Information Conveyance and the Make-or-Buy Decision

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Abstract

At its core, cost accounting aims to develop estimates of resources utilized by firms in making inputs and then converting these inputs into final products. Not surprisingly, the cost estimate for making inputs is useful data in evaluating whether or not a firm should outsource input production. In this paper, we demonstrate that a firm's ability to develop estimates of conversion costs also plays a critical role in its sourcing decision even when such costs are invariant to the sourcing choice itself. In particular, we show that when a firm gathers pertinent information about its conversion costs, any input procurement order it places with an outside party conveys information that is both stochastic and strategic in nature. *Stochastic information conveyance* refers to the fact that the order informs the input seller of the firm's conversion costs and, thus, its relative ability to compete in the marketplace. *Strategic information conveyance* refers to the fact that the order also informs the input seller of the firm's chosen strategic posturing in the marketplace. We demonstrate that both sources of information conveyance can point to a firm (i) preferring to buy inputs externally even when it can make them internally at a lower cost; and (ii) preferring to outsource input production to a supplier that also competes with it in the output market.
1. Introduction

The development of cost estimates to aid firm decisions is an oft discussed tenet of managerial accounting. A notable case in point is the decision of whether to outsource input production or establish internal capacity. This make-or-buy choice is typically viewed as one that amounts to contrasting the external market price for an input with a firm's estimate of the cost of producing that input. Accounting obviously seeps in here – a precise estimate of the relevant costs of input production can sharpen a firm's decision making, particularly when such estimates are adequately adjusted to reflect opportunity costs (Balakrishnan et al. 2009; Horngren et al. 2009). In this paper, we highlight a more nuanced role for accounting information in the make-or-buy decision by considering input conversion costs, even when such costs are invariant to the chosen procurement method. In particular, the nature of a firm's downstream costs and its ability to estimate them proves critical in the firm's upstream operations because of differential information conveyed by make and buy decisions.

To elaborate, the paper demonstrates that when a firm is successful in gathering conversion cost estimates (be it production costs, sales and administrative costs, transportation costs, etc.) any order it places with an external supplier serves to communicate information upstream. Such information transmission has two components, stochastic and strategic, each of which plays a crucial role in the firm's procurement choice in the first place. Stochastic information conveyance refers to the fact that the size of the firm's order with its supplier depends on its estimates of profitability of the products it will create with the input; as such, the supplier learns about the firm's conversion costs from the order it receives. Strategic information conveyance refers to the fact that the firm's order with its supplier also tips its hand about its strategic intentions in the output market.

Information conveyance of the firm's conversion costs and/or strategic posture are irrelevant if the supplier is an uninterested observer of output market proceedings – as a
result, the input price charged by such a supplier needs to be below the firm's cost of making the input to induce buying, much as traditional analysis would dictate. However, we demonstrate that precisely because of information conveyance, when the firm opts to outsource input production it will do so with a supplier who also has a stake in the output market. And, the information gains that come from such outsourcing translate into the firm being willing to buy even when the supplier's price exceeds the firm's cost of making the input internally.

The reasoning behind the result that information conveyance points to more outsourcing, specifically outsourcing to a rival, is roughly as follows. Take first stochastic information conveyance. When rivals in the output market are unaware of a firm's conversion costs, they must rely on expectations of a firm's efficiency when choosing their own quantities. When they learn of the firm's conversion costs, they can condition competitive response on the cost – when the firm is more efficient, they back away in competition and when the firm is less efficient they become more aggressive. This translates into rivals ceding power some of the time (when the firm is most efficient) and the firm ceding power other times (when the firm is less efficient); the end result is that competition in the output market is lowered. When the firm buys inputs from a rival, that rival becomes privy to some of the firm's conversion costs knowledge via information naturally embedded in the order size. As a result, outsourcing to a rival becomes an indirect means of conveying information and, thus, coordinating competitive behavior.

Second, consider the effect of strategic information conveyance. When a firm opts to outsource and places an input quantity order with a rival, it also conveys its output quantity as well. As such, the input supplier unwittingly becomes a Stackelberg follower in subsequent competition. Of course, the firm relishes claiming the role of Stackelberg leader that accompanies outsourcing. As it turns out, the supplier too can benefit from the sequencing because of its presence in both the input and output arenas. In particular, while the supplier suffers in the output realm by being a follower, its buyer's newfound
competitive strength translates into a greater willingness to pay for inputs and, thus, greater input market profits for the supplier. Further, when competition is characterized by multiple rivals, the output market downside of being a Stackelberg follower is spread among all rivals, despite the fact that only the supplier is privy to input the order. In contrast, the input market benefit from securing higher input prices is reaped by the supplier alone.

Given these forces, the question is when the information conveyance role of purchases will lead a firm to outsource input production to a rival. As discussed above, stochastic information conveyance is particularly useful when a firm's purchases communicate substantial information about its conversion costs. Consistent with this, we demonstrate that the firm opts to outsource if and only if its information advantage (i.e., the degree of rival uncertainty) is sufficiently large. Further, as also discussed above, strategic information conveyance is particularly appealing when a firm encounters several rivals. Consistent with this, we demonstrate that the firm opts to outsource even in the absence of information advantage when the output market is sufficiently competitive.

While perhaps surprising at first blush, the result herein that a firm may opt to outsource to its own competitor for strategic reasons is more than just a modeling novelty. In fact, the practice of relying on competitors for inputs is quite common, albeit not fully understood. Outsourcing to competitors has been documented in many arenas, including the aircraft, automobile, computer, glass, household appliances, telecommunications, and trucking industries (e.g., Arrunada and Vazquez 2006; Baake et al. 1999; Chen et al. 2011; Spiegel 1993). While some have viewed the practice of outsourcing to competitors as an option of last resort, its prevalence (and success) suggests there is more to the story. This paper posits that a firm's accounting system and information more broadly play a role in explaining the practice.

The existing literature in accounting, economics, and operations also provides other factors that work both for and against outsourcing. Long-term dynamics of supplier-buyer
interactions (Anderson et al. 2000; Demski 1997), institutional pressures to keep particular inputs in-house (Balakrishnan et al. 2010), and the importance of learning-by-doing (Anderson and Parker 2002; Chen 2005) are key considerations. In terms of strategic effects in outsourcing, the noted downsides include concerns of misappropriation of innovation by suppliers (Baiman and Rajan 2002) and technology spillovers that benefit rivals (Van Long 2005), while the benefits include exploiting differential cost structures, avoiding redundant fixed costs, influencing rivals' wholesale prices when reliant on a common supplier, and fostering retail price collusion under decreasing returns to scale (Arya et al. 2008; Baake et al. 1999; Buehler and Haucap 2006; Shy and Stenbacka 2003; Spiegel 1993).

In this paper, the extant reasons for outsourcing (as briefly summarized above) are intentionally excluded in order to highlight the novel role played by information. In particular, the desire to convey both stochastic and strategic information to a rival may point to outsourcing despite the fact that the prevailing outsourced price exceeds the cost of making the input internally. The desire to convey stochastic information identified here fits more broadly with the notion that, depending on the type and behavior of the uncertain information, a firm may wish to disclose information to competitors (see, e.g., Darrough 1993; Bagnoli and Watts 2011). Such findings also necessitate discussion of whether the information can be credibly communicated without a costly audit (e.g., Bagnoli and Watts 2010). In this case, information pooling and credible communication are moot since the firm's placement of input order with an external supplier automatically transmits the information.

In terms of the desire to convey strategic information via outsourcing, our result is broadly related to Chen et al. (2011), which notes that quantity pre-orders can promote a first-mover advantage. In that setting, with no uncertainty and a simple duopoly, however, it is concluded that the specter of such strategic effects leads a supplier to withhold inputs from its retail competitor, thereby forcing the firm to buy from another source. In contrast,
we demonstrate that a rival may willingly cede retail leadership by selling inputs to a firm, and a firm may gladly buy from the rival. Such a stark reversal arises due to the presence of uncertainty and/or multiple retail rivals that accentuate the mutual benefits of outsourcing.

The remainder of this paper proceeds as follows. Section 2 presents the basic model. Section 3 presents the results: 3.1 examines the equilibrium under the make option; 3.2 examines the outcome under outsourcing if the supplier does not have a presence in the output market (3.2.1) and if it does (3.2.2); 3.3 presents the main results by deriving the precise conditions under which the firm opts to outsource input production, and the nature of the party from which it procures. Section 4 concludes.

2. Model

A firm, denoted firm 0, is deciding whether to make or buy a critical input. To encompass the range of outsourcing options, we presume that if the firm opts to buy its inputs, it can either rely on a supplier that only operates in the input market or it can rely on a supplier that also has a presence in the output market. In particular, denote the representative supplier that operates only in the input market by \( I \), and denote the supplier that also is a rival in the output market by \( R \). To eliminate standard reasons to make vs. buy inputs, we assume firms 0, \( I \) and \( R \) can each make the input at the same unit cost, normalized to zero. Denoting the per-unit input price set by firm \( j, j = I, R \), as \( w_j \), firm 0's choice is thus to make at cost zero or procure at \( w_j \). Of course, in setting their prices, firms \( I \) and \( R \) are well aware of firm 0's alternate options, including making the input.

Subsequent to its procurement choice, firm 0 faces (Cournot) competition in the output (retail) market. As noted, firm \( R \) represents one source of such competition; that said, we allow for the possibility that there can be other retail competitors as well (with costs also normalized to zero). Say firm 0 faces \( n \) rivals in total, and denote the set of
rivals by $N$. Each rival incurs a conversion cost of $c$, while firm 0 incurs a conversion cost of $c - \delta$, where $\delta \in [\underline{\delta}, \bar{\delta}]$ is a mean zero noise term with variance $\sigma^2$.

The retail demand for firm 0 is given by the standard linear (inverse) demand function $p_0 = a - q_0 - k \sum_{i \in N} q_i$, and retail demand for rival $i$, $i \in N$, is $p_i = a - q_i - k \left( \sum_{j \in N \setminus i} q_j + q_0 \right)$. In the demand functions, $p_i$ and $q_i$ reflect the retail price and quantity for firm $i$, $k$, $0 < k \leq 1$, reflects the degree of product differentiation, and $N \setminus i$ denotes the set $N$ less element $i$. As is standard, throughout the analysis we assume $a$ is sufficiently large to ensure nonnegative quantities and prices.

The focus of this paper is on how firm 0's chosen procurement source can have informational reverberations in subsequent competitive interactions. To capture this consideration, say firm 0 privately observes $\delta$ prior to retail competition. In the analysis that follows, we examine subgame perfect equilibria by working backwards in the game to determine outcomes. The timeline of events for the setting is summarized in Figure 1.

![Figure 1: Timeline](image)

3. Results

To determine the firm's sourcing choice, we will first consider the subgame equilibrium in each case. The firm's equilibrium sourcing decision will then follow from a comparison of expected profits. We begin with the outcome when the firm opts to install capacity to produce inputs internally.
3.1. EQUILIBRIUM IN THE MAKE REGIME

A firm that makes its own inputs at the same unit cost as its competitors places itself on level competitive footing when it comes to input production. With conversion costs considered, the firm retains private information about its own competitive capabilities. In particular, firm 0 can condition its production choices on its advance read of its conversion efficiency (i.e., $\delta$), whereas its competitors are left with less informed estimates of the firm's efficiency. In particular, denoting firm 0's conjecture of firm $i$'s equilibrium quantity by $\tilde{q}_i$, $i \in N$, firm 0 chooses $q_0$ to solve (1):

$\max_{q_0} \left[ a - c + \delta - q_0 - k \sum_{i \in N} \tilde{q}_i \right] q_0$.  \hspace{1cm} (1)

Uninformed of $\delta$, firm $i$ chooses $q_i$ to maximize its expected profit, as in (2). In (2), $\tilde{q}_0(\delta)$ denotes firm $i$'s conjecture of firm 0's equilibrium quantity as a function of $\delta$, and $\tilde{q}_j$, $j \in N_{-i}$, denotes firm $i$'s conjecture of firm $j$'s equilibrium quantity.

$\max_{q_i} E_{\delta} \left\{ a - c - q_i - k\tilde{q}_0(\delta) - k \sum_{j \in N_{-i}} \tilde{q}_j \right\} q_i$, $i \in N$. \hspace{1cm} (2)

Jointly solving the first-order conditions in (1) and (2) and the condition that conjectures are correct in equilibrium reveals the following proposition in which the superscript "$M$" denotes the make regime (complete proofs of all propositions are provided in the appendix).

PROPOSITION 1. When firm 0 opts to make, the equilibrium entails

(i) $q_0^M(\delta) = \frac{a - c}{2 + kn} + \frac{\delta}{2}$; and

(ii) $q_i^M = \frac{a - c}{2 + kn}$, $i \in N$.

The proposition reflects the standard Cournot quantities adjusted for firm 0's private information. In particular, each firm chooses a baseline quantity of $\frac{a - c}{2 + kn}$,
reflecting that greater demand \((a)\), lower conversion costs \((c)\), and/or lower competitive intensity \((k \text{ or } n)\) each lead a firm to produce more. Having private information about its conversion costs, firm 0 is the only firm that conditions its production on them, as reflected in \(\delta/2\); the others rely only on their conjecture of firm 0's conversion costs in assessing its competitive stance (recall, \(E\{\delta\} = 0\)). The net result is that each firm's expected profits are again the standard Cournot profits, with the exception that firm 0 gains from its ability to condition production on the uncertain conversion costs: the more uncertainty, the more such conditioning is useful. In particular, using quantities in Proposition 1, expected profits in the make regime for firm 0 and firm \(i\), \(i \in N\), respectively, are:

\[
\Pi_0^M = \left[ \frac{a - c}{2 + kn} \right]^2 + \frac{\sigma^2}{4} \quad \text{and} \quad \Pi_i^M = \left[ \frac{a - c}{2 + kn} \right]^2, \quad i \in N. \tag{3}
\]

We next consider the outcome under the presumption that firm 0 opts to buy the input from an external party.

3.2. EQUILIBRIUM IN THE BUY REGIME

As alluded to at the start, the case of external input procurement proceeds differently if the firm opts to rely on an independent provider \((I)\) or a firm that also is a retail rival \((R)\). We consider each case in turn.

3.2.1 EQUILIBRIUM WHEN BUYING FROM FIRM I

When a firm buys inputs from an outside party unaffiliated with downstream competition \((I)\), the firm's procurement of inputs has no effect on either the information environment downstream or firms' competitive posturing (after all, any purchases the firm makes are known only to its supplier who has no "skin in the game" afterwards). As a result, the competitive interactions proceed as before, except that firm 0 pays \(w_I\) for each unit of input. In particular, when buying from \(I\), firm 0's quantities solve (4).
Of course, each competitor's choice is precisely the same as in the make case, as reflected in (2). Jointly solving the first-order conditions of (4) and (2) and the condition that conjectures are correct in equilibrium reveals the following proposition (in the proposition the superscript "I" denotes the firm buys from an independent supplier).

**PROPOSITION 2.** When firm 0 opts to buy from firm I, the equilibrium entails

(i) \[ q_0^I(w_I; \delta) = \frac{a - c}{2 + kn} + \frac{\delta}{2} - \left(\frac{2 + k[n - 1]}{[2 - k][2 + kn]}\right)w_I; \]

and

(ii) \[ q_i^I(w_I) = \frac{a - c}{2 + kn} + \frac{kw_I}{[2 - k][2 + kn]}, \quad i \in N. \]

Comparing Propositions 1 and 2, note that quantities are equivalent when \[ w_I = 0, \]

i.e., firm 0 is indifferent between making or buying from I at cost. If, however, \[ w_I > 0, \]

the equilibrium is changed. Greater \[ w_I \] means firm 0's costs are higher and, as such, its quantities are lower: \[ \frac{\partial q_0^I(w_I; \delta)}{\partial w_I} = -(2 + k[n - 1])/([2 - k][2 + kn]) < 0. \] Not having to incur the added input costs, the remaining competitors then swoop in to pick up the slack:

\[ \frac{\partial q_i^I(w_I)}{\partial w_I} = kw_I/([2 - k][2 + kn]) > 0. \]

The net result is that buying inputs from an external party translates into the usual effects on profits. Profit for firm 0 is the same as under making the input with an adjustment to reflect the difference between cost of making (here, zero) and the cost of buying (\[ w_I \]). In other words, if making and buying from an independent supplier are the only options, firm 0's choice amounts to the simple textbook explanation of comparing the quoted outsourcing price with the insourcing cost. Specifically, using Proposition 2 values in (2) and (4) and taking expectations, expected profits for each firm when firm 0 buys inputs from an independent supplier are as in (5).
What is notably absent from traditional textbook discussions is that an outside supplier may also have a competitive presence in the output market. This possibility, and the informational reverberations, are considered next.

3.2.2 EQUILIBRIUM WHEN BUYING FROM FIRM R

When buying from a rival in the output market, firm 0's problem is similar to before except that it realizes its procurement order will tip its hand to the rival in question. The rival (R), in turn, can condition its own production choice on its observation of firm 0's input order size. In particular, given its chosen wholesale price, \( w_R \), and firm 0's input order, \( q_0 \), and its conjectures of the quantities of the other firms, \( \tilde{q}_j, \ j \in N^- \), \( R \) chooses \( q_R \) to solve:

\[
\max_{q_R} \left[ a - c - q_R - kq_0 - k \sum_{j \in N^-} \tilde{q}_j \right] + w_Rq_0.
\]

Taking the first-order condition of (6) reveals \( R \)'s reaction function to firm 0's input order:

\[
q_R(q_0, \tilde{q}_j, j \in N^-) = \frac{1}{2} \left[ a - c - kq_0 - k \sum_{j \in N^-} \tilde{q}_j \right].
\]

As can be expected, in (7) a greater order from firm 0 translates into a softened stance by \( R \), i.e., \( \frac{\partial q_R(q_0, \tilde{q}_j, j \in N^-)}{\partial q_0} = -k / 2 < 0 \). This feature reflects the consequence of strategic information conveyed by firm 0's purchase: a higher quantity purchased by firm 0 reduces the marginal revenues of \( R \) and, thus, reduces its propensity
to produce its own outputs. Given this response, and its conjectures of the quantities of the other firms, \( \tilde{q}_j, j \in N_{-R} \), firm 0 chooses \( q_0 \) to solve:

\[
\text{Max}_{q_0} \left[ a - c + \delta - q_0 - kq_R(q_0, \tilde{q}_j, j \in N_{-R}) - k \sum_{j \in N_{-R}} \tilde{q}_j \right] q_0 - w_R q_0. \quad (8)
\]

The problem in (8) is precisely as in the case of buying from I in (4), except that the strategic information conveyance effect is in place, as \( q_R \) reflects not a conjecture but the strategic response function. In effect, by placing its input order upfront gives firm 0 the Stackelberg leader position and, thus, in choosing its quantity it also accounts for the fact that the choice will change \( R \)'s response, now the de facto late mover.

As for the remaining competitors, even though they remain in the dark about firm 0's purchases, they are well-aware that \( R \) as a follower will respond to them. That is, they form conjectures about firm 0's purchases and, given these conjectures, recognize how \( R \) would respond to those purchases, i.e., using (7) with conjecture \( \tilde{q}_0(\delta) \). Continuing with the same notation for said conjectures, firm \( i, i \in N_{-R} \) chooses its quantity to solve:

\[
\text{Max}_{q_i} E_{\delta} \left\{ \left[ a - c - q_i - k\tilde{q}_0(\delta) - kq_R(\tilde{q}_0(\delta), \tilde{q}_j, j \in N_{-R}) - k \sum_{j \in N_{-R}, j \neq i} \tilde{q}_j \right] q_i \right\}. \quad (9)
\]

In (9), the second informational consequence of purchasing from \( R \) is apparent. Unlike its competitors, \( R \) becomes aware of \( q_0 \) and, as a result, indirectly conditions its quantities on \( \delta \). Thus, while the other firms (\( i \in N_{-R} \)) choose quantities in expectation of \( \delta \), \( R \)'s quantities reflect \( \delta \), as in \( q_R(\tilde{q}_0(\delta), \tilde{q}_j, j \in N_{-R}) \). Jointly solving the first-order conditions of (8) and (9), and the condition that all conjectures are correct in equilibrium yields the equilibrium in when firm 0 procures its inputs from \( R \), as summarized in the following proposition.
PROPOSITION 3. When firm 0 opts to buy from firm R, the equilibrium entails

\[
\begin{align*}
(i) \quad q_0^R (w_R; \delta) &= \frac{2[(a-c-w_R)(2-k)-knw_R]}{8+k[4-k^2][n-1]-2k^2[n+1]} + \frac{\delta}{2-k^2}; \\
(ii) \quad q_R^R (w_R; \delta) &= \frac{[a-c][4-2k-k^2] + 2kw_R}{8+k[4-k^2][n-1]-2k^2[n+1]} - \frac{\delta k}{2(2-k^2)}; \text{ and} \\
(iii) \quad q_i^R (w_R) &= \frac{[a-c][4-2k-k^2] + 2kw_R}{8+k[4-k^2][n-1]-2k^2[n+1]}, \quad i \in N_R.
\end{align*}
\]

From the proposition, three key features emerge. To see them most succinctly, say \(w_R = 0\) (procurement from \(R\) is at cost), \(n = 1\) (\(R\) is the only rival), and \(k = 1\) (competition is intense). In this case, relative to the case of making the input, firm 0's expected quantity is greater when it buys \(([a-c]/2\) vs. \([a-c]/3\)) due to its de facto Stackelberg leader advantage. Similarly, \(R\)'s expected quantity is lower as the de facto Stackelberg follower \(([a-c]/4\) vs. \([a-c]/3\)). This reflects the first feature: strategic information conveyance.

The second critical feature, stochastic information sharing, is reflected in the fact that \(R\)'s quantity is now a function of \(\delta\): for this case, \(q_R^R (0; \delta) = [a-c]/4 - \delta / 2\), reflecting that when firm 0 is more (less) efficient, \(R\) backs away (becomes more aggressive) in competition. Of course, though the information is stochastic in nature, it too has strategic repercussions. Since firm 0 can convince \(R\) to back away when \(\delta\) is higher, it will take advantage by increasing quantities even more. In this case, \(q_0^R (0; \delta) = [a-c]/2 + \delta\), whereas \(q_0^M (\delta) = [a-c]/3 + \delta / 2\). Thus, the second key feature too has a notable strategic consequence.

A final crucial feature of the equilibrium is how it affects the rivals who do not provide firm 0 inputs. That is, when \(n > 1\), not only are firms 0 and \(R\) affected by the procurement choice but so too are the "innocent" bystanders. Not being privy to firm 0's purchases, these firms garner no ability to condition quantities on \(\delta\). One would think they are also unaffected by strategic information sharing in that firm 0's Stackelberg advantage extends only to \(R\). However, the remaining rivals are at least aware of the fact that firm 0 holds a Stackelberg advantage over \(R\) and, as a result, firm 0 will be more
aggressive in its quantity choices. Being aware of this extra aggressiveness means that the remaining firms unwittingly are Stackelberg followers of sorts as well. In fact, the Stackelberg follower disadvantage is equally shared among firm 0’s rivals. The only difference between firm $R$ and the other rivals is that $q^R_R(w_R; \delta)$ is contingent on $\delta$, whereas $q^i_i(w_R)$ is not, i.e., $E[q^R_R(w_R; \delta)] = q^i_i(w_R)$, $i \in N_R$.

As we will shortly see, the above three key features are the crux to determining the equilibrium procurement option. To demonstrate this formally, using the outcomes in Proposition 3 in the profit expressions of firms 0 and $R$ and taking expectations, the expected profits of each are presented in (10).

$$
\Pi^R_0(w_R) = 2(2-k^2) \left[ \frac{[a-c-w_R(2-k) - knw_R]}{8 + k[4-k^2][n-1] - 2k^2 [n+1]} \right]^2 + \frac{\sigma^2}{2[2-k^2]}; \text{ and}
$$

$$
\Pi^R_R(w_R) = \left[ \frac{[a-c][4-2k-k^2] + 2kw_R}{8 + k[4-k^2][n-1] - 2k^2 [n+1]} \right]^2 + \frac{2kw_R[(a-c-w_R)(2-k) - knw_R]}{8 + k[4-k^2][n-1] - 2k^2 [n+1]} + \frac{\sigma^2 k^2}{4[2-k^2]^2}. \quad (10)
$$

Using expected profit expressions in (3), (5), and (10), we next derive firm 0’s equilibrium procurement policy.

### 3.3. Make vs. Buy

Given the firm’s three choices, it is perhaps easiest to dispense with the one that is least attractive in that it is sure to be dominated. In particular, if the firm opts to buy the input from $I$, it gains no competitive advantage but only suffers from nontrivial input costs. That is, the setting is one where the usual comparison of a firm’s internal production ability to that of outsiders is moot. Formally, from (3) and (5), $\Pi^M_0 > \Pi^I_0(w_I)$ for any $w_I > 0$. Of course, $I$ will sell the input only if it can do so above its own cost, so in equilibrium, the make option will surely be preferred to buying from $I$. As a result, if firm 0 opts to
outsource input production, it will only do so to a rival. The question remains if and when such rival procurement occurs.

Recall from the previous discussion that buying from a rival has both strategic and stochastic information effects. As far as the strategic information effect, the Stackelberg advantage it provides firm 0 has clear benefits for the firm. As far as the stochastic effect, this too works in favor of outsourcing. To elaborate, with stochastic information conveyance, when the firm's conversion costs are low (its profitability is high), it is able to convince its rival to reduce its own quantities and can thus dominate the market. In contrast, when the firm's conversion costs are high (its profitability is low), the rival realizes that it has an opportunity to dominate the market. The net effect is that competition is lower, and firm 0 reaps the benefits of lower competition precisely when it is most profitable. Both information effects together translate into firm 0's willingness to pay for inputs from its rival being above its own cost. In particular, comparing $\Pi_0^M$ and $\Pi_0^R(w_R)$, firm 0 is willing to pay up to $\bar{w} > 0$ in order to buy, where

$$\bar{w} = \frac{[a - c][2 - k]}{2 + k[n - 1]} - \frac{[2 - k^2(2 + k) + kn(4 - k(2 + k))]}{2\sqrt{2}[2 - k^2][2 + k(n - 1)][2 + kn]} \sqrt{\frac{4[a - c]^2(2 - k^2) - k^2(2 + kn)^2 \sigma^2}{2\sqrt{2}[2 - k^2][2 + k(n - 1)][2 + kn]}}. \quad (11)$$

Of course, since firm 0 buying from $R$ puts the seller at a strategic disadvantage as a de facto Stackelberg follower, it is conceivable that $R$ does not want firm 0's input purchasing and will price it out of the market. Before addressing this specifically, consider the broader question of what $R$ would like to charge firm 0 for inputs if it were guaranteed to have firm 0 as a customer. That is, what is the value of $w_R$ that maximizes $\Pi_R^R(w_R)$? When it comes to competitive positioning, higher $w_R$ is better. However, $R$ also benefits from firm 0 being a nontrivial participant in the output market, since it gleans wholesale (input market) profit from firm 0. If $w_R$ is too high, $R$ risks winning the battle for retail supremacy but losing the war by forgoing substantial wholesale profit. Due to these
effects, $R$'s preferred input price is interior in nature. In particular, setting $\partial \Pi^R_R(w_R)/\partial w_R = 0$ reveals $R$'s preferred price is $\tilde{w}$, where

$$\tilde{w} = \frac{(a-c)[16 - 2k^2(4n-k+2) + k(8 + k^3)(n-1)]}{2[16 + 16k(n-1) - k^4(n-1)^2 - 2k^2(1 + 6n - 2n^2) - 2k^3(n^2 + n - 2)]}. \quad (12)$$

Taken together, (11) and (12) determine the equilibrium input price in the event firm 0 is induced to buy. That is, firm 0 is willing to pay up to $\bar{w}$ to buy from $R$. If $R$ wants to sell to firm 0, it must charge no more than this. It can, however, charge less should it wish to. So, if $\tilde{w} < \bar{w}$, $R$ would charge $\tilde{w}$. This result on the equilibrium input price in the event of buying is summarized in Proposition 4.

**PROPOSITION 4.** If the equilibrium outcome entails firm 0 buying the input, (i) it buys from $R$, and (ii) the wholesale price is $w^* = \text{Min}\{\tilde{w}, \bar{w}\}$.

The question that remains is whether $R$ would, in fact, choose a price so as to entice firm 0 into buying or would it prefer firm 0 makes? Recall, though firm 0 would be happy to buy at zero cost since doing so gives it a Stackelberg advantage, $R$ would not be a willing participant since this would put it at a disadvantage. Of course, firm 0 is willing to pay a premium for this advantage, but will that be enough for $R$ to willingly take a back seat in competition? To get a feel for the answer, take first the limiting case of $\sigma^2 = 0$ and $n = 1$. In this case, stochastic information conveyance is absent as is the strategic information conveyance effect on other rivals, and, thereby we can highlight the direct effect of strategic information conveyance on $R$. In this event, it is also readily confirmed that $w^* = \bar{w}$, i.e., $R$'s preferred price is the maximum firm 0 is willing to pay. To $R$, the benefit of selling at $w^* = \bar{w} > 0$ is that it gains non zero wholesale (input) profit; the downside is the loss of retail (output) profit. Comparing $\Pi^R_R(\bar{w})$ and $\Pi^M_R$ at $\sigma^2 = 0$ and $n = 1$ reveals that the downside is more pronounced. Thus, for $\sigma^2 = 0$ and $n = 1$, the equilibrium entails firm 0 making the inputs. This limiting case is broadly consistent with
the message of Chen et al. (2011), which notes that a rival would be unwilling to sell inputs to a firm since doing so may provide too much strategic advantage to the buyer.

Importantly, the limiting case of $\sigma^2 = 0$ and $n = 1$ excludes two of the key features discussed above, stochastic information conveyance and the effect of strategic information conveyance on other rivals. It turns out that each of these effects is critical in determining the efficacy of outsourcing to a rival. Consider the consequence of $\sigma^2 > 0$. This introduces the effect of stochastic information conveyance. As discussed before, the potential for stochastic information conveyance makes buying from a rival more attractive for firm 0, as manifest in its willingness to pay: $\partial w / \partial \sigma^2 > 0$. This increased willingness to pay bodes well for the willingness of $R$ to sell. Also, recall that firm 0 benefits from stochastic information conveyance since it reduces competition and gives it an edge precisely when it is most profitable. The same too goes for $R$: with information conveyance, $R$ cedes market share precisely when it is (relatively) less efficient and grabs market share when it is more efficient. Thus, not only does stochastic information conveyance increase firm 0's willingness to pay, it also reduces the price $R$ would require in order to sell. The end result is that the more pronounced this effect, i.e., the greater $\sigma^2$, the more attractive is outsourcing. The next proposition states this formally.

**PROPOSITION 5.** There exists $\hat{\sigma}^2(k,n)$ such that the equilibrium outcome entails firm 0 buying the input from $R$ if $\sigma^2 \geq \hat{\sigma}^2(k,n)$; and making the input if $\sigma^2 < \hat{\sigma}^2(k,n)$.

Note from the proposition that the intuition provided above, ostensibly for the case of $n = 1$, applies for all $n$. That said, $n > 1$ introduces another consideration. In particular, a feature discussed previously is that strategic information conveyance under outsourcing has repercussions for other rivals (those not providing inputs to firm 0). Recall, from $R$'s perspective, being a Stackelberg follower is a net disadvantage absent uncertainty: though firm 0 will pay more to be a leader, it is not enough to justify the distinct disadvantage of effectively moving last. This reasoning applies to the case of $n = 1$, but for $n > 1$, there is
also an added subtle effect on other rivals. Though not privy to the strategic information conveyed by firm 0's purchases, they are aware that such purchases are being made and, as such, find themselves as de facto Stackelberg followers too. From $R$'s perspective, this means that the disadvantage of being a follower is shared among the $n$ rivals, whereas the advantage of firm 0's increased willingness to pay is its own to reap. As a result, the more rivals to share the cost of being at a competitive disadvantage, the more attractive is the added wholesale profit. This feature is reflected in $\hat{\sigma}^2(k,n)$ decreasing in $n$.

To summarize the results in Proposition 5, Figure 2 provides a pictorial depiction of the cutoffs: the left panel plots $\hat{\sigma}^2(k,n)$ as a function of $n$ for various $k$-values; the right panel plots $\hat{\sigma}^2(k,n)$ as a function of $k$ for various $n$ values. In each panel, the feature that both greater $\sigma^2$ and greater $n$ point to outsourcing being more attractive is apparent.

Panel A: Preference as $\sigma^2$ and $n$ vary. Panel B: Preference as $\sigma^2$ and $k$ vary.

Figure 2. Make vs. Buy Preference as a function of $\sigma^2$, $k$, and $n$.

One tantalizing feature in the figure is that in both panels, for large enough $k$ and $n$ outsourcing is optimal even absent uncertainty. It turns out that this feature persists more
generally. That is, greater \( k \) reflects greater competitive intensity and, thus, a greater desire for firm 0 to get an edge via outsourcing (i.e., a greater potential wholesale premium for \( R \)), while a greater \( n \) reflects that the competitive cost of such outsourcing is spread among more firms. As a result, with enough competition, outsourcing is preferred even in the absence of any stochastic information conveyance.

PROPOSITION 6. There exists \( \hat{n}(k) \) such that \( \hat{\sigma}^2(k,n) = 0 \) if and only if \( n \geq \hat{n}(k) \). Thus, when firm 0 faces enough competitors, the equilibrium entails firm 0 buying the input even under cost certainty.

While the proposition notes that a sufficiently large \( n \) ensures outsourcing even without stochastic information conveyance, the reasoning provided above also relied on large values of \( k \). As it turns out, \( \hat{n}(k) \) is decreasing in \( k \), consistent with this view. Figure 3 depicts this graphically: the left panel plots profits under making vs. buying as a function of \( n \) for the case of \( k = 1 \), and the right panel then plots \( \hat{n}(k) \) as a function of \( k \).

![Figure 3](image_url)

**Panel A:** Profits in Each Regime as \( n \) varies.  
**Panel B:** \( \hat{n}(k) \) as \( k \) varies.

**Figure 3.** Information-Induced Outsourcing in the Absence of Uncertainty
In short, the results indicate that outsourcing to a rival may be fully rational for both the firm and the rival, solely on informational grounds. Interestingly, the information in question is not directly related to the input production process itself but instead pertains to the costs of conversion. Thus, accounting information and its precision affects not only the question at hand (here, the efficiency of selling outputs) but also other, seemingly unrelated questions (the efficiency of outsourcing inputs).

The informational benefits of outsourcing here rely on the firm's ability to convey both its strategic posture to its rival and indirectly signal its private information in the process all through its quantity procurement process. Though the strategic effect may seem to harm the rival on its face by placing the rival as a de facto Stackelberg follower, the net effect is more subtle since the costs are borne by all rivals (not just the seller), whereas such selling also reaps wholesale profits.

As a final note, the above discussion suggests that the only losers in the firm's decision to outsource to a rival are the remaining rivals who don't reap benefits from selling to firm $0$ but have to realize some of the costs. This suggests the other rivals may too wish to get in the input selling business. While the analysis here considers firm $R$ as the representative rival selling inputs for simplicity, a more general model wherein all $n$ firms can compete for firm $0$'s business is conceivable. Interestingly, the equilibrium procurement choices identified herein can persist in that case, although firm $0$'s added bargaining power may shift more profits its way. That is, consider an equilibrium in which none of the $n$ firms are willing to offer a price low enough that firm $0$ would buy from them. In that case, the analysis above confirms that for $\sigma^2 < \tilde{\sigma}^2(k,n)$, none would be willing to deviate and offer a price to ensure buying by firm $0$ (by symmetry, if $R$ doesn't want to coax buying, neither would any other want to unilaterally do so). Similarly, for $\sigma^2 > \tilde{\sigma}^2(k,n)$ it is in $R$'s best interest to set a price so as to ensure firm $0$ would buy from it provided no other rivals choose to do so. Of course, given this, another rival may offer an even lower price to ensure that if buying occurs, at least wholesale profits go to them.
Whatever the ultimate price in this competition for firm 0's input market business, the net effect is the same – for $\sigma^2 > \hat{\sigma}^2(k,n)$, firm 0 opts to buy from one of its rivals.

4. Conclusion

A firm's make-or-buy choice is a well documented management problem that has attracted the attention of academics and practitioners from diverse fields. The accountant's role in this choice also has a storied past, one rooted in the desire to develop accurate in-house production cost estimates to compare to external prices. The simple textbook explanation of the role of accounting information is quite staid, despite the fact that the information age has brought about a much more nuanced and strategic role of accounting in most other decisions a firm makes. In this paper, we revisit the role of cost information in the make-or-buy decision in light of the fact that firm decisions, and the information conveyed therein, often have notable strategic repercussions. In particular, we note that a firm's estimate of production cost is not the only cost number that proves crucial to the make-or-buy choice. A firm's estimate of conversion costs too can influence the decision of whether or not to outsource, even when those conversion costs themselves are not affected by the decision.

The reason for this result is that the information gathered about conversion costs by a firm is inevitably conveyed to a supplier, albeit indirectly, by purchasing choices the firm makes. In particular, with outsourcing, a supplier comes to learn of both the firm's belief about its efficiency and its choice of strategic posturing. While not all suppliers care about this information, we show that the fact that such information is on the horizon means a firm may prefer to buy from an input supplier who has "skin in the game" via a presence in the output market.

By indirectly conveying information on its efficiency to its supplier through its purchasing decisions, a firm can soften competition with its supplier's output market arm. And, by conveying information about its output market quantity choices through its input
orders, a firm can gain a de facto Stackelberg advantage over its supplier (and even other rivals). Both effects point to a strategic role of outsourcing, one rooted in information conveyance and supportive of procurement from rivals. Admittedly, this point was made in a model that excludes other traditional considerations in the make-or-buy choice to highlight the novelty of the result. Future work could layer in these other factors to better parse the critical features that promote outsourcing as well as the determinants of who to outsource from and when to initiate outsourcing.
Proof of Proposition 1. If firm 0 opts to make, the firms engage in Cournot competition, with only firm 0 being able to condition its quantity on $\delta$, its private information. In particular, given observation $\delta$, and Cournot conjecture of firm $i$'s quantity, denoted $\tilde{q}_i$, $i \in N$, firm 0 chooses quantity to maximize its profit. In writing the profit expressions for firms, it is convenient to use the net demand intercept, $\alpha$, where $\alpha = a - c$. Thus, firm 0's problem is as follows:

$$\max_{q_0} \left[ \alpha + \delta - q_0 - k \sum_{i \in N} \tilde{q}_i \right] q_0. \quad (A1)$$

Similarly, given firm $i$'s, $i \in N$, conjecture of the quantities of its rivals, denoted $\tilde{q}_0(\delta)$ and $\tilde{q}_j$, $j \in N_{-i}$, firm $i$ solves:

$$\max_{q_i} E_{\delta} \left[ \alpha - \tilde{q}_i - k \tilde{q}_0(\delta) - k \sum_{j \in N_{-i}} \tilde{q}_j \right] q_i, \quad i \in N. \quad (A2)$$

The first-order conditions of (A1) and (A2) are given below:

$$q_0(\tilde{q}_i, i \in N; \delta) = \frac{1}{2} \left[ \alpha + \delta - k \sum_{i \in N} \tilde{q}_i \right] \quad \text{and} \quad q_i(\tilde{q}_0(\delta), \tilde{q}_j, j \in N_{-i}) = \frac{1}{2} \left[ \alpha - kE_{\delta} \{ \tilde{q}_0(\delta) \} - k \sum_{j \in N_{-i}} \tilde{q}_j \right] , \quad i \in N. \quad (A3)$$

Jointly solving the $n + 1$ linear equations in (A3), along with the $n + 1$ equilibrium conditions, $q_0(\delta) = \tilde{q}_0(\delta)$ and $q_i = \tilde{q}_i$, $i \in N$, yields the quantities in (A4), where the superscript "M" denotes the make regime:

$$q_0^M(\delta) = \frac{\alpha}{2 + k\delta} + \frac{\delta}{2} \quad \text{and} \quad q_i^M = \frac{\alpha}{2 + kn}, \quad i \in N. \quad (A4)$$

Substituting (A4) into (A1), and taking expectation with respect to $\delta$, yields $\Pi_0^M$, expected profit of firm 0; using (A4) in (A2) yields $\Pi_i^M$, $i \in N$, expected profit of firm $i$:

$$\Pi_0^M = \left[ \frac{\alpha}{2 + kn} \right]^2 + \frac{\alpha^2}{4} \quad \text{and} \quad \Pi_i^M = \left[ \frac{\alpha}{2 + kn} \right]^2, \quad i \in N. \quad (A5)$$

This completes the proof of Proposition 1.
Proof of Proposition 2. If firm 0 opts to procure from firm \(I\), the firms engage in Cournot competition as in the proof of Proposition 1 except for the fact that firm 0's input procurement cost is \(wq_0\) rather than 0. Thus, while the problem for firm \(i\), \(i \in N\), is as in (A2), firm 0's problem is as in (A6):

\[
\max_{q_0} \left[ \alpha + \delta - q_0 - k \sum_{i \in N} \tilde{q}_i \right] q_0 - wq_0. \tag{A6}
\]

With (A6) replacing (A1), the rest of the analysis follows precisely the proof of Proposition 1 yielding \(q_0^I(w; \delta)\) and \(q_i^I(w)\) in Proposition 2. Using these in (A2) and (A6), and taking expectation with respect to \(\delta\), yields \(\Pi_0^I(w)\) and \(\Pi_i^I(w), i \in N:\)

\[
\Pi_0^I(w) = \left[ \frac{\alpha}{2 + kn} - \left( \frac{2 + k[n - 1]}{2 - k[2 + kn]} \right) w \right]^2 + \frac{\sigma^2}{4} \text{ and }
\]

\[
\Pi_i^I(w) = \left[ \frac{\alpha}{2 + kn} + \frac{kw}{2 - k[2 + kn]} \right]^2, i \in N. \tag{A7}
\]

This completes the proof of Proposition 2. \(\blacksquare\)

Proof of Proposition 3. If firm 0 opts to buy from its rival, firm \(R\), its placement of order puts it in the position of a Stackelberg leader vis-a-vis \(R\). Thus, in this case, we begin with the quantity choice of firm \(R\). Given wholesale price \(w\), order \(q_0\) from firm 0, and conjecture \(\tilde{q}_j\) of the quantity of firm \(j\), \(j \in N_{= R}\), firm \(R\) chooses quantity to solve:

\[
\max_{q_R} \left[ \alpha - q_R - kq_0 - k \sum_{j \in N_{= R}} \tilde{q}_j \right] q_R + wq_0. \tag{A8}
\]

The first-order condition of (A8) yields:

\[
q_R(q_0, \tilde{q}_j, j \in N_{= R}) = \frac{1}{2} \left[ \alpha - kq_0 - k \sum_{j \in N_{= R}} \tilde{q}_j \right]. \tag{A9}
\]

Anticipating the response in (A9), and given wholesale price \(w\) and conjecture \(\tilde{q}_j\) for firm \(j\)'s quantity, \(j \in N_{= R}\), firm 0 solves:

\[
\max_{q_0} \left[ \alpha + \delta - q_0 - kq_R(q_0, \tilde{q}_j, j \in N_{= R}) - k \sum_{j \in N_{= R}} \tilde{q}_j \right] q_0 - wq_0. \tag{A10}
\]

The first-order conditions of (A10) is given below:
Finally, firm $i$, given its conjectures $\bar{q}_0(\delta)$ and $\bar{q}_j$, $j \in N_{(R,i)}$, and the response in (A9), chooses its quantity to solve:

$$
\max_{q_i} E_{\delta} \left\{ \alpha - q_i - k\bar{q}_0(\delta) - kq_R(\bar{q}_0(\delta), \bar{q}_j, j \in N_{(R)}), j \in N_{(R,i)} - k \sum j \bar{q}_j \right\}, i \in N_{-R}. \tag{A12}
$$

The first-order conditions of (A12) is as follows:

$$
q_i(\bar{q}_0(\delta), \bar{q}_j, j \in N_{(R,i)}) = 
\frac{1}{2} \left\{ \alpha - kE_{\delta} \{ \bar{q}_0(\delta) \} - kE_{\delta} \{ q_R(\bar{q}_0(\delta), \bar{q}_j, j \in N_{(R)}) \} - k \sum j \bar{q}_j \right\}, i \in N_{-R}. \tag{A13}
$$

Jointly solving the first-order conditions in (A9), (A11), and (A13), along with the equilibrium conditions, $q_0(\delta) = \bar{q}_0(\delta)$, $q_i = \bar{q}_i$, $i \in N_{-R}$, yields the quantities in (A14), where the superscript "$R$" denotes buying from the rival firm $R$:

$$
q_0^R(w; \delta) = \frac{2[\alpha - k(2 - k) - k])}{8 + k[4 - k^n][n - 1] - 2k^n[n + 1]} + \frac{\delta}{2 - k^n};
$$

$$
q_R^R(w; \delta) = \frac{\alpha[4 - 2k - k^2] + 2kw}{8 + k[4 - k^n][n - 1] - 2k^n[n + 1]} - \frac{\delta k}{2 - k^n}; \text{ and}
$$

$$
q_i^R(w) = \frac{\alpha[4 - 2k - k^2] + 2kw}{8 + k[4 - k^n][n - 1] - 2k^n[n + 1]}, i \in N_{-R}. \tag{A14}
$$

Substituting (A14) in (A8) and (A10), and taking expectation with respect to $\delta$, yields $\Pi_0^R(w)$ and $\Pi_R^R(w)$, expected profit of firm 0 and firm $R$ in the buy regime:

$$
\Pi_0^R(w) = 2\left(2 - k^2\right) \left( \frac{[(\alpha - w)(2 - k) - k]w}{8 + k[4 - k^n][n - 1] - 2k^n[n + 1]} \right)^2 + \frac{\sigma^2 k^2}{2 - k^n}; \text{ and}
$$

$$
\Pi_R^R(w) = \left( \frac{\alpha[4 - 2k - k^2] + 2kw}{8 + k[4 - k^n][n - 1] - 2k^n[n + 1]} \right)^2 + \frac{2w((\alpha - w)(2 - k) - k)w}{8 + k[4 - k^n][n - 1] - 2k^n[n + 1]} + \frac{\sigma^2 k^2}{4[2 - k^n]^2}. \tag{A15}
$$

This completes the proof of Proposition 3.
Proof of Proposition 4. From (A5) and (A7), firm 0 prefers to buy from \( I \) rather than make inputs if and only if \( \Pi_0^B(w) - \Pi_0^M > 0 \), which is equivalent to \( w < 0 \). Of course, for any \( w < 0 \), supplier \( I \) makes negative profit and, thus, does not sell. In short, in equilibrium, outsourcing by firm 0 does not involve buying from \( I \), as noted in part (i).

Turning to part (ii), suppose firm 0 is induced to buy from firm \( R \). In this case, the wholesale price is firm \( R \)'s preferred price (denoted \( \tilde{w} \)) assuming firm 0 is willing to procure at this price rather than make inputs. However, if \( \tilde{w} \) is excessive in that firm 0 prefers to make, then firm \( R \) is restricted to charging the maximum price firm 0 is willing to pay (denoted \( \bar{w} \)). In other words, \( w^* = \text{Min}\{\tilde{w}, \bar{w}\} \).

The wholesale price \( \tilde{w} \) is the \( w \)-value that maximizes \( \Pi_R^R(w) \) in (A15). The first-order condition of (A15) yields:

\[
\tilde{w} = \frac{\alpha[16 - 2k^2(4n - k + 2) + k(8 + k^3)(n - 1)]}{2[16 + 16k(n - 1) - k^4(n - 1)^2 - 2k^2(1 + 6n - 2n^2) - 2k^3(n^2 + n - 2)]}.
\] (A16)

Using (A5) and (A15), the wholesale price \( \bar{w} \) is the \( w \)-value that solves \( \Pi_0^R(w) - \Pi_0^M = 0 \). Thus, \( \bar{w} \) equals:

\[
\bar{w} = \frac{\alpha[2 - k]}{2 + k[n - 1]} - \frac{[(2 - k)^2(2 + k) + kn(4 - k(2 + k))]\sqrt{4\alpha^2(2 - k^2) - k^2(2 + kn)^2}}{2\sqrt{2}[2 - k^2][2 + k(n - 1)][2 + kn]}. \] (A17)

This completes the proof of Proposition 4.

Proof of Proposition 5. From \( w^* = \text{Min}\{\tilde{w}, \bar{w}\} \), (A5), and (A15), \( \Pi_R^M \) is free of \( \sigma^2 \) while \( \Pi_R^R(w) \) is increasing in \( \sigma^2 \). Thus, there exists a variance cut-off, \( \hat{\sigma}^2(k,n) \), above which firm \( R \) induces firm 0 to buy and, below which, firm 0 is induced to make. For now, assume that at \( \sigma^2 = \hat{\sigma}^2(k,n) \), \( w^* = \text{Min}\{\tilde{w}, \bar{w}\} = \bar{w} \), a claim we will confirm subsequently. Using (A5) and (A15), firm 0 is induced to buy by firm \( R \) if and only if:

\[
\Pi_R^R(\bar{w}) - \Pi_R^M \geq 0 \iff \sigma^2 \geq \frac{2\alpha^2[4 - 2k^2 - A^2(k,n)]}{k^2[2 + kn]^2},\]

where

\[
A(k,n) = \frac{[2 - k^2][4 + k(-6 - k(n - 1) + 2n)][2 + kn] + [2 + k(n - 1)]\sqrt{B(k,n)}}{20 - k[20 + k - 4k^2 + k^3 - 2(2 - k)(5 - k - k^2)n - k(5 - k(2 + k)n^2]}
\] and

\[
B(k,n) = \frac{4k^2(-k^2 + 4k + 4) + k^2(4k + 4)[2 + kn] + [2 + k(n - 1)]\sqrt{B(k,n)}}{20 - k[20 + k - 4k^2 + k^3 - 2(2 - k)(5 - k - k^2)n - k(5 - k(2 + k)n^2]}
\]
\[ B(k,n) = [2 - k^2][72 - k(24 - 72n + k(22 + 2k(4 - k)(2 - k^2) + 36n + 4k(5 - k^3)n + (-18 + k(12 + (2 - k^2)k)n^2))]. \]  

(A18)

From (A18), if \( 4 - 2k^2 - A^2(k,n) < 0 \), then \( \Pi^R_R(\bar{w}) - \Pi^M_M > 0 \) for all \( \sigma^2 \geq 0 \). Thus, the equilibrium outcome entails firm 0 buying the input if and only if:
\[
\sigma^2 \geq \hat{\sigma}^2(k,n) = \text{Max}\left\{ \frac{2\alpha^2[4 - 2k^2 - A^2(k,n)]}{k^2[2 + kn]^2}, 0 \right\}. \]  

(A19)

Finally, from (A16) and (A17), note that \( \hat{w} - \bar{w} \) is decreasing in \( \sigma^2 \). Some tedious algebra verifies that \( \hat{w} - \bar{w} \big|_{\sigma^2=0} > 0 \) and \( \hat{w} - \bar{w} \big|_{\sigma^2=0} = \frac{2\alpha^2[4 - 2k^2 - A^2(k,n)]}{k^2[2 + kn]^2} > 0 \). That is, \( \hat{w} - \bar{w} > 0 \) at \( \sigma^2 = \hat{\sigma}^2 \) verifying our initial claim that \( w^* = \text{Min}\{\tilde{w}, \bar{w}\} = \bar{w} \) at the variance cutoff. This completes the proof of Proposition 5. 

**Proof of Proposition 6.** From (A19), \( \hat{\sigma}^2(k,n) = 0 \) if and only if \( 4 - 2k^2 - A^2(k,n) \leq 0 \). Using the expression for \( A(k,n) \) noted in (A18):
\[
\hat{\sigma}^2(k,n) = 0 \iff 4 - 2k^2 - A^2(k,n) \leq 0 \iff n \geq \hat{n}(k), \text{ where }
\]
\[
\hat{n}(k) = \frac{\sqrt{4 - 2k^2} - 2[1-k]}{2k} + \frac{\sqrt{4 + k} \left\{ 4 - 3k \right\} \left\{ 4 - k^2 - 2\sqrt{4 - 2k^2} \right\}}{\sqrt{2} \left\{ (2-k)\sqrt{4 - 2k^2} + k(2+k) - 4 \right\}}. \]  

(A20)

This completes the proof of Proposition 6. 

\[ \blacksquare \]
References


