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Composition of Robust Equity Portfolios

Jang Ho Kim\textsuperscript{a}, Woo Chang Kim\textsuperscript{b}, and Frank J. Fabozzi\textsuperscript{c,*}

Abstract

Robust portfolios resolve the sensitivity issue identified as a concern in implementing mean-variance analysis. Because robust approaches are not widely used in practice due to a limited understanding regarding the portfolios constructed from these methods, we present an analysis of the composition of robust equity portfolios. We find that compared to the Markowitz mean-variance formulation, robust optimization formulations form portfolios that contain a fewer number of stocks, avoid large exposure to individual stocks, have higher portfolio beta, and show low correlation between weight and beta of the stocks composing the portfolio. These properties are also found for global minimum-variance portfolios.

Keywords: robust portfolio, mean-variance model, global minimum-variance portfolio, stock beta, Ellsberg paradox

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1. Introduction

Markowitz (1952, 1959) presents a portfolio optimization method for finding the optimal portfolio based on the risk-return tradeoff using the expected return, variance, and covariance of asset returns. Despite the model’s simplicity, it is still used in asset allocation. One of the main criticisms of the approach is its lack of robustness to the input values (Michaud, 1989, Best and Grauer, 1991a, 1991b, Broadie, 1993, and Chopra and Ziemba, 1993). Among many approaches to increase the robustness of mean-variance portfolios, robust optimization focuses on finding the optimal portfolio under the worst-case situation. Although there have been many studies on formulating robust portfolio problems (Lobo and Boyd, 2000, and Goldfarb and Iyengar, 2003), robust optimization has not been adopted by practitioners as an approach to manage portfolio risk.

One of the reasons that limit the use of robust portfolios is the weak understanding on the characteristics of portfolios formed from robust optimization methods. Recently, some studies analyze whether robust portfolios are more dependent on fundamental factors when compared against mean-variance portfolios. Kim et al. (2012b) derive an analytical explanation for the robust formulation with an ellipsoidal uncertainty set that explains why portfolios with high robustness become closer to the portfolio whose variance is maximally explained by fundamental factors. Furthermore, Kim et al. (2012a) provide empirical evidence under various settings that robust portfolios from robust optimization show higher correlation with the Fama-French three factors (Fama and French, 1993). They both conclude that increased robustness leads to higher dependency on fundamental factors. Although they reach a meaningful conclusion at the portfolio-level, a complete analysis will also require examining the behavior of individual stocks that are included in robust portfolios. For example, Tütüncü and Koenig (2004) experiment with indices of five to 10 asset classes to document the change in the allocation of assets as they move along the efficient frontiers and find that robust portfolios contain fewer asset classes. Pflug and Wozabal (2007), in contrast, find that robustness leads to more diversified portfolios but their analysis includes only six stocks. Even though these two studies provide some preliminary observations, they use only a limited number of candidate assets and focus on the diversification level because the primary objectives of these papers were not to investigate the composition of robust portfolios. Although Lu (2011) also examines the level of diversification along with performance measures such as the rate of growth of wealth and transaction cost, he focuses on assessing two robust approaches that
use separable and joint uncertainty sets without including a comparison to the mean-variance model.

Therefore, in this paper, we present a thorough analysis of the composition of robust equity portfolios. By focusing on robust portfolio optimization methods based on worst-case optimization, we find several characteristics of robust equity portfolios and observe these properties in the global minimum-variance portfolio.

The organization of the paper is as follows. Section 2 briefly reviews robust portfolio formulations. Section 3 introduces the data and test model used for analyzing those robust models and the results are included in Section 4. Section 5 summarizes our analysis on robust equity portfolios and Section 6 concludes.

2. Robust Portfolio Formulations

Among several robust approaches, we focus on robust formulations based on worst-case optimization. Worst-case robust problems search for the optimal portfolio while assuming the worst possible situation within a predetermined set for the uncertain inputs (Lobo and Boyd, 2000, and Goldfarb and Iyengar, 2003). The robust behavior of investors can be explained by the Ellsberg paradox (1961) where decision makers are shown to be highly affected by their aversion to uncertainty. The maxmin expected utility decision rule describes this behavior because the minimum utility of each case is compared (Gilboa and Schmeidler, 1989).

In order to construct portfolios with the same level of risk aversion under mean-variance and robust models, we use the following objective function throughout the analysis,

\[ \min_{\omega} \mu' \omega - \lambda \omega' \Sigma \omega \]

where \( \omega \) is the portfolio weights, \( \Sigma \) is the covariance matrix of asset returns, \( \mu \) is the expected asset returns, \( \lambda \) is the risk-seeking coefficient, and \( \Omega \) is the set of feasible portfolio weights. For the robust formulations, the objective function is first maximized within an uncertainty set to find the worst case. We only consider uncertainty in expected return of assets because it is known to have a stronger effect on portfolio return than the variance or covariance of asset returns (Chopra and Ziemba, 1993). In our study, the uncertainty set for expected return of assets is considered to follow a box or an ellipsoidal shape.\(^1\) The two uncertainty sets are the two most researched models and they are also used by Kim et al. (2012a) for

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\(^1\) For the covariance matrix of estimation errors in the ellipsoidal uncertainty set, we assume the off-diagonal terms to be zero (Stubbs and Vance, 2005).
performing regression analysis between robust portfolios and the Fama-French three factors.  

3. Analysis Model for the Composition of Portfolios

3.1. Data

For our empirical analysis, we use 100 portfolios formed on size and book equity to market equity (B/M) ratio. These 100 portfolios are intersections of 10 portfolios formed on size and 10 portfolios formed on B/M ratio. The 100 portfolios are a good representation of the U.S. stock market since they include all stocks from the NYSE, AMEX, and NASDAQ that have data for book equity and market equity. Therefore, in our analysis, we use the daily returns of the 100 portfolios to represent stock-level returns where each stock has a unique level of size and B/M ratio. By forming mean-variance and robust portfolios from these 100 stocks, it allows us to present a complete analysis of the composition of portfolios based on market capitalization and B/M ratio of the individual stocks and also a clear understanding of the allocation among the four “asset classes” obtained on the basis of size and B/M ratio (large-cap value, large-cap growth, small-cap value, and small-cap growth). Moreover, the number of candidate stocks is large enough to look for patterns in the diversification levels as well as additional characteristics about the stocks that are included in the portfolios.

The overall period of our analysis is from July 1980 to June 2012. A time-series that includes 30 years of historical data will give us a confident understanding of robust equity portfolios. Moreover, because this period includes several bear markets, we also perform analysis during the two notable periods of market downturn: the dot-com bubble burst and the financial crisis of 2008. Because the 100 portfolios used in our analysis to represent stock-level returns are constructed at the end of June each year, when constructing mean-variance and robust portfolios, we use July as the beginning and June as the end of the yearly data. The excess return of the U.S. market is also collected for the same period and we use this for computing the betas of the 100 stocks.

3.2. Test Measures

We use several measures to find and compare properties of mean-variance, global minimum-variance, and robust portfolios. First, we observe the composition of portfolios based on two factors: size and B/M ratio. We

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2 For further details on the construction of uncertainty sets and robust formulations, see Fabozzi et al. (2007a, 2007b), Fabozzi et al. (2010), Kim et al. (2012c) and the references therein.

3 The data for the 100 portfolios and market excess returns are obtained from the online data library of Kenneth R. French (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).
also examine the proportion of portfolio weight invested in the four major asset classes. Computing these two is straightforward since the data we use for the experiment are already allocated based on size and B/M ratio. We also investigate the level of diversification of portfolios by the number of stocks contained in the portfolios. Since some stocks are given very small but not necessarily zero weight when solving the optimization problem, we count only the number of stocks with absolute weight of at least 1%. Furthermore, in order to check the fraction of stocks with negative weight, we count the number of stocks with weight less than -1%.

Although under simpler settings, the above measures were investigated by a few previous studies. In this paper, we extend the test to thoroughly analyze the properties based on stock beta. We focus on the conventional definition of stock beta that originates from capital asset pricing model; we calculate stock betas using the single index market model (Sharpe, 1963) estimated from regression analysis with the only factor being the excess market return.\(^4\) We first compute the portfolio beta, which is the weighted sum of the betas of the stocks in the portfolio where the weights are the portfolio allocations given to individual stocks.

The major contribution of our study comes from measuring the correlation between portfolio weights given to stocks and the betas of those stocks. We look at pairs of weight and beta for all stocks comprising the portfolios to see if robustness has any effect on the composition based on individual stock beta. For example, if robust portfolios show lower correlation than mean-variance portfolios, it would mean that worst-case optimization results in robust portfolios that invest more in low-beta stocks.

### 3.3. Model Settings

We impose constraints on portfolio weights by setting the upper bound as 100% and the lower bound as -100% for each stock. These bounds on weights are a valid representation of practical trading settings since high exposure on individual stocks is often avoided to reduce the risk of portfolios. We also find that unconstrained mean-variance portfolios have many stocks with an absolute weight exceeding 200% whereas most stocks in robust portfolios have an absolute weight below 20%. Therefore, the upper and lower bounds of 100% and -100% allow us to analyze the effect of increased robustness assuming a reasonable boundary.

For completeness, we perform all of our experiments using 1-year, 3-year, and 5-year rebalancing periods.

\(^4\) For the regression analysis, the independent variable is the excess market return and the dependent variable is the return of a stock.
We also use two values, 0.05 and 0.1, of the risk-averse coefficient, \( \lambda \). The risk-averse coefficient of 0.05 represents the more risk-averse situation because the value set to 0 computes the global minimum-variance portfolio. The higher value of the risk-averse coefficient is set to 0.1 because values that are greater than 0.2 result in the construction of aggressive mean-variance portfolios that have a majority of the stocks with 100% or -100% weights. The confidence levels of robust portfolios that range from 25% to 95% are used for the experiment. Since the observed characteristics are consistent for all confidence levels, we intentionally present the results for a high confidence level of 95% in order to magnify its difference with mean-variance portfolios, which are basically robust portfolios with a 0% confidence level.

4. Results for the Composition of Robust Portfolios

We summarize our empirical results for the composition of mean-variance and robust equity portfolios in this section. Throughout this section, we refer to portfolios from the mean-variance problem as MV, the global minimum-variance problem as GMV, the robust problem with a box uncertainty set as R1, and the robust problem with an ellipsoidal uncertainty set on the expected stock returns as R2. Even though the results are similar under various rebalancing periods, we primarily present figures when using a 3-year rebalancing period because we later include analysis during two historical bear markets, which both took place over 3-year periods.

4.1. Composition of R1 and R2

Before making a comparison between robust portfolios and mean-variance portfolios, we first look at characteristics of robust portfolios. By observing the proportion allocated amongst the 100 stocks, we find that more weights are given to stocks with small-cap and high B/M ratio and negative weights are given to stocks with small-cap and low B/M ratio. The average weights of 100 stocks for the entire period for robust portfolios with a 3-year rebalancing period are shown in Figure 1. Average allocation in each stock is represented by a circle where a positive weight is in gray and a negative weight is in black. The figures for R1 at the top and R2 at the bottom of Figure 1 clearly show larger positive allocations towards the upper-left corner and larger negative allocations towards the lower-left corner. The weights invested in the four asset classes are shown in Table 1, which contains average values for the entire test period. Both R1 and R2 put most weight on small-cap value stocks followed by large-cap value stocks. Small-cap growth stocks are given the least weight and, in fact, are often given negative weights. In addition, between R1 and R2, we find that
R1 is more concentrated in small-cap value stocks.

Figure 2 shows weights and betas of the 100 stocks for MV, GMV, R1, and R2 for three separate 3-year periods. Note that stocks are ordered in ascending order based on beta in order to easily display any pattern in correlation between weights and betas. For all three periods, we notice that R1 and R2 assign weights close to 0 (between -10% and 10%) to most of the stocks. One interesting finding is that R1 and R2 both seem to put more weight on stocks with the lowest betas. The negative correlation between weight and beta for robust portfolios are reported in Table 1. The average correlation of the overall period is lower for R2 than R1. In fact, R2 shows a correlation below -0.6, indicating a strong negative relationship between weights and betas of individual stocks. However, the opposite is true for portfolio betas; R2 shows high positive portfolio beta closely followed by R1. Table 1 also provides information about the diversification properties of robust portfolios. From the number of stocks with non-zero weights, it is seen that robust portfolios invest in less than 60% of the stocks. Moreover, R1 is shown to be more concentrated than R2 because R1 has fewer stocks with non-zero weight and fewer stocks with negative weights.

4.2. Comparison between R1, R2, and MV

Here we describe the difference between robust portfolios and mean-variance portfolios. First, we can easily notice from Figure 2 that the weights of MV deviate a lot more than R1 and R2. It is also difficult to find any trend in weights based on the value of stock betas. Allocations for MV as shown at the top of Figure 3 are much larger than those of R1 and R2 from Figure 1. Besides the scale of weights, MV has many negative weights on the right-hand side where large-cap stocks are represented. The same pattern can be observed in Table 1; weights for the four asset classes are higher in magnitude than R1 and R2 and negative weights are often given to large-cap stocks. On the other hand, MV appears to have some similarities with R1 and R2; all portfolios assign the highest weight to small-cap value stocks and the smallest weight (large negative weight) to small-growth stocks.

Surprisingly, but analogous to the results of Tütüncü and Koenig (2004), MV shows higher diversification than R1 and R2 as indicated in Table 1. MV puts at least a 1% absolute weight on almost all 100 stocks. Furthermore, MV takes short positions in almost half of the stocks in the portfolio whereas R1 and R2 take short positions in less than a third of the stocks in their portfolios. Even though MV contains more stocks, the portfolio beta of MV is a lot smaller than R1 and R2. This means that MV has virtually no
dependency on excess market returns because the average of the weighted sum of betas is less than 0.1 for all rebalancing periods and risk-aversion levels investigated in our test. Conversely, the correlation between weight and beta of MV is higher than the correlations for R1 and R2.

4.3. Characteristics of GMV

In Figure 2, weights for GMV indicate less deviation from a 0% weight than MV but slightly more deviation than R1 and R2. This is also observed in the allocations for GMV as shown at the bottom of Figure 3 where the allocations seem to be a scaled-down version of MV; they both have large positive weights towards the upper-left corner, large negative weights towards the lower-left corner, a few negative weights towards the upper-left side, and many negative weights on the right half of the graph. Allocations for GMV also look similar to Figure 1, especially with the allocations for R1. From Table 1, we observe that the allocation among the four asset classes is remarkably similar between GMV and R1. GMV has comparable asset divisions with robust portfolios, but the number of stocks in GMV is a lot higher than R1 and R2 and has more stocks with negative weights. With respect to portfolio beta, the value of GMV is larger than MV but smaller than those of R1 and R2. Nevertheless, the correlation between weight and beta for GMV is as small (large negative) as R1 and R2.

4.4. Results during Bear Markets

Robustness of portfolios is required most during bear markets and market crashes because stock returns in these periods will differ from the returns in the years prior to the downturn, which are used as inputs when forming portfolios and therefore can lead to large losses without a robust model. Hence, we check the composition of robust portfolios as well as mean-variance portfolios during periods of market downturn in order to determine if there are any deviations from our overall results. The two major bear markets are the dot-com bubble burst and the global financial crisis that began with the subprime mortgage crisis in the United States in the summer of 2007. Thus, we investigate the composition of portfolios from July 2000 to June 2003 (for the dot-com bubble) and from July 2007 to June 2010 (for the global financial crisis).

The results during these two periods do not appear to be any different from the results during the overall period. Weights during these periods display the same pattern as in Figure 2; weights of MV show the most deviation followed by GMV, and the robust portfolios seem to invest more in stocks characterized by low betas. Likewise, the observations for the number of stocks, short positions (i.e., negative weights), portfolio
5. **Analysis on the Properties of Robust Portfolios**

From the comparison between mean-variance and robust portfolios, we define the characteristics of robust portfolios as portfolios that have a decreased number of stocks (i.e., more concentrated), smaller magnitude of weight invested in each stock (i.e., not much exposure to any individual stock), higher portfolio beta (i.e., higher correlation with excess market return), and lower correlation between weight and beta for the individual stocks included in the portfolio (i.e., tendency to put more weight on low-beta stocks).

We can apply this analysis to the global minimum-variance portfolio. A GMV portfolio can be considered to be robust under our framework because even though the global minimum-variance problem does not explicitly incorporate uncertainty sets, it finds the optimal portfolio without using the expected return of stocks, which is the uncertain parameter in our robust formulations. In fact, the GMV portfolio demonstrates the four characteristics of robustness reported above when compared with mean-variance portfolios. While the observations of these four properties are not as apparent in the GMV portfolio as they are in robust portfolios, the GMV portfolio can be explained as a scaled-down version of mean-variance portfolios that becomes closer to the composition of robust portfolios during the scaling-down process. More importantly, from the opposite perspective, we can understand robust formulations as optimization methods that construct portfolios with increased robustness by forming portfolios with a composition similar to the GMV portfolio.

6. **Conclusion**

In this paper, we present a comprehensive analysis of the composition of robust equity portfolios. The main contribution of our work is identifying the major characteristics of the stocks that are selected in the construction of robust portfolios. Although Kim *et al.* (2012a) and Kim *et al.* (2012b) investigate the portfolio dependency on fundamental factors, in this paper we provide a more detailed analysis of the behavior of robust equity portfolios by looking at the weights given to individual stocks. We find evidence that worst-case optimization leads to robust equity portfolios with four characteristics compared to mean-variance portfolios: (1) fewer stocks, (2) less exposure to each stock, (3) higher portfolio beta, and (4) large negative correlation between weight and stock beta. These characteristics are observed for several risk levels and rebalancing periods for stocks in the U.S. equity market from 1980 to 2012 including the dot-com bubble and the global financial crisis that began in the summer of 2007. In addition, we find a connection between the global
minimum-variance portfolio and robust portfolios. When compared with mean-variance portfolios, the global minimum-variance portfolio maintains the four characteristics found for robust portfolios, thereby providing a more thorough understanding of the effect of minimizing risk for robust portfolios.

Even though our work focuses on robust portfolio optimization, the analysis on properties could be extended to other robust approaches. For example, Dow and Werlang (1992) look into the model of expected-utility maximization with nonadditive probabilities and find robust investors to be more conservative by showing regions of no asset holdings. Stutzer (2003) considers robustness based on the probability of the realized growth rate of wealth exceeding the target growth rate and compares the composition and performance of portfolios formed from 10 industries with different levels of risk aversion. Maenhout (2004) observes dynamic portfolios and illustrates how robustness reduces participation in risky assets. Further comprehensive analyses of robust methods based on additional approaches will extend the understanding and use of robust portfolios.

References


Fabozzi, F.J., Kolm, P.N., Pachamanova, D.A., Focardi, S.M., 2007b. Robust Portfolio Optimization and


Figure 1  Average weights of 100 stocks for R1 (top) and R2 (bottom) with a 3-year rebalancing period and $\lambda$ set to 0.1 (gray: positive, black: negative)
Figure 2  Weights and betas of 100 stocks for MV, GMV, R1, and R2 with a 3-year rebalancing period and λ set to 0.1 (primary vertical axis: beta, secondary vertical axis: weight)
Figure 3  Average weights of 100 stocks for MV (top) and GMV (bottom) with a 3-year rebalancing period and $\lambda$ set to 0.1 (gray: positive, black: negative)
Table 1  Average allocations, diversification levels, and measures based on stock beta

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<th>Small-Growth</th>
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<th>Large-Growth</th>
<th>Large-Value</th>
<th>Stocks with non-zero weight</th>
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<th>Weighted beta</th>
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<td>0.161</td>
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<td>MV</td>
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<td>53.1</td>
<td>12.6</td>
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Composition of Robust Equity Portfolios

- We find properties of robust equity portfolios at the individual stock level.
- There are four clear properties of portfolios from robust optimization.
- These properties are clearly observed when compared with mean-variance portfolios.
- Composition of global minimum-variance portfolios also shows these properties.
Keywords: robust portfolio, mean-variance model, global minimum-variance portfolio, stock beta, Ellsberg paradox

JEL Classification: G11, C61