Improving Cycle Time in Sculptured Surface Machining Through Force Modeling
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Abstract
In this paper, an enhanced mathematical model is presented for the prediction of cutting force system in ball end milling of sculptured surfaces. This force model is also used as the basis for off-line feed rate scheduling along the tool path in order to decrease the cycle time in sculptured surface machining. As an alternative for setting a constant feed rate all along the tool path in rough machining of sculptured surfaces, resultant cutting forces are aimed to be kept under a pre-set threshold value along the tool path by off-line scheduled piecewise variable feed rates. In this paper, it is shown that machining time, depending on complexity of sculptured surfaces, can be decreased significantly by scheduling feed rate along the tool path. The model is tested under various cutting conditions and some of the results are also presented and discussed in the paper.

Keywords:
End Milling, Force, Feed rate Scheduling

1 INTRODUCTION
Sculptured surface machining is an important process commonly used in various industries such as automotive, aerospace, and die/mold industries. Due to increasing competitiveness in the market, decreasing production time and cost without sacrificing from part quality are becoming more vital nowadays.

Although recently new scientific studies on milling tools have been carried out and focused on different aspects [1-6], most of the ball-end milling models are applicable to either machining of simple and limited workpiece geometries, such as 2½ axis milling cases, but not relatively complex sculptured surfaces, or can not detect the cutter/workpiece engagement regions automatically and can not simulate multi-pass machining operations.

Unfortunately, there is a lack of scientific tools in the selection of 'appropriate' feed rate values in the machining of sculpture surfaces that depend on a reliable force model. Therefore, conservative constant values of feed rates have been mostly used up to now. Currently and commonly used CAD/CAM programs and NC code generators are based on only the geometric and volumetric analyses, but not on the mechanics of the machining processes, yet. The CAD/CAM programs, in which sculpture surfaces are created, and the NC code generators suggest preset feed rate values for tool-material combinations from their databases for roughing, semi-finishing and finishing. These databases usually contain general and conservative values based on trial-error tests and experiences. Although the information can be valuable, in order to avoid undesirable results such as chipping, excessive tool deflections and tool breakage, they can be quite conservative. As production engineers in most of the cases in industry have no scientific tools based on the mechanics of the sculpture surface machining processes, they can not predict cutting forces, therefore, during the process planning stage, they have no choice but to be conservative in the selection of feed rate.

Additionally, setting a constant feed rate all along the tool path will result in losses in productivity in sculptured surface machining where tool-workpiece engagement changes continuously along the tool path. These changes create potentials for using variable and increased feed rate values in certain sections of tool path. However, varying feed rate should be in such a way that it should not affect the resultant cutting force levels due to the reasons explained above. Therefore, knowledge on the cutting force levels, and having a reliable model of force system are critical for the selection of varying feed rates along the tool path.

Therefore, in this paper, rather than setting the feed rate to a constant value all along the tool path, setting the resulting cutting force values along the tool path to a pre-set value is proposed. This 'appropriate' pre-set value can be selected by considering the desired surface accuracy, tool deflections, chipping, tool breakage, etc. This paper will not discuss the selection process of the 'appropriate' constant force value, since it depends on the workpiece, tool, desired surface quality, etc. However, the paper presents an enhanced model to predict the cutting force values all along the cutter path for a given set of inputs such as cutter geometry, workpiece materials, process parameters as explained in the following sections. Moreover, by using the force model predictions, in this paper it is shown that feed rate values can be changed piecewise along the tool path in sculptured surface machining in order to improve cycle time and productivity.

2 FORCE MODEL IN SCULPTURED SURFACE MACHINING
Cutting force system model consists of various modules such as cutter/workpiece intersection, kinematics/chip load, cutting force modules. The force system model employs a Boolean approach for given cutter, workpiece geometry, process parameters and tool path to determine instantaneous cutting forces.
2.1 Cutter / Workpiece intersection module

In 3D sculpture surface machining, the cutter/workpiece engagement region does vary along the cutter path and in general, unless some specific and very simple workpiece geometry is machined, it is difficult to find an exact analytical representation for the engagement region. The model detects and outputs the engagement regions using its own self-sufficient algorithm for a given cutter path.

Chip load and force calculations are based on the cutter/workpiece engagements; therefore the output of this module is very critical. The cutter and workpiece are meshed into small elements whose projection into the X-Y plane is a square (Figure 1). Due to this meshing, automatic determination of the engagement regions along the tool path is possible. It should be noted that there is a trade-off between computation time and accuracy of the predictions in the selection of grid sizes. Based on the inputs of the model (tool path, cutter and workpiece geometry), the cutter is translated at the nodal points of the mesh elements in the workspace according to the tool and workpiece kinematics. Considering spindle speed and feed rate together with the other inputs, all the locations of discrete points for each region of the ball-end mill during the machining process are calculated in terms of X-Y-Z global coordinates (Figure 1). Thereafter, the instantaneous workpiece heights at each of the X-Y coordinates of the cutting region are also determined, and based on the difference between the cutting edge and workpiece height at these points, the algorithm determines which portion of the cutter is in contact with the workpiece.

This can also be explained as the following: let D and W be defined as the cutter domain and workpiece surface at any instant of machining, respectively, and let \( P(X, Y, Z) \) be a discrete point on the cutting nose, \( Q(X_m, Y_m, Z_m) \) be a nodal point on the workpiece surface. If at any time, the condition,

\[
[ Z_d \leq Z_w | P \in D, Q \in W, X_d = X_w, Y_d = Y_w ]
\]

is satisfied then the point \( P \) is said to be in cutting with point \( Q \) and considered to be in the engagement domain at that instant.

2.2 Kinematics and chip load module

In order to determine the differential cutting forces at any cutter point in the engagement domain, the first step to be taken is to find the infinitesimal chip load for such a discrete element. Thereafter, for the chip load to be evaluated, chip thickness must be known. For a ball-end mill cutter, the instantaneous undeformed chip thickness is found as follows,

\[
(t_c)_{ku} = t_x \cdot \sin(\theta) \cdot \sin(\psi) \cdot \varepsilon_{ku} \tag{2}
\]

where \( t_x \) [mm/tooth] is the feed-per-tooth-per-revolution, and it is determined from feed rate [mm/min] \( f \), \( N_r \) [rpm]. \( N_r \) and \( \omega \) represent the number of flutes and spindle speed, respectively. \( \psi \) is the cutting element rotation angle, and \( \psi \) is the cutting element position angle (Figure 2). \( \varepsilon_{ku} \) takes the value of 1, if the \( k^\text{th} \) discrete point on the \( n^\text{th} \) cutting edge is in engagement with the workpiece. Otherwise, it takes the value of 0. Therefore, instantaneous infinitesimal chip load for each discrete element that is in contact with the workpiece can be written as the following,

\[
dA_k = (t_c)_{ku} \cdot (dz)_n \tag{3}
\]

where \((dz)_n\) represents differential chip height along the longitudinal cutting axis at the \( k^\text{th} \) discrete point on the \( n^\text{th} \) cutting edge.

2.3 Cutting Force Module

For a differential chip load \((dA_k)\) in engagement domain, the differential radial \((dF_r)\), axial \((dF_a)\) and tangential \((dF_t)\) cutting forces can be written as follows,

\[
dF_r = K_{rc} \cdot dA_k + K_{rc} \cdot dz \cdot dF_a = K_{wa} \cdot dA_k + K_{wa} \cdot dz \cdot dF_t \tag{4}
\]

where \( K_{rc}, K_{wa} \) are the radial, axial and tangential (Figure 2) cutting constants, and \( K_{rc}, K_{wa} \) are the related edge coefficients, respectively. These specific cutting energy coefficients vary based on the cutter/workpiece material combination and vary along the cutting edge in ball-end milling. Once \( dF_r, dF_a \) and \( dF_t \) were obtained through use of Equation 4, these cutting force components can be easily transformed into the X-Y-Z global coordinate system as the following.

\[
\begin{bmatrix}
\frac{dF_X}{df_t} \\
\frac{dF_Y}{df_t} \\
\frac{dF_Z}{df_t}
\end{bmatrix}
= A \begin{bmatrix}
\frac{dF_X}{df_t} \\
\frac{dF_Y}{df_t} \\
\frac{dF_Z}{df_t}
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
\sin(\psi) \cdot \sin(\theta) & -\cos(\psi) \cdot \sin(\theta) & -\cos(\theta) \\
\cos(\psi) \cdot \cos(\theta) & \cos(\psi) \cdot \sin(\theta) & -\sin(\theta) \\
\cos(\psi) & -\sin(\psi) & 0
\end{bmatrix}
\]

\[\theta = \Omega \cdot \frac{(n-1) \cdot 2\pi}{N_f} - \beta_k; \ n = 1 \ldots N_f \]

\[
\begin{bmatrix}
F_X \\
F_Y \\
F_Z
\end{bmatrix}
= \sum_{n=1}^{N_f} \sum_{k=1}^{K} \begin{bmatrix}
\frac{dF_X}{df_t} \\
\frac{dF_Y}{df_t} \\
\frac{dF_Z}{df_t + dF_{pres}}
\end{bmatrix}
\]

where \( K \) represents the total number of discrete points on a cutting edge, \( \Omega \) is the cutting edge rotation angle (Figure 2). One important aspect of the model to mention here is the additional \( dF_{pres} \) force that is added to \( dF_z \). This force is assumed to result from a constant pressure value existing over the workpiece as long as the cutter moved down into the workpiece in the Z direction. Its amplitude equals this constant pressure times the area of the cutter/workpiece contact region. From the experiments, it was observed that value of constant pressure is mainly dominated by feed rate. Therefore, in the calibration stage, the pressure...
(in MPa) was determined as a function of feed rate (in mm/min) for Al7039 which was used in the experiments as the following:

\[ P(f) = 0.0004 \cdot f^3 - 0.0409 \cdot f^2 + 1.8767 \cdot f + 4.3248 \]

\[ \beta = z_c \cdot \tan(\gamma_0)/r = z_c \cdot \tan(\gamma_0) \cdot \sqrt{2R_z - z_c^2} \quad (8) \]

where \( z_c \) is the distance from the tip, \( r \) is the radius of a point on the cutting edge on a plane perpendicular to the cutter longitudinal axis and \( \beta \) is the lag angle between the line which connects this point to the tip and the line which is tangent to the cutting edge at the tip (Figure 2). Calibration tests have been performed in advance to obtain the cutting constants and edge coefficients to be used in the mathematical model. Since chip thickness, cutting velocity, and therefore the cutting coefficient values, change along the cutting edge in the ball part, the 6 mm radius cutter was divided into seven intervals along the cutter axis. For each feed rate, seven incremental slot-cutting tests, which correspond to these intervals, have been performed. In order to find the cutting forces of a specific interval, the difference of cutting forces for two subsequent intervals have been found first. The radial, axial and tangential differential forces have been plotted versus the average chip thickness per revolution-per-tooth [7] in order to obtain the cutting force \((K_{tc}, K_{rc}, K_{tc})\) and edge force \((K_{ta}, K_{rc}, K_{tp})\) in (Mpa) for the respective interval (Table 1).

### Table 1: Cutting and edge force coefficients for Al7039.

<table>
<thead>
<tr>
<th>Intervals from tool tip [mm]</th>
<th>0-0.5</th>
<th>0.5-1</th>
<th>1-1.5</th>
<th>1.5-2</th>
<th>2-3</th>
<th>3-4</th>
<th>4-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{tc} )</td>
<td>10478</td>
<td>3327</td>
<td>2376</td>
<td>1805</td>
<td>1404</td>
<td>1364</td>
<td>1032</td>
</tr>
<tr>
<td>( K_{rc} )</td>
<td>6156</td>
<td>2191</td>
<td>741</td>
<td>398</td>
<td>212</td>
<td>173</td>
<td>16</td>
</tr>
<tr>
<td>( K_{ta} )</td>
<td>110</td>
<td>255</td>
<td>433</td>
<td>190</td>
<td>143</td>
<td>123</td>
<td>168</td>
</tr>
<tr>
<td>( K_{te} )</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>7</td>
<td>11</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( K_{re} )</td>
<td>24</td>
<td>0</td>
<td>6</td>
<td>12</td>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>( K_{te} )</td>
<td>14</td>
<td>19</td>
<td>13</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

In order to perform concave and convex machining cases, three sine waves each with the amplitude of 3 mm, the wavelength of 50 mm and the pick feed of 3 mm/track were aimed to be created on 150mmx50mm blank workpiece surface (Figure 3). In these cutting tests, spindle speed and feed rate were 1000 rpm and 150 mm/min, respectively. In the simulation, ball-part of the cutter was discretized into disks of 0.1 mm height, and the force calculations were performed every 3.6 degrees of cutter rotation. Amplitude and waveforms of simulated force components agreed very well with the experimental data collected at 2000 Hz (Figure 4). A zoomed view of resultant cutting force between two maximum points of the sine waves on the workpiece was shown in Figure 5c for constant feed rate case. Maximum resultant force amplitude was 550 N along the 50 mm long tool path.

In order to decrease the cycle time, feed rate scheduling is implemented in the machining of the sine waves. While scheduling the feed rate along the tool path according to

![Figure 2: Illustration of the cutting force components and angular relationships.](Image 72x441 to 291x718)
Equation 7, it was desired that the resultant cutting force should not exceed 600 N. NC code was modified to accommodate the piecewise varying feed rate values as shown in Figure 5b.

Figure 3: Illustration of machined sculptured surface.

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Figure 4: Simulated and measured force components for the constant feed rate case.

Figure 4: Simulated and measured force components for the constant feed rate case.

During the implementation of machining with the scheduled feed rate values, all the other cutting conditions were kept constant, and cutting forces were sampled at 2000 Hz. The zoomed view of resultant force for one period of sine wave is shown in Figure 5d. The resultant force amplitude was kept under the threshold limit of 600 N along the tool path. Moreover, it was also observed from the collected data that in this typical example, with the scheduled feed rate, machining time is decreased 41% compared to the constant feed rate case.

The only observable difference between the constant and scheduled feed rate cases was the tooth marks on the machined workpiece surface, due to the high feed values, near the maximum points of the sine waves. This may create problems in finishing operations. However, in the rough milling case, this can be tolerated.

5 CONCLUSIONS

A force model was presented for sculptured surface machining with ball-end mill. Based on this force model, while keeping resultant cutting force under a pre-set threshold value, an off-line feed rate scheduling method was introduced in order to decrease the cycle time in rough sculptured surface machining processes.

Figure 5: Zoomed view of experimental resultant forces for constant and scheduled feed rate cases.

It was shown that off-line scheduling feed rate along the tool path based on a reliable force model has a potential to substantially decrease cycle time in sculptured surface machining.

6 REFERENCES


