Bayesian equilibrium: agents act “rationally” at their information sets under all contingencies.

Equilibrium consists of

- informed trader’s strategies: a decision on whether or not to rumor-monger, a signaling strategy determining the content of the rumor, demand strategies for each contingency of rumor-mongering decision and signaling strategy;

- sophisticated traders’ strategies: demand strategies for each contingency of informed trader’s rumor-mongering decision and signaling strategy;

- naive traders' strategies: demand strategies for each contingency of informed trader's rumor-mongering decision and signaling strategy;
- market maker's strategies[13]: pricing strategies for each contingency of informed trader's rumor-mongering decision.

In the trading round, informed and uninformed traders choose their demand strategies by maximizing their own expected trading profits conditional on their information and beliefs. The market maker sets the price equal to the expected value of the asset payoff conditional on the total order flow. In the signaling round, the informed trader first decides whether or not to send a rumor after having observed the asset payoff v. If she decides to send a rumor, then she also chooses her signaling strategy that determines the content of the rumor by maximizing her expected payoff given other agents' demand strategies and the asset payoff v.

Formally, an equilibrium of the described market game is a list of strategies

$$\Big\{\hat{\delta}(\cdot), \hat{f}(\cdot), \left(\hat{x}^R(\cdot,\cdot), \hat{x}^{NR}(\cdot)\right), \left(\{\hat{s}_j^R(\cdot)\}_{j=1}^{N_s}, \{\hat{s}_j^{NR}\}_{j=1}^{N_s}\right), \\ \left(\{\hat{g}_j^R(\cdot)\}_{j=N_s+1}^{N}, \{\hat{g}_j^{NR}\}_{j=N_s+1}^{N}\right), \left(\hat{P}^R(\cdot), \hat{P}^{NR}(\cdot)\right)\Big\}$$

such that

1. for any given $v \in \mathbb{R}$, each demand strategy $\hat{z}^{NR} \in \left\{\hat{x}^{NR}(v), \{\hat{s}_j^{NR}\}_{j=1}^{N_s}, \{\hat{g}_j^{NR}\}_{j=N_s+1}^{N}\right\}$ solves
$$\max_z \mathrm{E}\left[\left(\tilde{v} - \hat{P}^{NR}\left(z + \hat{X}^{NR}(\tilde{v}, \tilde{L}) - \hat{z}^{NR}\right)\right) z \,\middle|\, I_{\hat{z}^{NR}}\right],$$
where $\hat{X}^{NR}(\tilde{v}, \tilde{L}) := \hat{x}^{NR}(\tilde{v}) + \sum_{j=1}^{N_s} \hat{s}_j^{NR} + \sum_{j=N_s+1}^{N} \hat{g}_j^{NR} + \tilde{L}$ and $I_{\hat{z}^{NR}}$ denotes the posterior belief of the trader whose demand strategy is $\hat{z}^{NR}$,

2. for any given $(v, r) \in \mathbb{R}^2$, each demand strategy $\hat{z}^R \in \left\{\hat{x}^R(v, r), \{\hat{s}_j^R(r)\}_{j=1}^{N_s}, \{\hat{g}_j^R(r)\}_{j=N_s+1}^{N}\right\}$ solves
$$\max_z \mathrm{E}\left[\left(\tilde{v} - \hat{P}^R\left(z + \hat{X}^R(\tilde{v}, \tilde{r}, \tilde{L}) - \hat{z}^R\right)\right) z \,\middle|\, I_{\hat{z}^R}\right],$$

[13]Recall that we assume "whether a rumor is sent or not" is observable by the market maker although the content of the rumor itself is not observable to the market maker.

where $\hat{X}^R(\tilde{v},\tilde{r},\tilde{L}) := \hat{x}^R(\tilde{v},\tilde{r}) + \sum_{j=1}^{N_s} \hat{s}_j^R(\tilde{r}) + \sum_{j=N_s+1}^{N} \hat{g}_j^R(\tilde{r}) + \tilde{L}$ and $I_{\hat{z}^R}$ denotes the posterior belief of the trader whose demand strategy is $\hat{z}^R$,

3. for any given $(X^{NR}, X^R) \in \mathbb{R}^2$, the pricing strategies $\hat{P}^{NR}(X^{NR})$ and $\hat{P}^R(X^R)$ satisfy

   (a) $\hat{P}^{NR}(X^{NR}) = \mathrm{E}\left[\tilde{v} \,\middle|\, I_{m^{NR}}\right]$

   (b) $\hat{P}^{R}(X^{R}) = \mathrm{E}\left[\tilde{v} \,\middle|\, I_{m^{R}}\right]$

   where $I_{m^{NR}}$ and I_{m^R} denote the posterior beliefs of the market maker after observing the total order flows X^{NR} and X^R, respectively,

4. the signaling strategy $\hat{f}(v)$ solves

$$\max_{q\in\mathbb{R}} \mathrm{E}\left[\left(\tilde{v} - \hat{P}^R\left(x^*(q) + \sum_{j=1}^{N_s} \hat{s}_j^R(q) + \sum_{j=N_s+1}^{N} \hat{g}_j^R(q) + \tilde{L}\right)\right) x^*(q) - \alpha\,(q-\tilde{v})^2 \,\middle|\, \tilde{v} = v\right],$$

   where

$$x^*(q) = \arg\max_{x\in\mathbb{R}} \mathrm{E}\left[\left(\tilde{v} - \hat{P}^R\left(x + \sum_{j=1}^{N_s} \hat{s}_j^R(q) + \sum_{j=N_s+1}^{N} \hat{g}_j^R(q) + \tilde{L}\right)\right) x \,\middle|\, \tilde{v} = v\right],$$

5. the rumor-mongering decision $\hat{\delta}(v)$ solves $\max_{\delta\in\{R,NR\}} \mathrm{E}\left[u_i(\delta)\right]$, where

$$\begin{aligned}
\mathrm{E}\left[u_i\left(R\right)\right] &= \mathrm{E}\Bigg[\Bigg(\tilde{v} - \hat{P}^R\Big(\hat{x}^R(\tilde{v},\hat{f}(\tilde{v})) + \sum_{j=1}^{N_s} \hat{s}_j^R(\hat{f}(\tilde{v})) + \sum_{j=N_s+1}^{N} \hat{g}_j^R(\hat{f}(\tilde{v})) + \tilde{L}\Big)\Bigg)\hat{x}^R(\tilde{v},\hat{f}(\tilde{v})) \\
&\qquad - \alpha(\hat{f}(\tilde{v}) - \tilde{v})^2 \,\Bigg|\, \tilde{v} = v\Bigg], \\
\mathrm{E}\left[u_i\left(NR\right)\right] &= \mathrm{E}\left[\left(\tilde{v} - \hat{P}^{NR}\left(\hat{x}^{NR}(\tilde{v}) + \sum_{j=1}^{N_s} \hat{s}_j^{NR} + \sum_{j=N_s+1}^{N} \hat{g}_j^{NR} + \tilde{L}\right)\right)\hat{x}^{NR}(\tilde{v}) \,\middle|\, \tilde{v} = v\right].
\end{aligned}$$

To complete the definition of the equilibrium, we now define the equilibrium posterior beliefs of the agents. The equilibrium posterior beliefs consist of agents' observations and their equilibrium conjectures (anticipations) of others' strategies. Specifically,

- the equilibrium posterior belief of the trader, whose demand strategy is $\hat{z}^{NR}$, is given by

$$I_{\hat{z}^{NR}} = \begin{cases} \{\tilde{v} = v,\, \delta(\tilde{v}) = NR\} & \text{if } \hat{z}^{NR} = \hat{x}^{NR}(v) \\ \{\delta(\tilde{v}) = NR\} & \text{if } \hat{z}^{NR} = \hat{s}^{NR}(v) \\ \{\delta(\tilde{v}) = NR\} & \text{if } \hat{z}^{NR} = \hat{g}^{NR}(v), \end{cases}$$

- the equilibrium posterior belief of the trader, whose demand strategy is $\hat{z}^{R}$, is given by

$$I_{\hat{z}^{R}} = \begin{cases} \{\tilde{v} = v,\, \tilde{r} = r,\, \delta(\tilde{v}) = R;\, \hat{f}(\tilde{v}) \equiv \tilde{r}\} & \text{if } \hat{z}^{R} = \hat{x}^{R}(v, r) \\ \{\tilde{r} = r,\, \delta(\tilde{v}) = R;\, \hat{f}(\tilde{v}) \equiv^{s} \tilde{r}\}^{14} & \text{if } \hat{z}^{R} = \hat{s}^{R}(r) \\ \{\tilde{r} = r,\, \delta(\tilde{v}) = R;\, \tilde{v} \equiv^{g} \tilde{r}\}^{15} & \text{if } \hat{z}^{R} = \hat{g}^{R}(r), \end{cases}$$

- the equilibrium posterior belief of the market maker, when no rumor is sent and the total order flow is X^{NR}, is given by

$$I_{m^{NR}} = \left\{ \hat{X}^{NR}(\tilde{v}, \tilde{L}) \;=\; X^{NR},\; \delta(\tilde{v}) = NR \right\},$$

- the equilibrium posterior belief of the market maker, when a rumor is sent and the total order flow is X^{R}, is given by

$$I_{m^{R}} = \left\{ \hat{X}^{R}(\tilde{v}, \tilde{r}, \tilde{L}) \;=\; X^{R},\; \delta(\tilde{v}) = R;\; \hat{f}(\tilde{v}) \equiv^{m} \tilde{r} \right\}.^{16}$$

In our equilibrium definition, conditions (1) and (3.a) give the requirements to be satisfied in the trading round when the informed trader decides not to send any rumor. Conditions (2) and (3.b) specify the requirements to be satisfied in the trading round when the informed trader

[15] In equilibrium sophisticated traders correctly conjecture that the rumor equals $\hat{f}(v)$, and the notation $\hat{f}(\tilde{v}) \equiv^{s} \tilde{r}$ represents this conjecture.

[15] In equilibrium naive traders erroneously conjecture that the rumor equals v, and the notation $\tilde{v} \equiv^{g} \tilde{r}$ represents this conjecture.

[16] In equilibrium market maker correctly conjectures that the rumor equals $\hat{f}(v)$, and the notation $\hat{f}(\tilde{v}) \equiv^{m} \tilde{r}$ represents this conjecture.

decides to send a rumor and chooses the content of her rumor to be $r = \hat{f}(v)$. Condition (4) states that the informed trader optimally chooses the content r of her rumor given that everyone else plays equilibrium trading round strategies. Finally, condition (5) states that the rumor-mongering decision $\hat{\delta}(v)$ is optimal for the informed trader given that everyone plays the rest of the game prescribed by their equilibrium strategies.

We call a list of strategies

$$\{\hat{x}^{NR}(\cdot), \hat{\{}s_j^{NR}\}_{j=1}^{N_s}, \{\hat{g}_j^{R}\}_{j=N_s+1}^{N}, P^{NR}(\cdot)\}$$

an *NR-equilibrium* if they satisfy conditions (1) and (3.a). Similarly, we call a list of strategies

$$\left\{\hat{f}(\cdot),\ \hat{x}^{R}(\cdot,\cdot),\ \{\hat{s}_j^{R}(\cdot)\}_{j=1}^{N_s},\ \{\hat{g}_j^{R}(\cdot)\}_{j=N_s+1}^{N},\ \hat{P}^{R}(\cdot)\right\}$$

an *R-equilibrium* if they satisfy conditions (2), (3.b) and (4).

The *equilibrium outcome* depends on the equilibrium disclosure decision $\hat{\delta}$. If $\hat{\delta} = NR$, then demands and price attained at equilibrium are determined by

$$\left(\hat{x}^{NR}(\cdot), \{\hat{s}_j^{NR}\}_{j=1}^{N_s}, \{\hat{g}_j^{NR}\}_{j=N_s+1}^{N}, \hat{P}^{NR}(\cdot)\right).$$

If $\hat{\delta} = R$, then demands and price attained at equilibrium are determined by

$$\left(\hat{x}^{R}(\cdot,\cdot), \{\hat{s}_j^{R}(\cdot)\}_{j=1}^{N_s}, \{\hat{g}_j^{R}(\cdot)\}_{j=N_s+1}^{N}, \hat{P}^{R}(\cdot)\right).$$

Also, if $\hat{\delta} = R$, then the rumor attained at equilibrium is $r = \hat{f}(v)$.

In the followinf analysis of equilibrium, we will establish the existence of a *symmetric linear* equilibrium, i.e. an equilibrium in which the trading strategies are linear;

$$\begin{aligned}
\hat{x}^{R}(v,r) &= a^{R}\,v + b^{R}\,r, \\
\hat{s}_j^{R}(r) = \hat{s}^{R}(r) &= c^{R}\,r, \quad j = 1, ..., N_s, \\
\hat{g}_j^{R}(r) = \hat{g}^{R}(r) &= d^{R}\,r, \quad j = N_s+1, ..., N, \\
\hat{P}^{R}(X) &= \lambda^{R}\,X,
\end{aligned} \tag{1}$$

and

$$\begin{aligned}
\hat{x}^{NR}(v) &= a^{NR}v, \\
\hat{s}_j^{NR} = s^{NR} &= c^{NR}, \quad j=1,...,N_s, \\
\hat{g}_j^{NR} = g^{NR} &= d^{NR}, \quad j=N_s+1,...,N, \\
\hat{P}^{NR}(X) &= \lambda^{NR}X.
\end{aligned} \tag{2}$$

The exclusive analysis of symmetric linear equilibria is for the sake of tractability, and it is common in the market microstructure literature.

3 Hype and Dump

3.1 Rumor-mongering, demands, and price

In this section, we examine how rumor-mongering affects asset demands and asset price. To that end, we first analyze trading strategies when there is no rumor-mongering (i.e. NR-equilibrium) and then signaling and trading strategies when there is rumor-mongering (i.e. R-equilibrium). One particular problem we face in this analysis is the information revealed by the informed trader's rumor-mongering strategy, i.e. her initial decision on whether or not to send a rumor. Throughout this section, we assume that the rumor-mongering strategy of informed trader, $\delta(v) \in \{NR, R\}$, does not reveal any information about the asset payoff v. In the proceeding sections, we will establish the existence of an equilibrium in which the rumor-mongering strategy of informed trader satisfies the said assumption.

When there is no rumor-mongering, the game reduces to a standard trading game as in Kyle (1985):

Proposition 1 *Suppose the rumor-mongering strategy $\delta(v) = NR$ is not informative about the* *asset payoff v. There exists a unique symmetric linear NR-equilibrium, which is given by*

$$\begin{aligned} \hat{x}^{NR}(v) &= \frac{\sigma_L}{\sigma_v}\, v, \\ \hat{s}^{NR} = \hat{g}^{NR} &= 0, \\ \hat{P}^{NR}(X) &= \frac{\sigma_v}{2\,\sigma_L}\, X. \end{aligned}$$

When there is no rumor-mongering in the market, sophisticated and naive traders are at an informational disadvantage relative to the market maker: the market maker can infer information from the net total order flow whereas sophisticated and naive traders stay completely uninformed throughout the game. As a result, only the informed trader, liquidity traders, and the market maker participate in trading.

However the situation changes when we move on to consider a market with rumor-mongering. In such a market, sophisticated and naive traders are no more at an informational disadvantage compared to the market maker; after all, they observe the rumor sent by the informed trader. Therefore, the trading game in this market significantly differs from the one analyzed in Proposition 1.

Proposition 2 *Suppose the rumor-mongering strategy $\delta(v) = R$ is not informative about the asset payoff v. If there is at least one naive trader (i.e. $N > N_s$) and the intensity of regulatory enforcement α is non-zero, then there exists a symmetric linear R-equilibrium in which the signaling strategy is linear, i.e.*

$$\hat{r} = \hat{\rho}\, v, \tag{3}$$

where $\hat{\rho}$ satisfies

$$-\frac{2+N+N_s}{N-N_s} \;<\; \hat{\rho} \;<\; -\frac{N_s}{N-N_s}. \tag{4}$$

and demand and pricing strategies satisfy

$$
\begin{aligned}
\hat{x}^R(v,r) &= \frac{1}{2\,\hat{\lambda}^R}\,v - \frac{\hat{\rho}(N-N_s)+N_s}{2\hat{\rho}(2+N)\,\hat{\lambda}^R}\,r, \\
\hat{s}^R(r) &= \frac{2+(1-\hat{\rho})(N-N_s)}{2\hat{\rho}(2+N)\,\hat{\lambda}^R}\,r, \\
\hat{g}^R(r) &= \frac{2\hat{\rho}-(1-\hat{\rho})N_s}{2\hat{\rho}(2+N)\,\hat{\lambda}^R}\,r, \\
\hat{P}^R(X) &= \hat{\lambda}^R\,X,
\end{aligned}
\tag{5}
$$

where

$$
\hat{\lambda}^R = \frac{\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\sigma_v}{2(2+N)\sigma_L},
$$

and $\mathcal{A}(N,N_s,\hat{\rho}) := (2+(1-\hat{\rho})(N-N_s))\,(2+(1+\hat{\rho})N+(1-\hat{\rho})N_s)$.

The linear trading and pricing strategies in equilibrium are common in the literature. Our novelty here is to show the existence of linear signaling at equilibrium. When there is rumor-mongering in the market, there are two stages, each with a different game: the stage of signaling game, where the signaling strategy is chosen by the informed trader, and the stage of trading game, in which the demands and price are determined. The R-equilibrium we consider in Proposition 2 presents a plausible signaling strategy: the informed trader's rumor is in opposite direction to her private information on the asset payoff. The informed trader herds the naive traders in the wrong direction by telling them to buy if the asset has a high payoff and vice versa, which, in turn, affects the information inferred by the market maker from the net total order flow, hence the price set by the market maker falls far short of reflecting the asset payoff. As a result, the informed trader manages to increase her profit margin.

In the determination of R-equilibrium signaling strategy, the intensity α of regulatory enforcement plays a critical role. Recall that the choice of signaling strategy is only observable to the informed trader, and other market participants are only endowed with beliefs (anticipations) regarding the equilibrium strategy. Therefore when the informed trader deviates from the equilibrium strategy, the other market participants continue to believe that she is still following the equilibrium path. In the case of no regulatory enforcement, the described belief structure would cause the informed trader to lie as much as possible, i.e. her choice of r would converge to $\mp\infty$, which would in turn imply that the informed trader would never be able to reach an equilibrium when it comes to her choice of signaling strategy. The regulatory enforcement helps the informed trader to commit to an equilibrium signaling strategy, because deviation from equilibrium by lying also brings a disutility due to the cost associated with dishonest rumor-mongering. That is why we need α to be non-zero when we show the existence of R-equilibrium in Proposition 2.

A particularly visible implication of rumor-mongering is the increase in the participation in trading. As we mentioned earlier, since the sophisticated and gullible traders receive information on the asset payoff in the form of a rumor, they trade. Not only they trade but also trade a great deal. The sophisticated traders trade aggressively because they fully infer the value of the asset payoff from the rumor in equilibrium. The naive traders also trade aggressively, however for the wrong reasons: they take the rumor at its face value and wrongly believe that they fully know the asset payoff, which leads them to intense trading.

3.2 Rumor-mongering, hype and dump, and policy implications

We next characterize those economies in which the informed trader will send a rumor to uninformed traders. We also discuss the policy implications of rumor-mongering in financial markets.

Proposition 3 establishes that rumor-mongering can be attained in equilibrium.

Proposition 3 *If there is at least one naive trader and at least one sophisticated trader (i.e. $0 < N_s < N$) and the intensity of regulatory enforcement α is greater than or equal to $\frac{(N-N_s)^2}{N\,N_s}\frac{\sigma_L}{2\,\sigma_v}$, then there exists an equilibrium in which the rumor-mongering strategy is R. In this equilibrium, the rumor sent by the informed trader is of the form $r = \hat{\rho}\,v$, where $\hat{\rho}$ satisfies (4), and the attained demands and price are as given by (5). Moreover, the equilibrium rumor-mongering strategy, $\hat{\delta}(v) = R$, does not reveal any information about the asset payoff v.*

From now on, we will refer to the equilibrium considered in Proposition 3 as *hype and dump equilibrium*, because in such an equilibrium the informed trader first creates a false hype about the asset by sending a rumor, which is in the opposite direction to her private information, and then makes use of the hype during the trading stage.

Proposition 3 also sheds some light on the economic circumstances under which the hype and dump technique is employed by the informed trader. First of all, Proposition 3 validates our analysis in Section 3.1, which was carried out under the assumption that the decision on whether or not to rumor-monger is non-revealing: there indeed exists an equilibrium in which the rumor-mongering strategy, $\hat{\delta}$, does not reveal information about the asset payoff. Second, Proposition 3 shows that rumor-mongering can be attained in equilibrium even if there is only one naive trader in the market. The trade-off on rumor-mongering comes from three different sources: (i) regulatory enforcement makes rumor-mongering costly, even though it also helps the informed trader to commit to an equilibrium rumor-mongering strategy as discussed earlier in Section 3.1, (ii) the presence of naive traders encourages the informed trader to rumor-monger since their presence allows the informed trader to manipulate the asset price, (iii) the presence of sophisticated traders puts the informed trader off from rumor-mongering because they fully infer the value of the asset payoff from her rumor and then increase the competition. It would not have been surprising to see rumor-mongering attained in equilibrium if all the traders were naive. However, it is not straightforward to see whether or not the informed trader would optimally choose to rumor-monger in an economy that contains both sophisticated and naive traders. Nevertheless Proposition 3 accomplishes to show that the presence of sophisticated traders may not be sufficient to deter the informed trader from rumor-mongering.

Also, Proposition 3 offers a paradoxical policy implication: it suggests that regulatory enforcement may not prevent rumor-mongering and hype and dump manipulation. This implication essentially follows from the fact that regulatory enforcement creates not only a deterrence mechanism but also a commitment mechanism for the informed trader when it comes to her signaling strategy. As we discussed earlier, regulatory enforcement can prove to be helpful for the informed trader because only in the presence of enforcement does the informed trader find deviation from equilibrium by lying costly.

Table I

Rumor-mongering and policy implications

$\alpha = 0.6$

N_s	R-equilibria ($\hat{\rho}$)	equilibria ($\hat{\delta}$)
10	{-1.03, 0.273, 0.537}	$\{NR,\ NR,\ NR\}$
50	{-3.021, 0.429, 0.703}	$\{R,\ NR,\ NR\}$
90	{-19.199, 0.216, 0.886}	$\{R,\ NR,\ NR\}$

$\alpha = 1$

N_s	R-equilibria ($\hat{\rho}$)	equilibria ($\hat{\delta}$)
10	{-1.170, 0.075, 0.863}	$\{R,\ NR,\ NR\}$
50	{-3.033, 0.183, 0.904}	$\{R,\ NR,\ NR\}$
90	{-19.2, 0.120, 0.941}	$\{R,\ NR,\ NR\}$

$\alpha = 2$

N_s	R-equilibria ($\hat{\rho}$)	equilibria ($\hat{\delta}$)
10	{-1.226, 0.028, 0.958}	$\{R,\ NR,\ NR\}$
50	{-3.038, 0.079, 0.966}	$\{R,\ NR,\ NR\}$
90	{-19.2, 0.057, 0.973}	$\{R,\ NR,\ NR\}$

This table reports how the intensity of regulatory enforcement and the number of sophisticated traders affect equilibrium outcomes. Each panel is set for a different value of α. The first column reports the number N_s of sophisticated traders, the second column reports R-equilibria characterized by the signaling strategy $\hat{\rho}$, and the third column reports equilibria characterized by the rumor-mongering strategy $\hat{\delta}$. All panels below share the following model parameters: the variance σ_v^2 of asset payoff is 1, the variance σ_L^2 of liquidity demand is 1, and the total number N of uninformed traders is 100. The equilibrium analysis is confined to symmetric linear equilibria in which the rumor-mongering strategy (i.e. the decision on whether or not to send a rumor) does not reveal any information about the asset payoff.

However, the fact that regulatory enforcement does not necessarily put a curb on the hype and dump manipulation, by no means suggests that employing enforcement measures on rumor-mongering is a bad policy overall. Proposition 3 only warns that policy proposals that only focus on the deterrence brought by enforcement, and ignore the other incentive implications for the market participants, can be dangerously misleading.

Numerical examples show that there can be multiple equilibria. In some of these equilibria, enforcement measures can deliver a market with no rumor-mongering. This strengthens our earlier emphasis that enforcement may not be a bad policy when all aspects are considered. Table I reports some of the multiple equilibria obtained through numerical examples.

3.3 Who benefits from hype and dump?

In a stock market manipulation, someone makes a profit at the expense of other traders. The proposition shows that, in the hype and dump equilibrium, the informed trader and the sophisticated traders are the winners, and the naive traders are the losers.

Proposition 4 *Suppose that there is at least one naive trader and at least one sophisticated trader (i.e. $0 < N_s < N$) and the intensity of regulatory enforcement α is greater than or equal to $\frac{(N-N_s)^2}{N\,N_s}\frac{\sigma_L}{2\,\sigma_v}$.*

(a) Given the asset payoff v and the rumor r, the informed trader's expected trading profit in a hype and dump equilibrium is greater than that attained if the informed trader did not send any rumor.

(b) Given the asset payoff v and the rumor r, a sophisticated trader's trading profit in a hype and dump equilibrium is strictly greater than that attained if the informed trader did not send any rumor.

(c) Given the asset payoff v and the rumor r, a naive trader's expected trading profit in a hype and dump equilibrium is strictly less than that attained if the informed trader did not send any rumor.

The informed trader is bound to benefit from hype and dump since she is the one who optimally chooses to rumor-monger and create the false hype. It is also natural that naive traders lose in the hype and dump equilibrium, because they incorrectly believe that the rumor truthfully reveals the asset payoff. Sophisticated traders benefit from hype and dump since they correctly infer the asset payoff from the sent rumor.

Proposition 4 is also broadly consistent with empirical evidence. Frieder and Zittrain (2006) analyze 75,000 unsolicited e-mails sent between January 2004 and July 2005 and conclude that spammers could make a return of 4.9%-6% by using this method, while recipients who act on the spam message typically lose 5.25% of their investment within two days.

3.4 Market depth, trading volume and market efficiency

In this section we investigate the asset pricing implications of hype and dump manipulation. First, we analyze the impact of hype and dump on market depth. In the market microstructure models based on Kyle (1985), market depth is, in general, the reciprocal of the price impact of the informed trader's demand. Therefore it essentially measures what Harris (1990) refers to as the *resiliency* dimension of liquidity, i.e. the ability to trade at a minimal price impact. In our model, $\frac{1}{\lambda}$ is the measure of market depth, where λ is the price coefficient of the net total order flow received by the market maker.

Proposition 5 *The market depth in a hype and dump equilibrium is greater than that attained if the informed trader did not send any rumor.*

Informed trader spreads a rumor in order to create higher trading activity, hence higher market depth, so that she can better camouflage her private information against the market maker. Proposition 5 confirms this argument and shows that market depth rises in a hype and dump equilibrium.

Recent empirical studies find that rumor-mongering leads to higher trading volume. Frieder and Zittrain (2006) show that trading volume responds positively and significantly to heavy stock touting. Also, Hanke and Hauser (2006) show that trading volume in spammed stocks is significantly higher on and around spam days. In our model, we measure trading volume by summing up the absolute values of all traders' demands. In particular, given $(v, L) \in \mathbb{R}^2$, the trading volume is given by

$$|x^R(v, r)| + \sum_{j=1}^{N_s} |s_j^R(r)| + \sum_{j=1}^{N_s} |g_j^R(r)| + |L|$$

when there is rumor-mongering in the form of $r = \rho\, v$, and by

$$|x^{NR}(v)| + \sum_{j=1}^{N_s} |s_j^{NR}| + \sum_{j=1}^{N_s} |g_j^{NR}| + |L|$$

when there is no rumor-mongering. Our model is able to rationalize the empirical findings, mentioned above, with the following proposition.

Proposition 6 *For any given $(v, L) \in \mathbb{R}^2$, the trading volume in a hype and dump equilibrium is higher than that attained if the informed trader did not send any rumor.*

It is straightforward to observe from Propositions 1 and 2 that participation in trading increases in the hype and dump equilibrium, however this does not necessarily imply that trading volume would also increase. Proposition 6 shows that trading volume does increase in the hype and dump equilibrium.

Finally, we inspect the effect of hype and dump manipulation on market efficiency, i.e. the informational efficiency of asset price. We measure market efficiency by the reciprocal of the variance of the asset payoff conditional on the asset price. This measure gives us an indication of the amount of information revealed by the asset price. The proposition below clearly shows that market efficiency is lower in a hype and dump equilibrium.

Proposition 7 *The informational efficiency of price in a hype and dump equilibrium is lower than that attained if the informed trader did not send any rumor.*

This result is not surprising given that the informed trader's objective in rumor-mongering is to mislead the market maker in her pricing decision. Proposition 7 confirms this logic.

4 Conclusion

We have developed a model with stages of signaling and asset trading to better understand the economic circumstances under which hype and dump manipulation can occur. In our model, the informed trader, who knows the asset payoff, has the option to send a rumor (a stock tip) to the uninformed traders. Dishonest rumor-mongering is costly due to regulatory enforcement. If a rumor is sent, the uninformed traders construe that rumor differently depending on their types: The sophisticated traders, in equilibrium, know the relationship between the rumor and the asset payoff, hence they fully infer the private information of the informed trader. The naive ones take the rumor at its face value, and act as if the rumor truthfully reveals the asset payoff.

This paper has shown that hype and dump manipulation can be an equilibrium outcome provided that there is at least one naive trader in the market and the cost of dishonest rumor-mongering is high enough. The fact that hype and dump can occur in the presence of intense regulatory enforcement (i.e. high rumor-mongering cost) may seem paradoxical. However the effect of enforcement on the informed trader's incentives is two-folded; on the one hand, it deters the informed trader from a great deal of dishonesty by imposing a punishment, on the other hand it makes the informed trader attain some level of dishonesty in her rumor by creating favorable market beliefs regarding her intentions.

Our model also yields a number of welfare and asset pricing implications. These implications include: (1) the informed trader increases her net profit by hyping an dumping at the expense of naive traders, (2) market depth rises after hyping and dumping, (3) trading volume increases after hyping and dumping, (4) hype and dump decreases market efficiency.

On the theoretical side, we have established that endogenous rumor-mongering can significantly alter trading behavior in financial markets. On the policy side, our results warn that policy proposals that solely focus on the deterrence brought by enforcement and ignore other incentive implications for the market participants involved, can be misleading.

One natural question to pursue in future research is whether or not an uninformed trader, who pretends to be informed, can successfully engage in hype and dump manipulation in equilibrium. It also remains to be seen whether there are optimal cost schemes that can be enforced by financial regulatory bodies so as to prevent potential manipulators from dishonest rumor-mongering.

Appendix A: Mathematical Preliminary

Lemma 1 *Let* $(\tilde{x}, \tilde{y})$ *be an* m*-dimensional jointly normally distributed random vector with mean* $\mu \in \mathbb{R}^m$ *and variance-covariance matrix* $\Sigma \in \mathbb{R}^{m \times m}$*. Suppose that*

$$\mu = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}_{m \times 1} \quad \textit{and} \quad \Sigma = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{sx} & \Sigma_{yy} \end{pmatrix}_{m \times m}$$

where μ_a *denotes the mean of random vector* $\tilde{a} \in \{\tilde{x}, \tilde{y}\}$ *and* Σ_{ab} *denotes the variance-covariance matrix of random vector* $(\tilde{a}, \tilde{b}) \in \{\tilde{x}, \tilde{y}\} \times \{\tilde{x}, \tilde{y}\}$*. Then the conditional distribution of* $\tilde{x}$ *given* $\tilde{y} = y$ *is normal with mean* $\mu_x + \Sigma_{xy}\Sigma_{yy}^{-1}(y - \mu_y)$ *and variance* $\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}$*.*

Appendix B: Proofs

Proof of Proposition 1.

This result follows directly from Kyle (1985), therefore we skip the proof.

Lemma 2 *Suppose the disclosure strategy* $\delta = R$ *does not reveal any information about the asset payoff and the rumor is of the form* $r = \hat{\rho}\, v$*, i.e* $\hat{f}(v) = \hat{\rho}\, v$ *for some* $\hat{\rho} \in \mathbb{R}$*. If* $\left(\hat{f}, \hat{x}^R(\cdot,\cdot), \hat{s}^R(\cdot), \hat{g}^R(\cdot), \hat{P}^R(\cdot)\right)$ *is a symmetric linear* R*-equilibrium, then*

$$\begin{aligned} \hat{x}^R(v,r) &= a^R\, v + b^R\, r, \\ \hat{s}^R(r) &= c^R\, r, \\ \hat{g}^R(r) &= d^R\, r, \\ \hat{P}^R(X) &= \lambda^R\, X, \end{aligned} \tag{6}$$

where

$$\begin{aligned} a^R &= \tfrac{1}{2\,\lambda^R}, \\ b^R &= -\tfrac{\hat{\rho}(N-N_s)+N_s}{2\hat{\rho}(2+N)\,\lambda^R}, \\ c^R &= \tfrac{2+(1-\hat{\rho})(N-N_s)}{2\hat{\rho}(2+N)\,\lambda^R}, \\ d^R &= \tfrac{2\hat{\rho}-(1-\hat{\rho})N_s}{2\hat{\rho}(2+N)\,\lambda^R}, \\ \lambda^R &= \tfrac{\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\ \sigma_v}{2\,(2+N)\,\sigma_L}, \end{aligned}$$

with

$$\mathcal{A}(N,N_s,\hat\rho)=(2+(1-\hat\rho)(N-N_s))\ (2+(1+\hat\rho)N+(1-\hat\rho)N_s)\,.$$

Proof. Since $\left(\hat\rho,\hat x^R(\cdot,\cdot),\hat s^R(\cdot),\hat g^R(\cdot),\hat P^R(\cdot)\right)$ is a symmetric linear R-equilibrium, demand and pricing strategies are of the forms given by (6).

The informed trader's optimal demand strategy $\hat x^R(v,r)$ solves

$$\max_z \mathrm{E}\left[\left(\tilde v-\lambda^R\left(z+N_sc^R\tilde r+(N-N_s)d^R\tilde r+\tilde L\right)\right)z-\alpha(\tilde r-\tilde v)^2\,\middle|\, v,r,\delta=R\right]. \tag{7}$$

The first order condition (foc) of (7) gives us

$$\hat x^R(v,r)\;=\;\frac{v-\lambda^R\left(N_sc^R+(N-N_s)d^R\right)r}{2\,\lambda^R}. \tag{8}$$

(6), (8), and the second order condition (soc) of (7) yield

$$\begin{aligned} a^R &= \tfrac{1}{2\,\lambda^R},\\ b^R &= -\tfrac{N_sc^R+(N-N_s)d^R}{2},\\ \lambda^R &\geq 0. \end{aligned} \tag{9}$$

A sophisticated trader optimally chooses her demand strategy $\hat s^R(r)$ to solve

$$\max_z \mathrm{E}\left[\left(\tilde v-\lambda^R\left(z+a^R\tilde v+b^R\tilde r+(N_s-1)c^R\tilde r+(N-N_s)d^R\tilde r+\tilde L\right)\right)z\,\middle|\, r,\delta=R\,;r\equiv^s\hat\rho v\right]. \tag{10}$$

Since a sophisticated trader correctly anticipates that v equals $\hat\rho\, r$ while taking expectation, using the foc of (10), we obtain

$$\hat s^R(r)\;=\;\frac{\frac{r}{\hat\rho}-\lambda^R\left(a^R\,\frac{r}{\hat\rho}+b^R\,r+(N_s-1)c^R\,r+(N-N_s)d^R\,r\right)}{2\lambda^R}. \tag{11}$$

(6), (11), and the soc of (10) yield

$$\begin{aligned} c^R &= \tfrac{\frac{1}{\hat\rho}-\lambda^R\left(\frac{a^R}{\hat\rho}+b^R+(N-N_s)d^R\right)}{(N_s+1)\lambda^R},\\ \lambda^R &\geq 0. \end{aligned} \tag{12}$$

A naive trader optimally chooses her demand strategy $g^R(r)$ to solve

$$\max_z \mathrm{E}\left[\left(\tilde v-\lambda^R\left(z+a^R\tilde v+b^R\tilde r+N_sc^R\tilde r+(N-N_s-1)d^R\tilde r+\tilde L\right)\right)z\,\middle|\, r,\delta=R\,;r\equiv^g v\right]. \tag{13}$$

Since a naive trader incorrectly anticipates that v equals r while taking expectation, the foc of (13) implies

$$g^R(r) \;=\; \frac{\left(1-\lambda^R\left(a^R+b^R+N_s c^R+(N-N_s-1)d^R\right)\right)r}{2\lambda^R}. \tag{14}$$

(6), (14), and the soc of (13) yield

$$\begin{aligned} d^R &= \tfrac{1-\lambda^R\left(a^R+b^R+N_s c^R\right)}{(N-N_s+1)\,\lambda^R}, \\ \lambda^R &\geq 0. \end{aligned} \tag{15}$$

The market maker sets the asset price so that

$$\begin{aligned} P^R(X) &= \mathrm{E}\left[\tilde{v}\,\middle|\, X = x^R(v,r)+N_s\,s^R(r)+(N-N_s)\,g^R(r)+L,\, \delta=R;\, r\equiv^m \hat{\rho}\,v\right] \\ &= \mathrm{E}\left[\tilde{v}\,\middle|\, X = a^R\,v+(b^R+N_s\,c^R+(N-N_s)\,d^R)\,r+L,\,,\delta=R;\, r\equiv^m \hat{\rho}\,v\right] \\ &= \frac{\left(a^R+(b^R+N_s\,c^R+(N-N_s)\,d^R)\,\hat{\rho}\right)\,\sigma_v^2}{\left(a^R+(b^R+N_s\,c^R+(N-N_s)\,d^R)\,\hat{\rho}\right)^2\sigma_v^2+\sigma_L^2}\,X, \end{aligned}$$

where the latter equality follows from Lemma 1. Following (6) and the equation above,

$$\lambda^R \;=\; \frac{\left(a^R+(b^R+N_s\,c^R+(N-N_s)\,d^R)\,\hat{\rho}\right)\,\sigma_v^2}{\left(a^R+(b^R+N_s\,c^R+(N-N_s)\,d^R)\,\hat{\rho}\right)^2\sigma_v^2+\sigma_L^2}. \tag{16}$$

Using the second order conditions, i.e. that $\lambda^R > 0$, and solving (9), (12), (15), and (16) together yields

$$\begin{aligned} a^R &= \tfrac{1}{2\,\lambda^R}, \\ b^R &= -\tfrac{\hat{\rho}(N-N_s)+N_s}{2\hat{\rho}(2+N)\,\lambda^R}, \\ c^R &= \tfrac{2+(1-\hat{\rho})(N-N_s)}{2\hat{\rho}(2+N)\,\lambda^R}, \\ d^R &= \tfrac{2\hat{\rho}-(1-\hat{\rho})N_s}{2\hat{\rho}(2+N)\,\lambda^R}, \\ \lambda^R &= \tfrac{\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\;\sigma_v}{2\,(2+N)\,\sigma_L}, \end{aligned} \tag{17}$$

where

$$\mathcal{A}(N,N_s,\hat{\rho}) = (2+(1-\hat{\rho})(N-N_s))\;(2+(1+\hat{\rho})N+(1-\hat{\rho})N_s)\,. \quad \square$$

Proof of Proposition 2. Let $N_s < N$. We will conjecture a linear equilibrium signaling strategy $r = \hat{\rho}\,v$ and verify it.

Since the rumor r is a choice variable for the informed trader, it solves

$$\max_{r\in\mathbb{R}} \mathrm{E}\left[\left(\tilde{v} - \hat{P}^R\left(x^* + N_s\,\hat{s}^R(r) + (N-N_s)\hat{g}^R(r) + \tilde{L}\right)\right)x^* - \alpha(r-\tilde{v})^2 \,\middle|\, v,\,\delta = R\right], \tag{18}$$

where

$$x^* = \arg\max_{x} \mathrm{E}\left[\left(\tilde{v} - P^R\left(x + N_s\,\hat{s}^R(r) + (N-N_s)\,\hat{g}^R(r) + \tilde{L}\right)\right)x - \alpha(r-\tilde{v})^2 \,\middle|\, v,\,\delta = R\right] \tag{19}$$

Using Lemma 2, when sophisticated traders and the market maker anticipate that $r = \hat{\rho}\,v$, we have

$$x^* = \frac{v - \lambda^R\left(N_s c^R + (N-N_s)d^R\right)r}{2\lambda^R}. \tag{20}$$

Plugging (20) into (18), we obtain

$$\max_{r\in\mathbb{R}} \mathrm{E}\left[\frac{\left(v - d^R\,N\,r\,\lambda^R + \left(-c^R + d^R\right)\,r\,\lambda^R\,N_s\right)^2}{4\,\lambda^R} - \alpha\,(v-r)^2 \,\middle|\, v,\,\delta = R\right] \tag{21}$$

Given that sophisticated traders and the market maker anticipate that $r = \hat{\rho}\,v$, using Lemma 2, (21) becomes

$$\max_{r\in\mathbb{R}} \mathrm{E}\left[\frac{\left((N\,(r-v) - 2\,v)\,\hat{\rho} + (r - r\,\hat{\rho})\,N_s\right)^2\sigma_L}{2\sqrt{\mathcal{A}(\mathcal{N},\mathcal{N}_f,\hat{\rho})}\,(2+N)\,\hat{\rho}^2\,\sigma_v} - \alpha\,(v-r)^2 \,\middle|\, v,\,\delta = R\right] \tag{22}$$

Clearly the objective function in (22) is a quadratic polynomial of r. From FOC of (22) solving for r yields

$$\hat{r} = -\left(\frac{2\,\alpha + \frac{(-2-N)\,(N\,\hat{\rho}+(1-\hat{\rho})\,N_s)\,\sigma_L}{\sqrt{\mathcal{A}(\mathcal{N},\mathcal{N}_f,\hat{\rho})}\,(2+N)\,\hat{\rho}\,\sigma_v}}{-2\,\alpha + \frac{(N\,\hat{\rho}+(1-\hat{\rho})\,N_s)^2\ ,\sigma_L}{\sqrt{\mathcal{A}(\mathcal{N},\mathcal{N}_f,\hat{\rho})}\,(2+N)\,\hat{\rho}^2\,\sigma_v}}\right)v \tag{23}$$

And the SOC of (22) is

$$-2\,\alpha + \frac{\left(N\,\hat{\rho} + (1-\hat{\rho})\,N_s\right)^2\sigma_L}{\sqrt{\mathcal{A}(\mathcal{N},\mathcal{N}_f,\hat{\rho})}\,(2+N)\,\hat{\rho}^2\,\sigma_v} \,<\, 0 \tag{24}$$

From (25), we see that, given that sophisticated traders and the market maker anticipate that $r = \hat{\rho}\, v$, the best response function of the informed trader is a linear function of v. Thus o establish the existence of a symmetric linear R-equilibrium, we need to show that there exists $\hat{\rho} \in \mathbb{R}$ which satisfies,

$$\hat{\rho} = -\left(\frac{2\,\alpha + \frac{(-2-N)\,(N\,\hat{\rho}+(1-\hat{\rho})\,N_s)\,\sigma_L}{\sqrt{\mathcal{A}(\mathcal{N},\mathcal{N}_f,\hat{\rho})}\,(2+N)\,\hat{\rho}\,\sigma_v}}{-2\,\alpha + \frac{(N\,\hat{\rho}+(1-\hat{\rho})\,N_s)^2\;,\sigma_L}{\sqrt{\mathcal{A}(\mathcal{N},\mathcal{N}_f,\hat{\rho})}\,(2+N)\,\hat{\rho}^2\,\sigma_v}} \right) \tag{25}$$

$$0 > -\,2\,\alpha + \frac{(N\,\hat{\rho} + (1-\hat{\rho})\;N_s)^2\,\sigma_L}{\sqrt{\mathcal{A}(\mathcal{N},\mathcal{N}_f,\hat{\rho})}\;(2+N)\;\hat{\rho}^2\,\sigma_v} \tag{26}$$

[17]To establish the existence of $\hat{\rho}$ satisfying (25), first notice that when $\rho = 0$, the rumor reveals no information about the asset payoff v. Prior to the choice regarding the rumor content, the informed trader has the option not to spread any rumor to the uninformed traders, which captures the desired effect that $\rho = 0$ provides without incurring any rumor-mongering cost. Therefore we can assume that $\hat{\rho} \neq 0$. Simplifying (25) we obtain

$$1 \;\;=\;\; \frac{(2+N)\;\Big((\hat{\rho}\,N - (-1+\hat{\rho})\;N_s)\;\sigma_L - 2\,\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\hat{\rho}\,\alpha\,\sigma_v\Big)}{(\hat{\rho}\,N - (-1+\hat{\rho})\;N_s)^2\,\sigma_L - 2\,\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,(\hat{\rho})^2\;(2+N)\;\alpha\,\sigma_v}, \tag{27}$$

which is equivalent to

$$\begin{aligned} 0 \;\;=\;\; & (\hat{\rho}\,N - (-1+\hat{\rho})\;N_s)^2\,\sigma_L - 2\,\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,(\hat{\rho})^2\;(2+N)\;\alpha\,\sigma_v \\ & -(2+N)\;\Big((\hat{\rho}\,N - (-1+\hat{\rho})\;N_s)\;\sigma_L - 2\,\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\hat{\rho}\,\alpha\,\sigma_v\Big) \end{aligned} \tag{28}$$

provided that the numerator and the denominator of the right hand side (RHS) of equation (27) are not simultaneously equal to zero at $\hat{\rho}$. Let

$$\begin{aligned} f(x) \;\;=\;\; & (x\,N - (-1+x)\;N_s)^2\,\sigma_L - 2\,\sqrt{\mathcal{A}(N,N_s,x)}\,(x)^2\;(2+N)\;\alpha\,\sigma_v \\ & -(2+N)\;\Big((x\,N - (-1+x)\;N_s)\;\sigma_L - 2\,\sqrt{\mathcal{A}(N,N_s,x)}\,x\,\alpha\,\sigma_v\Big). \end{aligned} \tag{29}$$

It follows from (28) that the root of f essentially satisfies (25) if the numerator and the denominator of the RHS of equation (27) are not simultaneously equal to zero at $\hat{\rho}$. First, let us show

[17]Note that the maximization function, given in (22), would become convex if α were equal to 0.

that f has a root within the interval $\left(-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{N-N_s}\right)$. It is easy to check that f is continuous within the interval $\left[-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{N-N_s}\right]$. Moreover, $f\left(-\frac{2+N+N_s}{N-N_s}\right) = 2(2+N)^2\sigma_L$ and $f\left(-\frac{N_s}{N-N_s}\right) = -\frac{2\,\alpha\,N\,(2+N)^2\,N_s\,\sigma_v}{(N-N_s)^2}$. Thus, by intermediate value theorem, it follows that there exists

$$\hat{\rho}\in\left(-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{N-N_s}\right),$$

which is a root of f. Next we show that the numerator and the denominator of RHS of (27) are not simultaneously equal to zero at the root $\hat{\rho}\in\left(-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{N-N_s}\right)$ of f. Suppose to the contrary that both the numerator and the denominator of RHS of (27) are zero at $\hat{\rho}$. Let $w := 2\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\hat{\rho}\,\alpha\,\sigma_v$. Using our assumption that the denominator of RHS of (27) equals zero at $\hat{\rho}$, we derive

$$w=\frac{(\hat{\rho}\,N+N_s-\hat{\rho}\,N_s)^2\,\sigma_L}{\hat{\rho}\,(2+N)}.$$

Plugging w into numerator of RHS of (27), we obtain

$$\begin{aligned}
&(2+N)\,\left((\hat{\rho}\,N-(-1+\hat{\rho})\,N_s)\,\sigma_L-2\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\hat{\rho}\,\alpha\,\sigma_v\right)\\
=\;&(2+N)\,\hat{\rho}\,(w+(N\,\hat{\rho}+N_s-\hat{\rho}\,N_s)\,\sigma_L)\\
=\;&(-N\,\hat{\rho}+(-1+\hat{\rho})\,N_s)\,(N_s-\hat{\rho}\,(2+N_s))\,\sigma_L \qquad (30)
\end{aligned}$$

Clearly (30) is not equal to zero given that $\hat{\rho}\in\left(-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{N-N_s}\right)$. Hence, we have proved that both the numerator and the denominator of RHS of (27) are not equal to zero at $\hat{\rho}$. Therefore, $\hat{\rho}\in\left(-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{N-N_s}\right)$ satisfies (25).

Next we show that $\hat{\rho}\in\left(-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{N-N_s}\right)$ satisfies (26) if $\alpha>0$. Let

$$y:=\frac{(N\,\hat{\rho}+N_s-\hat{\rho}\,N_s)\,\sigma_L}{\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,(2+N)\,\hat{\rho}^2\,\sigma_v}.$$

Using the foc, (25), we obtain

$$y=\frac{-2\,\alpha\,(-1+\hat{\rho})}{-(\hat{\rho}\,N_s)+\hat{\rho}\,(2+N+\hat{\rho}\,(-N+N_s))}.$$

The soc, (26), can be then rewritten as follows:

$$\begin{aligned}
0>-\alpha+\frac{(\hat{\rho}\,N-(\hat{\rho}-1)N_s)^2\sigma_L}{2\,(\hat{\rho})^2\,(2+N)\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\sigma_v}&=-\,2\,\alpha+y\,(N\,\hat{\rho}+N_s-\hat{\rho}\,N_s)\\
&=\frac{2\,\alpha\,(N_s-\hat{\rho}\,(2+N_s))}{\hat{\rho}\,(2+N-N\,\hat{\rho}+(-1+\hat{\rho})\,N_s)}. \qquad (31)
\end{aligned}$$

The fact that $\hat{\rho} < 0$ implies that

$$\frac{2\,\alpha\,\left(N_s - \hat{\rho}\,\left(2+N_s\right)\right)}{\hat{\rho}\,\left(2+N-N\,\hat{\rho}+\left(-1+\hat{\rho}\right)\,N_s\right)} < 0$$

if $\alpha > 0$, and therefore (31) is satisfied as long as $\alpha > 0$. This in turn implies that (26) is satisfied by $\hat{\rho} \in \left(-\frac{2+N+N_s}{N-N_s}, -\frac{N_s}{N-N_s}\right)$ if $\alpha > 0$.

The rumor-mongering strategy, $\hat{\rho} \in \left(-\frac{2+N+N_s}{N-N_s}, -\frac{N_s}{N-N_s}\right)$, coupled with demand and pricing strategies, which are of the form given in Lemma 2, constitute the symmetric linear R-equilibrium given in Proposition 2 provided that $\alpha > 0$. $\square$

Proof of Proposition 3.

Let $(\hat{x}^{NR}(\cdot), \hat{s}^{NR}, \hat{g}^{NR}, \hat{P}^{NR}(\cdot))$ be the symmetric linear NR-equilibrium given in Proposition 1. Also, let $(\hat{\rho}, \hat{x}^{R}(\cdot,\cdot), \hat{s}^{R}(\cdot), \hat{g}^{R}(\cdot), \hat{P}^{R}(\cdot))$ be the symmetric linear R-equilibrium given in Proposition 2. We will show that

$$\left(R, \hat{\rho}, \left(\hat{x}^{R}(\cdot,\cdot), \hat{x}^{NR}(\cdot)\right), \left(\hat{s}^{R}(\cdot), \hat{s}^{NR}\right), \left(\hat{g}^{R}(\cdot), \hat{g}^{NR}\right), \left(\hat{P}^{R}(\cdot), \hat{P}^{NR}(\cdot)\right)\right) \tag{32}$$

is a (symmetric linear) equilibrium for $\alpha \geq \frac{(N-N_s)^2}{N N_s}\frac{\sigma_L}{2\sigma_v}$. Suppose for now that the equilibrium disclosure strategy $\hat{\delta} = R$ does not reveal any information about the asset payoff v. We will later verify this assumption.

In order to show that (32) is an equilibrium, we need to show that

$$\mathrm{E}\left[u_i\left(R\right)\right] \;\geq\; \mathrm{E}\left[u_i\left(NR\right)\right], \tag{33}$$

where

$$\begin{aligned}
\mathrm{E}\left[u_i\left(R\right)\right] &= \mathrm{E}\left[\left(\tilde{v}-P^{R}\left(\hat{x}^{R}(\tilde{v},\tilde{r})+N_s\hat{s}^{R}(\tilde{r})+(N-N_s)\hat{g}^{R}(\tilde{r})+\tilde{L}\right)\right)z-\alpha(\tilde{r}-\tilde{v})^2 \,\middle|\, v, r,\, \delta{=}R\right],\\
\mathrm{E}\left[u_i\left(NR\right)\right] &= \mathrm{E}\left[\left(\tilde{v}-P^{NR}\left(\hat{x}^{NR}(\tilde{v})+N_s\hat{s}^{NR}+(N-N_s)\hat{g}^{NR}+\tilde{L}\right)\right)z \,\middle|\, v, \delta{=}NR\right].
\end{aligned}$$

Using Propositions 1 and 2, we can rewrite (33) as

$$\frac{\left(2+N-\hat{\rho}\,N+\left(-1+\hat{\rho}\right)\,N_s\right)^2\sigma_L}{2\left(2+N\right)\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\sigma_v}\,v^2-\alpha(\hat{\rho}-1)^2\,v^2 \quad\geq\quad \frac{\sigma_L}{2\,\sigma_v}\,v^2,$$

which, of course, reduces to

$$\frac{\left(2+N-\hat{\rho}\,N+(-1+\hat{\rho})\;N_s\right)^2\sigma_L}{2\,(2+N)\,\sqrt{\mathcal{A}(N,N_s,\hat{\rho})}\,\sigma_v}-\alpha(\hat{\rho}-1)^2\quad\geq\quad\frac{\sigma_L}{2\,\sigma_v}. \tag{34}$$

Recall from the proof of Proposition 2 that $\hat{\rho}$ is a root of f, which is given by (29). Using the fact that $f(\hat{\rho})=0$, we derive

$$\mathcal{A}(N,N_s,\hat{\rho})\;=\;\left(\frac{(-2+(-1+\hat{\rho})\;N-(-1+\hat{\rho})\;N_s)\;(\hat{\rho}\,N-(-1+\hat{\rho})\;N_s)\;\sigma_L}{2\;(-1+\hat{\rho})\;\hat{\rho}\;(2+N)\;\alpha\,\sigma_v}\right)^2.$$

The equality above and straightforward algebraic manipulations allow us to rewrite (34) as follows:

$$\begin{aligned}
\frac{(-1+\hat{\rho})\;\hat{\rho}\;(-2+(-1+\hat{\rho})\;(N-N_s))}{\hat{\rho}\,(N-N_s)+\;N_s}\,\alpha-\alpha(\hat{\rho}-1)^2\quad&\geq\quad\frac{\sigma_L}{2\,\sigma_v},\\
\iff\quad\alpha\,(1-\hat{\rho})\left(\frac{-\hat{\rho}\,(2+(1-\hat{\rho})\;(N-N_s))}{-\hat{\rho}\,(N-N_s)-\;N_s}-(1-\hat{\rho})\right)\quad&\geq\quad\frac{\sigma_L}{2\,\sigma_v}\\
\iff\quad\alpha\,(1-\hat{\rho})\left(\frac{-2\,\hat{\rho}+(1-\hat{\rho})\,N_s}{-\hat{\rho}\,(N-N_s)-\;N_s}\right)\quad&\geq\quad\frac{\sigma_L}{2\,\sigma_v}.
\end{aligned} \tag{35}$$

Since $\hat{\rho}\in\left(-\frac{2+N+N_s}{N-N_s},-\frac{N_s}{NsN_s}\right)$, we have

$$\alpha\,(1-\hat{\rho})\left(\frac{-2\,\hat{\rho}+(1-\hat{\rho})\,N_s}{-\hat{\rho}\,(N-N_s)-\;N_s}\right)\quad>\quad\alpha\,\frac{N\,N_s}{(N-N_s)^2}. \tag{36}$$

(35) and (36) together yield that (33) is satisfied if $\alpha\geq\frac{(N-N_s)^2}{N\,N_s}\,\frac{\sigma_L}{2\,\sigma_v}$. This proves that (32) is an equilibrium for sufficiently large α.

Next, observe that, in this equilibrium, the choice of disclosure strategy follows from (33), which is equivalent to (34). Since (34) is not dependent on the asset payoff v, the equilibrium disclosure strategy $\hat{\delta}=R$ does not reveal any information about the asset payoff, as claimed earlier. This concludes the proof. □

Proof of Proposition 4.

Suppose $0<N_s<N$ and $\alpha\geq\frac{(N-N_s)^2}{N\,N_s}$.

The result stated in part (a) essentially follows the proof of Proposition 3. In particular, this proof establishes that

$$\mathrm{E}\Big[\,\Big(\tilde v - P^R\left(\hat x^R(\tilde v,\tilde r) + N_s \hat s^R(\tilde r) + (N-N_s)\hat g^R(\tilde r) + \tilde L\right)\Big)\,\hat x^R(\tilde v,\tilde r) - \alpha(\tilde r - \tilde v)^2\,\Big|\,v,\,r\Big]$$
$$>\ \mathrm{E}\Big[\,\Big(\tilde v - P^{NR}\left(\hat x^{NR}(\tilde v) + N_s \hat s^{NR} + (N-N_s)\hat g^{NR} + \tilde L\right)\Big)\,\hat x^{NR}(\tilde v)\,\Big|\,v\Big] = \frac{\sigma_L}{2\,\sigma_v}\,v^2,$$

where $(\hat\rho, \hat x^R(\cdot), \hat s^R, \hat g^R, \hat P^R)$ is the hype and dump equilibrium and $(\hat x^{NR}(\cdot), \hat s^{NR}, \hat g^{NR}, \hat P^{NR})$ is the symmetric linear NR-equilibrium given in Proposition 1. The inequality above naturally implies that

$$\mathrm{E}\Big[\,\Big(\tilde v - P^R\left(\hat x^R(\tilde v,\tilde r) + N_s \hat s^R(\tilde r) + (N-N_s)\hat g^R(\tilde r) + \tilde L\right)\Big)\,\hat x^R(\tilde v,\tilde r)\,\Big|\,v,\,r\Big],$$
$$>\ \mathrm{E}\Big[\,\Big(\tilde v - P^{NR}\left(\hat x^{NR}(\tilde v) + N_s \hat s^{NR} + (N-N_s)\hat g^{NR} + \tilde L\right)\Big)\,\hat x^{NR}(\tilde v)\,\Big|\,v\Big] = \frac{\sigma_L}{2\,\sigma_v}\,v^2. \quad (37)$$

Hence, given v and r, the expected trading profit of the informed trader in a hype and dump equilibrium is higher than what she would attain if she did not send any rumor.

If there is no rumor, following Proposition 1, a sophisticated trader does not trade and thus his trading profit is zero. However, in a hype and dump equilibrium, a sophisticated trader's demand is equal to the demand of the informed trader (this can be observed from Proposition 2), which implies that his trading profit is also equal to that of the informed trader. We know from (37) that informed trader's expected trading profit is strictly positive. Therefore, given v and r, the expected trading profit of the sophisticated trader in a hype and dump equilibrium is higher than what he would attain if the informed trader did not send any rumor. This proves part (b).

If there is no rumor, following Proposition 1, a gullible trader does not trade and thus his trading profit is zero. However, in a hype and dump equilibrium, the sign of a gullible trader's demand is the opposite of the sign of the demand of the sophisticated trader (this can be observed from Proposition 2). Since a sophisticated trader's expected profit is strictly positive in the hype and dump equilibrium, the gullible trader's expected profit in the hype and dump equilibrium is bound to be strictly negative. Therefore, given v and r, the expected trading profit of the gullible trader in a hype and dump equilibrium is lower than what he would attain if the informed trader did not send any rumor. This proves part (c). $\square$

Proof of Proposition 5.

In order to prove this proposition, it suffices to show that $\lambda^R < \lambda^{NR}$, where λ^R and λ^{NR} are as given in Proposition 2 and Proposition 1, respectively. Following Propositions 1 and 2,

$$\begin{aligned}
\lambda^R &= \frac{\sqrt{(2+(1-\hat{\rho})(N-N_s))\ (2+(1+\hat{\rho})N+(1-\hat{\rho})N_s)}\,\sigma_v}{(2+N)\ (2+N_s)\ \sigma_L} \\
&\le \max_{\rho\in\mathbb{R}} \frac{\sqrt{(2+(1-\rho)(N-N_s))\ (2+(1+\rho)N+(1-\rho)N_s)}\,\sigma_v}{(2+N)\ (2+N_s)\ \sigma_L} \\
&= \frac{\sqrt{(2+(1-\rho)(N-N_s))\ (2+(1+\rho)N+(1-\rho)N_s)}}{(2+N)\ (2+N_s)\ \sigma_L}\Bigg|_{\rho=-\frac{N_s}{N-N_s}} \\
&= \frac{\sigma_v}{(2+N_s)\sigma_L} \\
&\le \frac{\sigma_v}{2\sigma_L} \ = \ \lambda^{NR}.
\end{aligned} \tag{38}$$

This proves the proposition. $\square$

Proof of Proposition 6.

In order to prove this proposition, it suffices to show that

$$\left|\hat{x}^R(v,r)\right| + N_s\left|\hat{s}^R(r)\right| + (N-N_s)\left|\hat{g}^R(r)\right| + |L| \ > \ \left|\hat{x}^{NR}(v)\right| + N_s\left|\hat{s}^{NR}\right| + (N-N_s)\left|\hat{g}^{NR}\right| + |L|\,,$$

where $(\hat{\rho}, \hat{x}^R(\cdot,\cdot), \hat{s}^R(\cdot), \hat{g}^R(\cdot), \hat{P}^R(\cdot))$ is the hype and dump equilibrium and

$$(\hat{x}^{NR}(\cdot), \hat{s}^{NR}, \hat{g}^{NR}, \hat{P}^{NR}(\cdot))$$

is the symmetric linear NR-equilibrium given in Proposition 1. Following Proposition 2,

$$
\begin{aligned}
& \left|\hat{x}^R(v,r)\right| + N_s\left|\hat{s}^R(r)\right| + (N-N_s)\left|\hat{g}^R(r)\right| + |L| \\
= \quad & (N_s+1)\left|\frac{2+(1-\hat{\rho})(N-N_s)}{2(2+N)\,\hat{\lambda}^R}\,v\right| + (N-N_s)\left|\frac{2\hat{\rho}-(1-\hat{\rho})N_s}{2(2+N)\,\hat{\lambda}^R}\,v\right| + |L| \\
\underbrace{\geq}_{\text{from (38)}} \quad & (N_s+1)\left|\frac{2+(1-\hat{\rho})(N-N_s)}{2+N}\,v\right|\,\frac{\sigma_L}{\sigma_v} + (N-N_s)\left|\frac{2\hat{\rho}-(1-\hat{\rho})N_s}{2+N}\,v\right|\,\frac{\sigma_L}{\sigma_v} + |L| \\
\underbrace{=}_{\text{since } \hat{\rho}<0} \quad & (N_s+1)\,\frac{2+(1-\hat{\rho})(N-N_s)}{2+N}\,\frac{\sigma_L}{\sigma_v}\,|v| + (N-N_s)\,\frac{-2\hat{\rho}+(1-\hat{\rho})N_s}{2+N}\,\frac{\sigma_L}{\sigma_v}\,|v| + |L| \\
\underbrace{>}_{-\frac{2+N+N_s}{N-N_s}<\hat{\rho}<-\frac{N_s}{N-N_s}} \quad & (N_s+1)\,\frac{\sigma_L}{\sigma_v}\,|v| + N_s\,\frac{\sigma_L}{\sigma_v}\,|v| + |L| \\
= \quad & (2\,N_s+1)\,\frac{\sigma_L}{\sigma_v}\,|v| + |L|. \qquad (39)
\end{aligned}
$$

On the other hand, following Proposition 1,

$$
\begin{aligned}
& \left|\hat{x}^{NR}(v)\right| + N_s\left|\hat{s}^{NR}\right| + (N-N_s)\left|\hat{g}^{NR}\right| + |L| \\
= \; & \frac{\sigma_L}{\sigma_v}\,|v| + |L|. \qquad (40)
\end{aligned}
$$

(39) and (40) together prove the proposition. $\square$

Proof of Proposition 7.

In order to prove this proposition, it suffices to show that

$$
\begin{aligned}
& \operatorname{var}\left(\tilde{v}|P^R\left(\hat{x}^R(v,r) + N_s\hat{s}^R(r) + (N-N_s)\hat{g}^R(r) + L\right)\right) \\
> \; & \operatorname{var}\left(\tilde{v}|P^{NR}\left(\hat{x}^{NR}(v) + N_s\hat{s}^{NR} + (N-N_s)\hat{g}^{NR} + L\right)\right),
\end{aligned}
$$

where $(\hat{\rho}, \hat{x}^R(\cdot,\cdot), \hat{s}^R(\cdot), \hat{g}^R(\cdot), \hat{P}^R(\cdot))$ is the hype and dump equilibrium and

$$
(\hat{x}^{NR}(\cdot), \hat{s}^{NR}, \hat{g}^{NR}, \hat{P}^{NR}(\cdot))
$$

is the symmetric linear NR-equilibrium given in Proposition 1. Following Lemma 1 and Proposition 2,

$$
\begin{aligned}
& \operatorname{var}\left(\tilde{v} | P^R\left(\hat{x}^R(v,r) + N_s\hat{s}^R(r) + (N-N_s)\hat{g}^R(r) + L\right)\right) \\
= \quad & \sigma_v^2 - \frac{\left(\lambda^R \operatorname{cov}\left(\tilde{v}, x^R(\tilde{v},\tilde{r}) + N_s\hat{s}^R(\tilde{r}) + (N-N_s)\hat{g}^R(\tilde{r}) + \tilde{L}\right)\right)^2}{(\lambda^R)^2 \operatorname{var}\left(x^R(\tilde{v},\tilde{r}) + N_s\hat{s}^R(\tilde{r}) + (N-N_s)\hat{g}^R(\tilde{r}) + \tilde{L}\right)} \\
= \quad & \sigma_v^2\left(1 - \frac{2+N+N_s+\hat{\rho}\,(N-N_s)}{2(2+N)}\right) \\
= \quad & \sigma_v^2\,\frac{2+N-N_s-\hat{\rho}\,(N-N_s)}{2(2+N)} \\
\underbrace{>}_{-\frac{2+N+N_s}{N-N_s}<\hat{\rho}<-\frac{N_s}{N-N_s}} \quad & \frac{\sigma_v^2}{2}.
\end{aligned} \tag{41}
$$

On the other hand, following Lemma 1 and Proposition 2,

$$
\operatorname{var}\left(\tilde{v} | P^{NR}\left(\hat{x}^{NR}(v) + N_s\hat{s}^{NR} + (N-N_s)\hat{g}^{NR} + L\right)\right) = \frac{\sigma_v^2}{2}. \tag{42}
$$

(41) and (42) together prove the proposition. □

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