A communication game on electoral platforms

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Abstract

This paper proposes a game to study strategic communication on platforms by parties. Parties’ platforms have been chosen in a multidimensional policy space, but are imperfectly known by voters. Parties strategically decide the emphasis they put on the various issues, and thus the precision of the information they convey on their position - and possibly that of their opponent - on each issue. The questions we address are the following: what are the equilibria of this communication game? Will parties talk about the same issues or not? Will they talk about consensual or divisive issues?

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1 Introduction

Electoral campaigns play a role in determining the winner, as witnessed by recent elections such as the 2000 presidential election in US or the 2002 presidential election in France. This suggests that relevant information is conveyed during the campaign. Indeed, there is a long tradition in political science on ignorant ill-informed voters (e.g. Campbell et al. 1960). Voters have little incentive to invest time and effort to gather all the relevant information. During the campaign, voters learn about the candidate’s personal characteristics and parties platforms. As a result, how much voters learn about parties platforms is partly determined by parties themselves. Even if the candidates do not ‘lie’, they may have incentives to make some of this information hard to obtain for voters, by making extremely vague and ambiguous statements for instance (Page 1978) or by avoiding to address an issue on which their position is quite at odd with the public opinion. So far, the key feature of the electoral campaign that has been mostly studied is the ‘where to stand’ question (see however the literature referred to below). But if parties strategically decide the emphasis they put on the various issues and the precision of the information they convey on each issue, another key question is: ‘what will they talk about’? And this may prove to be very important when voters mainly learn about the platforms through the campaign. By deciding which issues they want to emphasize, parties will determine the quality of voters’ information, the dividing lines in the electorate and the issues on which the election will eventually depend upon. The objective of this paper is precisely to address this question.

We develop a game that parties may play - or probably more accurately that candidates may play - once platforms have been chosen. The idea we want to capture here is that in the couple of weeks before an election, it may be impossible for a candidate to adjust a platform the way he wished he could. For instance, these platforms may have been decided by the party and officially written in a manifesto. Due to poorly informed voters, even though platforms are chosen, they is still a lot of room for the candidates to be strategic, regarding the features of their platform they want to put special emphasis on. When invited on a TV show, a candidate may want to speak mainly about law and order issues, or mainly about economic issues, or on the contrary, avoid as much as possible such issues. We assume that voters have a priori beliefs regarding where parties stand on the various issues. They are ready to update these beliefs when they get new information from the campaign. The more a candidate talks about an issue, the better-informed voters will be regarding his position on this issue. But in doing so, he may also convey information on his opponent’s position on this issue. Hence, the strategic variables chosen by the candidates during the campaign are how much time they will spend explaining their position, and possibly that of their opponent, on each issue, subject to a global time constraint. Since platforms have already been chosen, parties only care about their probability of winning the election (or alternatively about the vote share they get) and choose their strategies accordingly. The analysis is conducted in
a "probabilistic voting game" as introduced by Coughlin (1983). At the end of the electoral campaign, voters vote for the party they prefer. Besides their information on the policy platforms, their own preferred platforms, voters care about some unmodeled differences between parties (that may be other policy dimensions, or personal characteristics such as gender or race of the candidate).

The questions we want to answer are the following: what are the equilibria of this communication game? How much information is transmitted to voters through the campaign? Will parties talk about the same issues or not? Will they talk about issues on which they are close one to the other, or on very divisive issues? In our game, two factors determine how much attention (measured by some effort variable interpreted as time) parties devote to each potential issue. The first factor concerns the motives for speaking. A speech has two effects. It conveys information on the candidate’s true position and it reduces the uncertainty for the voters. These two effects may play in various directions, and possibly enter into conflict. Uncertainty reduction is unambiguously favorable to a party and may explain why both parties may both want to address a same issue. The information conveyed on where a candidate truly stands may or may not be beneficial to him since it depends on where the electorate stands. More surprisingly, this impact on positions may be favorable to both (or unfavorable to both) but this depends on the second factor, which is the type of debate. In practice, competitors are not always completely free and may have to respond to their opponents or to the journalists. This second factor is treated as a parameter in our model. The interaction between the positions, the a priori voters’ uncertainty and the type of debate leads to a variety of equilibrium configurations. In particular, the chances for both parties to address the same issues may be far from being negligible. But also, many issues are addressed by only one party.

We think that this sheds some light on previous puzzles. For example, when parties can freely and credibly choose their announcements the standard election competition model predicts the convergence of the announced platforms towards the median voter’s position (when it exists). This prediction has been seen as quite at odd with empirical evidence by many observers: Parties differentiate their platforms (see for example Budge et al. who compare estimates of the median voters with estimates of the candidates’ platforms). Another puzzle concerns issue convergence. Petronik (1996) argued that each candidate enjoys an advantage (‘ownership’) a priori on some issues. Viewing the election as a two person zero sum game, no issue can work to the advantage of both candidates, and opponents should address different ‘orthogonal’ issues (Austen-Smith 1993, Simon 2002). Again, this prediction is at odd with some evidence (Sigelman and Buell 2004).

Some distinctive modeling assumptions should be made precise. The first feature concerns parties’ sincerity. During the campaign, candidates are assumed to be truthful although voters may wrongly interpret their speeches. Specifically, parties’ speeches are interpreted as noisy but unbiased signals about the true parties’ positions. As a result, the strategic aspect bears on the allo-
cation of time (precision) spent on addressing the issues. A second important feature, related to the previous one, is about commitment. Voters vote according to their assessment about parties’ platforms. Hence, as in the standard spatial electoral competition game (Downs 1957, Hotelling 1929), it is implicit that platforms will be implemented (or that deviations are due to unforeseen circumstances). Sincerity and commitment are questioned by the literature that models an electoral campaign as a manipulation game. As here parties have 'true' platforms, which are imperfectly known by voters. A party’s platform may be interpreted as its preferred policy, the one it will implement once in office. Announcements serve to 'manipulate' voters' beliefs, and may be more or less effective in transmitting information depending on voters' reactions. No information is transmitted if the game is pure cheap talk zero-sum game. Introducing some cost born by the winning candidate not only makes communication possible but also induces a multiplicity of equilibria (Banks 1990). Our game is not a cheap talk game since speaking always conveys information.

Also recent papers have investigated the opposite case where voters preferences are unknown, and subject to a macro-economic shock. In that setting, Bernhard, Duggan and Squintani (2005) consider the impact of private or public polls on choice of the platforms and whether this may lead to divergence.

Section 2 introduces the model, Section 3 carefully analyzes the impact of the electoral campaign on voter’s beliefs. Sections 4 and 5 are devoted to the analysis of equilibria with a single issue and multiple issues respectively. Section 6 discusses the main assumptions of the model.

2 The electoral game

To keep the presentation of the model as simple as possible, we first present the model in a one-dimensional setting. The one-dimensional model straightforwardly extends to a multidimensional policy space, as will be described in section 5.

2.1 Voters’ and parties

We consider a unidimensional policy space $X$, which we take to be the real line: $X = \mathbb{R}$. Voter $i$’s utility if policy $x \in X$ is implemented is

$$u_i(x) = -(x - x_i)^2,$$

where $x_i$ is voter $i$’s bliss point in $X$. Bliss points are distributed with density $f(.)$.

There are two parties, party $A$ and party $B$. Parties have fixed platforms in the policy space $X$: party $J$’s platform is denoted by $x_J$, $J \in \{A, B\}$. Those are the platforms that parties will implement if elected. We take those platforms to be fixed; for example, they are contained in a written manifesto on which members of the party have reached a consensus. We do not explicit here how those platforms have been chosen. We take them to be given when the
electoral campaign starts. Each party knows its platform, as well as that of its opponent. But voters do not know any of those platforms with certainty. Before the electoral campaign starts, voters share the same a priori beliefs on these platforms, which they take to be independently distributed between parties, with \( x_A \sim \mathcal{N}(m_A, s_A^2) \) and \( x_B \sim \mathcal{N}(m_B, s_B^2) \). Those beliefs may come from past campaigns or from observing the policies chosen by the party in charge during the previous legislatures, or from what they have heard during the party congress. Acquiring a perfect information about parties’ platforms would be prohibitively costly. Yet, voters have some opportunity to learn about these platforms during the campaign, and to update their beliefs as to where parties stand. At the end of the electoral campaign, voters vote for party A or B, and the party that gets the higher number of votes is elected. When elected, a party implements its platform.

The campaign will be analyzed according to the following timing.

1. Candidates choose their emphasis strategy (or communication strategy). Candidates are only interested in getting as many votes as possible, in expectation.
2. Voters are receptive to the campaign. There may however be some variation in the speeches they listen to, the meetings they attend to, and how they interpret them. This may result in different ‘signals’, \( y_i \) for voter \( i \). Furthermore, voters may be affected by idiosyncratic bias \( \sigma_i \) as defined in next section.
3. Voters vote and the party getting the highest number of votes is elected.

Comments.

1. Since we are at the stage where platforms have already been chosen, the fact that candidates are trying to get as many votes as possible does not imply that they are purely office-motivated.
2. One may consider that \( x_A \) is a consensus reached within party A. Signals are noisy because they are conveyed by different party’s members. Depending on the electoral system (and the union within the party), the noise in the signals will be more or less important.
3. An alternative interpretation is that \( x_J \) is the candidate’s platform instead of the party’s platform. This may be a more sensible interpretation in some elections where the candidate is quite independent from the party, such as the US presidential election.

2.2 The electoral campaign stage

During the campaign, parties can not change their platform. But what they can do, is to decide how much information they want to convey on the issue at stake. More precisely, during the campaign, the candidates decide how much time they want to spend discussing the issue. A candidate who spends

\(^1\text{Having in mind these two interpretations, we indifferently use the term "party" or "candidate" to refer to this player.}\)
some time discussing an issue sends information to voters about where his party stands on this issue. But in doing so, he may also convey information on his opponent’s position on this issue. Here, we shall assume that the information conveyed on the opponent’s is ‘involuntary’, represented by an exogeneous parameter $\rho \in [0, 1]$. In one extreme case, a candidate speaks only on his own position. In the other extreme case, a candidate conveys as much information on his opponent’s position as on his own. This information is represented by signals. The variance of the signal on a party’s position depends on the times spent by the two candidates discussing this issue.

More precisely, during the campaign, party $J$ decides how much time, $t_J \in T = [0, 1]$, it wants to spend discussing the issue. When a party spends $t_J$ hours discussing an issue, a fraction $\frac{1}{1+\rho}$ of this time is devoted to exposing its own platform, and the remaining fraction $\frac{\rho}{1+\rho}$ of this time is devoted to exposing the opponent’s platform on this issue. Voter $i$ receives two imperfect signals: one on party $A$’s true position $y_{i,A}$, whose variance depends on the total time spent discussing the platform of party $A$, $\frac{t_A + \rho t_B}{1+\rho}$, and one signal on party $B$’s true position $y_{i,B}$, whose variance depends on the total time spent discussing the platform of party $B$, $\frac{t_A + \rho t_B}{1+\rho}$. Signals on parties’ positions are assumed to be unbiased and normally distributed:

$$y_{i,A} \sim N\left(x_A, \sigma^2_A \left(\frac{t_A + \rho t_B}{1+\rho}\right)\right),$$

$$y_{i,B} \sim N\left(x_B, \sigma^2_B \left(\frac{t_B + \rho t_A}{1+\rho}\right)\right),$$

with:

$$\sigma^2_J(0) = +\infty, \sigma'_J(t) \leq 0 \text{ for } t \in [0, 1], \text{ for } J = A, B.$$

Denote by $y_i = (y_{i,A}, y_{i,B})$ the vector of signals received by voter $i$. We make no specific assumptions regarding the correlations of signal across voters. They can be independently distributed (conditional on $x_A, x_B$) or correlated.

The case $\rho = 0$ corresponds to situations where a party only conveys information about its own position, and the case $\rho = 1$ corresponds to situations where a party, when tackling an issue, is constrained to spend the same amount of time exposing its view and that of its opponent. This parameter $\rho$ describes the technology of campaign communication, and is exogenous. It represents the extent to which spending some time talking about an issue - and thus conveying voters information about where one stands on an issue - might also send voters some information about where the other party stands on this issue. It may be so because once a party starts discussing an issue, it is constrained, say, by journalists, to comment on his opponent’s position on this issue. It might also be because when a candidate tackles an issue during a debate, or in a press conference, for example, his opponent has to answer, and also talks about this issue. In this interpretation, when party $A$ devotes $t_A$ units of time to discuss the issue, party $B$ is given $\rho t_A$ additional units of time (on top of his initial time budget) to answer and expose his own position on the issue. A party has
two distinct time budget: \( T = [0, 1] \) on which it has full control, and some additional time due to this "right to answer back" - which it does not control - and is imposed by its opponent’s communication strategy.

Note that when \( \rho = 0 \), choosing the time it spends discussing an issue is equivalent for a party to choosing the precision of the signals it will send to voters regarding its position. Therefore, an alternative interpretation of the model in that case is that parties decide how precise - or how vague and ambiguous - they want to be about an issue.

In section 6, we come back on some of these assumptions. In particular, we come back on the assumption that the quantity of information conveyed about the opponent is involuntary, by making explicit the choice of speaking about the opponents’ position. We also discuss the case where parties are not constrained to send unbiased signals about their position.

2.3 Voters’ treatment of information

Using the signals received during the campaign, voters update their beliefs regarding the parties’ platforms. Consider a party, say A. An identical analysis holds for B.

Each voter \( i \) receives signals \( y_i \) from the candidates, and she also perceives the time spent by each candidate \( t = (t_A, t_B) \in [0, 1]^2 \). What are the voter’s posterior beliefs regarding party A’s position, after the reception of these signals? Her posterior on party A’s true position follows \( \mathcal{N} (\hat{x}_A (y_i, t), \hat{s}_A^2 (t)) \), where:

\[
\frac{1}{\hat{s}_A^2 (t)} = \frac{1}{s_A^2} + \frac{1}{\sigma_A^2 (t)} \left( \frac{t_A + \rho t_B}{1 + \rho} \right),
\]

\[
\hat{x}_A (y_i, t) = \frac{\sigma_A^2 (t)}{s_A^2} \left[ m_A + \frac{y_i}{\sigma_A^2 (t)} \left( \frac{t_A + \rho t_B}{1 + \rho} \right) \right].
\]

Such a voter with bliss point \( x_i \) gets the expected utility if A is elected:

\[
U_A (y_i, t, x_i) = - \left[ (\hat{x}_A (y_i, t) - x_i)^2 + \hat{s}_A^2 (t) \right]. \tag{2}
\]

The level \( U_B (y_i, t, x_i) \) is similarly defined for party B.

Note that voters are assumed to be naive (although they are Bayesian), in the sense that they take at face values the messages sent by parties. They do not interpret the messages as stemming from parties’ strategies. For example, when the effective time spent discussing party A’s position is zero \( \left( \frac{t_A + \rho t_B}{1 + \rho} = 0 \right) \), the voter’s a posteriori beliefs regarding party A’s position coincide with his a priori beliefs. She does not interpret the fact that if a candidate does not talk about an issue, it might be because he has no incentive to do so. Section 6 considers the case of more sophisticated voters. The timing is summarized in Figure 1.
2.4 Voters’ behavior: the probabilistic voting model.

We assume the following simple version of a probabilistic voting model, following Persson and Tabellini (2000).\(^2\) Candidates not only differ with respect to the policy platforms that they put forward, but they also differ in some other dimension, unrelated to the policy issue at stake, and that parties do not influence through the campaign stage. It may involve some other attributes of the candidates, such as personal characteristics (gender, race, age, ...), on which voters also have preferences. Assume that voter \(i\) votes for party \(A\) iff

\[
U_A(y_i, t, x_i) - U_B(y_i, t, x_i) > \sigma_i
\]

where \(\sigma_i\) is an individual-specific bias in favor of candidate \(B\). Individual biases are taken to be i.i.d, with a uniform distribution on \([-\frac{1}{2\phi}, \frac{1}{2\phi}]\). We assume that the support of the distribution is wide enough so that whatever \((y_i, t, x_i)\),

\[
U_A(y_i, t, x_i) - U_B(y_i, t, x_i) \in \left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right].
\]

Parties know the distribution of these biases, but they do not know their realized values for each individual, at the time they have to choose their emphasis strategies. Conditional on receiving signals \(y_i\), the probability that voter \(i\) votes for \(A\) is

\[
\frac{1}{2} + \phi [U_A(y_i, t, x_i) - U_B(y_i, t, x_i)].
\]

Thus the probability that this voter votes for \(A\) conditional on \(t\) (before the reception of the signals) is given by the expectation of this expression over the signals. Thus denoting

\[
U_J(t, x_i) = E[U_J(y_i, t, x_i)]
\]

the probability that \(i\) votes for \(A\) is equal to

\[
\frac{1}{2} + \phi [U_A(t, x_i) - U_B(t, x_i)]
\]

\(^2\)Considering individuals’ shocks on preferences that are independent on preferences on platforms is known as the ”probabilistic voting game”. This was introduced in part to solve the existence problem in the standard model where purely office motivated parties choose their platforms. (See Coughlin (1983), Lindbeck and Weibull (1993)).
Now, in the probabilistic model as considered here, the expected vote share for a party only depend on the expectation of the probability of votes over the electorate.

Hence, taking the average of (4) over the electorate yields the expected vote share for party \( A \) as

\[
\frac{1}{2} + \phi [U_A(t) - U_B(t)]
\]

where

\[
U_J(t) = \int_u U_J(t, x) f(x) dx
\]

The game is analyzed backward. We have seen how voters vote in function of the received signals and the parties efforts. Recall however that parties do not control perfectly the signals. Their decisions to speak are taken on the basis of the vote shares that they expect. Before going to the equilibrium analysis, we first examine in some detail how the uncertainty in the signals affects these expected shares.

3 Impact of the campaign on parties’ expected vote shares

To analyze more closely how efforts (time) affect the vote shares that are expected by the parties, we need to evaluate voter’s expected utility for \( A \) being elected without knowing their signals, i.e. we need to evaluate (3).

Consider expression (2), which gives the value of \( U_A(y_i, t, x_i) \) upon the receipt on signals \( y_i \). Note that without campaign, this voter achieves the expected utility \(-[(m_A - x_i)^2 + s_A^2]\) if \( A \) is elected. The difference in expected utility if \( A \) gets elected after and before the campaign is:

\[
- \left[ (x_i - \hat{x}_A(y_i, t))^2 + (x_i - m_A)^2 \right] + \left[ s_A^2 - \hat{s}_A^2(t) \right].
\]

The precision of information on \( A \)’s position has two effects on the expected utility of \( A \) being elected. A first effect is to reduce uncertainty as measured by the positive term \( s_A^2 - \hat{s}_A^2(t) \), which is unambiguously favorable to \( A \). A second effect is a change the perception on \( A \)’s position from the prior \( m_A \) to the posterior \( \hat{x}_A(y_i, t) \). Hence the perception moves towards a combination of the signal \( y_i,A \) and the prior \( m_A \), which is on average towards the true position \( x_A \). This move may or may not be beneficial to \( A \) depending on the position of the bliss point. But note that both cases occur due to noisy signals.

\[3\]What matters for a candidate is his estimation of the number of votes. Hence the game is identical whether signals are identical or conditionally independent across voters (or more generally correlated). This result is due to our probabilistic setting and does not extend to a deterministic one. Without perturbation in preferences, individual \( i \) votes for \( A \) if \( U_A(y_i, t, x_i) - U_B(y_i, t, x_i) > 0 \). If signals are independent (and independent of the preferences \( x_i \)) the number of votes is independent of the sample of the signals \( y_i \) (provided that a law of large numbers approximately applies). Hence the impact of a speech is deterministic. If on the other hand signals are identical across voters, the impact of the speech is random because it depends on the realized common signal.
Given \( t \), the posterior value \( \hat{x}_A(y_i, t) \) is normally distributed, hence \( U_A(y_i, t, x_i) \) follows a mixture of a normal and the square of a normal variables. An important point is that the uncertainty in the posterior value \( \hat{x}_A \) lowers the expectation of \( U_A \), which decreases the chances that a voter votes for \( A \). To see this, taking expectation over the signals \( y_i \) conditional on \( t = (t_A, t_B) \), we have

\[
U_A(t, x_i) = -E\left[(\hat{x}_A - x_i)^2 + \hat{s}_A^2\right] = -\left[\left(E(\hat{x}_A|t) - x_i\right)^2 + var(\hat{x}_A|t) + \hat{s}_A^2(t)\right].
\]

Hence, the uncertainty in the impact of speeches lowers the expected utility for \( A \) by the variance of the posterior \( \hat{x}_A \). This is due to the concavity in the utility function (and is not related to risk aversion).\(^4\) More precisely, the benefit from a favorable shock in the signals, that is a shock that decreases the distance between the expected position of the party to the bliss point, is lower than the loss incurred from a shock of the same magnitude in the opposite direction.

Taking the average over the electorate yields the expected utility for \( A \) being elected. That is:

\[
U_A(t) = -\left[\left(E(\hat{x}_A|t) - \bar{x}\right)^2 + var(\hat{x}_A|t) + \hat{s}_A^2(t)\right] + \left[\bar{x}^2 - \bar{x}\right]
\]

where \( \bar{x} = \int x f(x)dx \), and \( \bar{x}^2 = \int x^2 f(x)dx \).

To go further, it is convenient to introduce a measure of the gain in the precision on party \( A \)'s position. Given total time \( t \in [0, 1] \) spent on issue \( A \) let us define

\[
H_A(t) = \frac{s_A^2}{s_A^2 + \sigma_A^2(t)}
\]

and

\[
h_A(t) = H_A \left(\frac{\rho t_B}{1 + \rho}\right).
\]

Note that \( h_A(0) = 0 \). With this notation,

\[
E(\hat{x}_A|t) = m_A + h_A(t) (x_A - m_A)
\]

\[
var(\hat{x}_A|t) = s_A^2 (1 - h_A(t)) h_A(t)
\]

\[
\hat{s}_A^2(t) = s_A^2 (1 - h_A(t)).
\]

Note that \( var(\hat{x}_A|t) \) is null whenever no information is conveyed (\( h_A(t) = 0 \)) - in which case all voters share the same a priori beliefs on party \( A \)'s platform, or

\(^4\)To see this note that \( u_i \) is not a VNM utility function. Consider a simple form of risk aversion represented by a mean variance criteria \( E[u_i] - \beta var(u_i) \) where \( \beta \) is the weight on the variance. The expected utility level (2) is now replaced by multiplying the term \( \hat{s}_A^2(t) \) by \( (1 + \beta u_i) \). Taking the party’s point of view, the expectation is

\[
-\alpha, \left[(x_i - E\hat{x}_A|t)^2 + var(\hat{x}_A|t) + (1 + \beta u_i)\hat{s}_A^2(t)\right].
\]
when full information is conveyed \((h_A(t) = 1)\) - in which case all voters share the same a posteriori beliefs on party A’s platform: they all know the true value of \(x_A\). This variance is maximal for \(h_A(t) = \frac{1}{2}\).

\[
U_A(t) = -[m_A + h_A(t)(x_A - m_A) - \bar{x}]^2 - [s^2_A(1 - h_A^2(t))] + \left[\bar{x}^2 - \bar{x}^2\right]
\]  

(6)

The value of \(U_A(t)\) depends on the strategies of both parties through the precision of the information as reflected by \(h_A(t)\). For the sequel, it is worth analyzing this expression as a function of \(h_A\), where \(h_A\) varies in an interval \([0, 1]\).

The first term in \(U_A(t)\) in the summation of (6),

\[-[m_A + h_A(t)(x_A - m_A) - \bar{x}]^2 = -[E(\hat{x}_A(t)) - \bar{x}]^2,
\]

results from the change in the average expected value of \(x_A\) in the electorate; this term is maximal when \(h_A\) is such that \(E(\hat{x}_A)\) is made as close as possible to \(\bar{x}\). More precisely, suppose without loss of generality that \(m_A \leq x\). In that case this term is decreasing in \(h_A\) whenever \(x_A \leq m_A\); indeed, giving information about party A’s platform in that case moves \(E(\hat{x}_A)\) away from \(m_A\) in the direction of \(x_A\), thus further away from the target \(\bar{x}\). On the contrary, this term is increasing in \(h_A\) whenever \(m_A \leq x_A \leq \bar{x}\), since giving information about party A’s platform in that case unambiguously moves \(E(\hat{x}_A)\) in the direction of \(\bar{x}\). When \(m_A < \bar{x} \leq x_A\), this term is first increasing in \(h_A\), up to a threshold value \(h_A = \frac{(\bar{x} - m_A)}{(x_A - m_A)} \in [0, 1]\) where there is perfect coincidence between \(E(\hat{x}_A)\) and the target \(\bar{x}\), and is then decreasing in \(h_A\). Note that the marginal benefit from increasing the precision \(h_A\) on the party’s position is always decreasing.

The second term in \(U_A(t)\)

\[-s^2_A(1 - h_A^2(t))\]

results from the change in the variance of the posterior \(\hat{x}_A\) in the electorate and the decrease in the uncertainty of party A’s platform as perceived by voters. Overall, this second term is increasing in \(h_A\), with increasing marginal benefit.

Now, the total effect on \(U_A\) of an increase of the precision \(h_A\) depends on the sign and of the strength of the two effects detailed above. Let us label the effect resulting from the first term "the average position effect" - which can be either positive of negative, and the effect resulting from the second term "the reduced variance effect" - which is always positive.

For the sequel, it is useful to introduce some further notation: Let us denote by \(P_A\) the marginal benefit of the first unit of precision:

\[P_A = (x_A - m_A)(\bar{x} - m_A);\]  

(7)

it is positive whenever \(x_A\) and \(\bar{x}\) are located on the same side of \(m_A\). Indeed, for this first unit of precision, the reduced variance effect is null, and the average
position effect is positive whenever talking moves the posterior beliefs in direction of the average ideal policy \( \bar{\pi} \). We shall say that the position is favorable if \( P_A \) is positive.

As we have seen, the marginal benefit from increasing the precision on the party’s position is decreasing for the position effect and increasing for the variance reduction effect. The over-all effect depends on

\[
Q_A = s_A^2 - (x_A - m_A)^2.
\]

When it is positive, the marginal benefit from increasing the precision increases; this occurs when the true position is not too far from the prior value, less than one standard error and we say that the issue is in standard position. It is decreasing when \( Q_A \) is negative; in that case we say that the issue is in non standard position. With this notation

\[
U_A(t) = U_A(0) + h_A(t) [2P_A + Q_A h_A(t)].
\]

To sum up, in the case \( m_A < \bar{x} \):

<table>
<thead>
<tr>
<th>Effect of ( h_A \in [0,1] )</th>
<th>( x_A \leq m_A )</th>
<th>( m_A \leq x_A \leq \bar{x} )</th>
<th>( \bar{x} &lt; x_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_A \leq 0 )</td>
<td>( h_A = 0 )</td>
<td>( h_A = 1 )</td>
<td>( h_A = \frac{\pi - m_A}{(x_A - m_A)} )</td>
</tr>
<tr>
<td>( P_A &gt; 0 )</td>
<td>( E(\hat{x}_A) = m_A )</td>
<td>( E(\hat{x}_A) = x_A )</td>
<td>( E(\hat{x}_A) = \bar{x} )</td>
</tr>
<tr>
<td>Average position maximal for with resulting ( E(\hat{x}_A) )</td>
<td>( \nabla ) in ( h_A )</td>
<td>( \nabla ) in ( h_A )</td>
<td>( \nabla ) then ( \nabla ) in ( h_A )</td>
</tr>
<tr>
<td>Reduced variance Total</td>
<td>( \nabla ) in ( h_A )</td>
<td>( \nabla ) in ( h_A )</td>
<td>( \nabla ) in ( h_A )</td>
</tr>
</tbody>
</table>

4 Equilibria

Recall that party A’s expected vote share is an affine increasing function of \( U_A(t) - U_B(t) \) as given by (3). We shall denote by \( \pi \) this difference. Using the expression (9) of \( U_A(t) \) (a similar expression holds for \( B \)) we obtain

\[
\pi(t) = U_A(t) - U_B(t) = h_A(t) [2P_A + Q_A h_A(t)] - h_B(t) [2P_B + Q_B h_B(t)].
\]

Parties are playing a zero-sum game with criterion \( \pi \). At some places, we shall use a linearity assumption that simplifies the analysis.

Definition Let us say that precision measure \( H_J \) is linear if \( H_J(t) = a_J \) for a positive scalar \( a_J \), with \( a_J \leq 1 \).

4.1 Equilibria for \( \rho = 0 \).

Without interaction in speeches, \( U_J \) depends only on \( t_J \). Equivalently, a party actually chooses the precision \( h_J \). As noted in section above, \( h_A \) affects \( U_A \) through the "the average position effect" and the "reduced variance effect".

\[5\]Linearity of \( H_J \) is satisfied if \( \frac{1}{(\sigma_j)^2(t)} = \frac{1}{1-a_Jt} - 1.\]
Denote by \( a \) the **maximal reachable precision** that candidate \( A \) can reach when he talks full time. When \( a \) is smaller than 1, **full precision** (\( h_A = 1 \)) is not reachable. Party \( A \) can choose any precision \( h_A \) in \([0,a]\).

**Proposition 1** The table below presents the optimal time speech, depending on the value of \( x_A \), as well as the resulting average perception on \( A \)’s position \( E(\hat{x}_A) \). The table presents results for the case \( m_A \leq \pi \).

<table>
<thead>
<tr>
<th>( x_A )</th>
<th>( c_1 &lt; x_A &lt; c_2 )</th>
<th>( c_2 &lt; x_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal ( h_A )</td>
<td>0</td>
<td>( a )</td>
</tr>
<tr>
<td>( E(\hat{x}_A</td>
<td>t_A) )</td>
<td>( m_A )</td>
</tr>
</tbody>
</table>

where

\[
c_1 = m_A + \left( \frac{\pi - m_A}{a} \right) - \sqrt{s_A^2 + \left( \frac{\pi - m_A}{a} \right)^2} < m_A,
\]

\[
c_2 = m_A + \left( \frac{\pi - m_A}{2a} \right) + \sqrt{s_A^2 + \left( \frac{\pi - m_A}{2a} \right)^2} > \pi
\]

**Comments.**

**Optimal communication strategies.** In the discussion that follows, we shall assume that \( m_A \leq \pi \). Also to keep the the comments simpler, we assume that full precision is reachable (\( a = 1 \)). The optimal strategy solves the trade-off between the position effect and the reduced variance effect when they are in conflict.

When the position is not favorable, \( (P_A < 0) \), here when the party’s true position \( x_A \) is below the prior \( m_A \), the optimal strategy for the party is to remain silent except if the reduced variance effect is dominant and the marginal incentives to speak are increasing (that is \( Q_A > 0 \)): this gives the first threshold value \( c_1 \) and explains why full speech is then optimal. (For this reason, when the maximal precision \( a \) is lower, the party may prefer not to speak at all for some of these positions: this also explains why the threshold increases with \( a \)).

When the position is favorable, \( (P_A > 0) \), here when the party’s true position \( x_A \) is above the prior \( m_A \), the optimal strategy for the party is to speak. It speaks full time and reveals its true position when the position is moderate enough, (that is when \( x_A \) is below a second threshold \( c_2 \) which is larger than \( \pi \)). This is clearly optimal when \( m_A \leq x_A \leq \pi \) since the average position effect and the reduced variance effect both play in the same direction. It is less clear when \( \pi \leq x_A \); if the party was only concerned with the average position effect, it would adjust its time speech so that the average posterior beliefs about its position exactly matches the average ideal position in the electorate \( \pi \): it would not speak full time. Now, the reduced variance effect induces it to speak full time instead when the position effect is not too detrimental that is when \( x_A \) is smaller than the second threshold \( c_2 \). There is some ’overshooting’, in the sense than during the campaign, the party moves from a prior value below the median \( (m_A \leq \pi) \) to a posterior above the median \( (E(x_A) > \pi) \). When the party’s true position is more extreme, \( x_A \) is above the threshold \( c_2 \), the optimal
strategy for the party is to talk only part time. Again, if the candidate was only concerned with the average position effect, he would match the median; the reduced variance effect induces an increase in time speech, but not to a point where full time speech is reached, so that the posterior value is above the median (there is some 'overshooting'). The optimal time speech is decreasing with $x_A$ and tends to zero as the position gets infinitely extreme.

A similar analysis holds when $m_A \geq \bar{x}$. The situation where a party’s prior coincides with the median ($d_A = 0$) is rather special since a speech can only deteriorate the perceived position and the only motive is to reduce uncertainty. The party speaks full time when its position is less than one standard error from the prior (which is also the median) and does not speak otherwise. To sum up a party strategy, a party speaks when its position is favorable or when its position, although not favorable, is close enough to the prior allowing for a reduction in voters’ uncertainty without too much impact on the posterior.

Consider now the average perception on party $A$’s platform. First, not surprisingly, there is a bias towards the median: The posterior is always closer to the median than the true position. Secondly, the perception is not monotonic with respect to the true position. In particular, there is a downward discontinuity in $c_1$ as the strategy jumps from silence to full speech and the average posterior decreases with the extremism of the party (for positions above $c_2$) and converges to the median average position $\bar{x}$ when the party gets infinitely extreme. This behavior is due to the assumption of voters ‘naively’ updating their beliefs and will be discussed in Section 6.

To go further it is convenient to normalize the variables, specifically to work on the deviations from the prior in terms of standard error. Define $e_A = \frac{x_A - m_A}{s_A}$ and $d_A = \frac{x - m_A}{s_A}$. A party may be said to be ‘far’ or ‘close’ to the median by looking at the values for $d_A$. For example, the chances for candidate $A$’s position to be at the median are very small when $d_A$ is larger than two. The incentives to speak depend only on the normalized variables.

For $d_A = 0$, the party speaks full time when its position is less than one standard error from the prior (which is also the median) and does not speak otherwise, i.e. $|e_A|$. For $d_A > 0$ a party speaks full time for $d_A - \sqrt{1 + d_A^2} \leq e_A \leq d_A + \sqrt{1 + d_A^2}$, does not speak for $e_A$ smaller than the lower bound, and speaks part time for $e_A$ larger than the upper bound. Hence, for $d_A > 0$, the probability that a party talks is equal to $F[-d_A + \sqrt{1 + d_A^2}]$ where $F$ denotes the cumulative distribution of a standard normal variable. This probability decreases with $d_A$ from 0.84 to 1/2. Thus, the chances for a party to talk are lower the further away from the median it is a priori (the larger $x - m_A$) and the smaller the uncertainty on their position (the smaller $s_A$).

For $d_A = 0$, the prior is at the median, and the probability that a party talks is 0.68 : there is a jump upward to 0.84 because a large set of position becomes favorable when the prior is not at the median.

When full precision cannot be reached, the same expression for the probability of speaking holds by replacing $d_A$ by $d_A/a$. As a result, lowering the maximal precision (or lowering the available time) decreases the chances that a
party speaks. This is explained by the fact that speaking when the position is standard but unfavorable becomes less attractive.

**Dialogue.** From this simple single issue case, it is clear that it is perfectly possible that both parties speak on the issue, a situation referred to as 'dialogue' by Simon (2002). They do so if speaking is favorable to both. (Such a situation is perfectly possible since the condition of favorability relates to the party’s positions -prior and true one- relative to the median.) They may also do so if the perceived uncertainty on their position is large. The motive for uncertainty reduction is more likely to prevail for a challenger (whose position is unknown) or on a new issue at stake. Note that it is also possible that no party talks about it.

From the computation above the probability that both parties engage in dialogue is at least $\frac{1}{4}$, and no more than $0.70$. It is also interesting to assess this probability as a function of the prior and median position. In what follows $A$ is always taken ‘on the left’ of $B$ that is $m_A \leq m_B$. Let us first interpret the prior positions in terms of divisive or congruent issues. Let us say that an issue is ‘divisive’ if the chances for the parties’ position to coincide are almost null. This occurs when $m_B - m_A$ is large compared to the standard error of $x_B - x_A$. Let us denote by $d$ the ratio $d = \frac{m_B - m_A}{\sqrt{s_A^2 + s_B^2}}$. A natural illustration of such a situation is when the prior values $m_A$ and $m_B$ are each on one side, and each far apart from the median, say for $d$-values larger than 2. Then the chances of dialogue are rather small, close to their minimum $\frac{1}{4}$. At the opposite when both parties are close to the median, the chances are rather large, close to their maximum $0.70$.

When both parties are on the same side and far from the median, parties agree between themselves but disagree with the electorate. This was the case for example for the European Union issue in the 2007 French presidential election. The same computation as for a divisive issue applies (since strategies do not depend at all on the relative positions of the parties): Parties do not speak much.

### 4.2 Equilibria for $\rho = 1$

Under full leakage, both criteria $U_A$ and $U_B$, and hence $\pi$, depend on $t = (t_A, t_B)$ only through the total time spent discussing each party’s position $\frac{t_A + t_B}{2}$. Denote by $\Pi(t)$:

$$
\Pi(t) = 2P_A H_A(t) + Q_A H_A^2(t) - 2P_B H_B(t) - Q_B H_B^2(t)
$$

where $t = \frac{t_A + t_B}{2} \in [0, 1]$.

**Pure strategies equilibrium.** The impact of a speech does not depend on which party is speaking. Hence if a party strictly benefits from an additional quantity of speech, its opponent is made strictly worse off by it. This property has strong implications on an equilibrium in pure strategies, provided that the payoff is not locally constant. It implies that a pure equilibrium is formed with
'corner' strategies, either (1, 0) or (0, 1), as stated in Proposition 2. Assuming \( \Pi \) to be locally non-constant is almost always satisfied when full precision \( (H_A = H_B = 1) \) is not reachable (unless maybe if both parties talk full time). Indeed in that case each precision is strictly increasing over \([0, 1]\), and \( \Pi \) can be locally constant only in very specific cases.

**Proposition 2** Let \( \rho = 1 \). Assume that \( \Pi(t) \) is not locally constant on \([0, 1]\).

At an equilibrium in pure strategies (assuming it exists), one candidate talks full time whereas the other remains silent.

The proof is given in the appendix because some care is needed at points with null derivative for \( \Pi \).

Proposition 2 fails when \( \Pi \) is locally constant. Assume that full precision is reached as soon as one party talks full time (so that \( \Pi \) is constant over \([1/2, 1]\)), then both parties talking full time gives (trivially) an equilibrium. One may further check that at any equilibrium, there is perfect revelation on both parties' platforms.

An equilibrium in pure strategies may fail to exist. To understand why it may be so, consider a situation where: \( \Pi(0) < \Pi(1/2) \) and \( \Pi(1) < \Pi(1/2) \). Party A's vote share is higher with partial information when a single party talks than with no information at all or than with maximal revelation of information. In that case, we are in a matching pennies game (focusing on the strategies 0 or 1 since we know that there are the only one to be played at a pure equilibrium): Party B is always better off by matching what party A does, speaking full time rather than remaining silent when A speaks and remaining silent rather than speaking when A is silent. There is no equilibrium in pure strategies.

Note that such a situation may occur. For example, suppose that there is very little uncertainty on party B's position \( (s_B \) close to zero, it may be so because B is the incumbent) and party B's a priori position as well as true position coincide with the average position in the electorate \( (m_B = x_B = \mu) \). In that case, speaking on the issue will have no average position effect on B, and very little variance effect (since \( s_B \) is close to zero). Speaking (by either party) will only affect the probability on winning the election through party A's average position effect and variance effect (equivalently, the payoff of the game moves as \( U_A \) since \( U_B \) is almost constant). As has been shown in the case \( \rho = 0 \), when party A's average position effect and variance effect play in opposite directions, some intermediate value of precision may be optimal for party A's vote share, and it may be the case that both inequalities \( \Pi(0) < \Pi(1/2) \) and \( \Pi(1) < \Pi(1/2) \) simultaneously hold.

To go further in characterizing condition under which a pure equilibrium exists, we consider the special case in which precision indices are linear. Denote by \( a \) and \( b \) the respective slopes of \( H_A \) and \( H_B \). Under the linearity of the
precision indices, we have
\[ H_A(t) = at, a \leq 1; \quad H_B(t) = bt, b \leq 1, \]
and \( \Pi(t) = t \left[ 2(aP_A - bP_B) + (a^2Q_A - b^2Q_B) t \right], \)
where \( t = \frac{t_A + t_B}{2} \in [0,1]. \)

Note that the criterion \( \Pi \) is either concave or convex in \( t \). This simplifies the conditions under which a pure equilibrium exists. In general, strategies \((1,0)\) is an equilibrium in the simultaneous game iff \( \Pi(t) \leq \Pi(1/2) \) for all \( t \in [0,1/2] \) (party A has no profitable deviation) and \( \Pi(t) \geq \Pi(1/2) \) for all \( t \in [1/2,1] \) (party B has no profitable deviation). Since \( \Pi \) is either concave or convex in \( t \), under the linear precision indices assumption, these conditions are equivalent to \( \Pi(0) \leq \Pi(1/2) \leq \Pi(1) \). Similarly, \((0,1)\) is an equilibrium iff \( \Pi(0) \geq \Pi(1/2) \geq \Pi(1) \).

One may want to go further and obtain quantitative indications as to when an equilibrium in pure strategies may fail to exist. To get closed-form formula, let us further assume that candidates are a priori completely symmetric (with the normalization \( \tau = 0 \):)

\[ -m_A = m_B = m \geq 0, s_A = s_B = s > 0, a = b \in [0,1]. \]

In that case, straightforward computation shows that there is no equilibrium in pure strategies if and only if:

\[ -\frac{2}{a} d < e < -\frac{2}{3a} d, \]

where \( e = \frac{x_B - x_A - 2m}{\sqrt{2} s} \) is normally distributed with mean 0 and variance 1, and \( d = \sqrt{2m} \) is the ratio of \( m_B - m_A \) over the standard error of \( x_B - x_A \) (this index \( d \) of division between parties was introduced in the previous subsection). For example, the probability for no equilibrium in pure strategies when \( a = 1 \) and \( d = 2 \) is 9.1%.

The probability that no equilibrium in pure strategies exists is bell-shaped with the congruence of parties: starting from 0, it is increasing in \( d \) up to some critical value - its maximal value is then 0.24 - and it is decreasing beyond this point, converging towards 0 as \( d \) gets infinitely large. In particular, it is very small when both parties are very close to the median or when the issue is very divisive.

When the simultaneous game does not admit an equilibrium in pure strategies, an equilibrium in mixed strategies exists. We find it more interesting to look instead at a game in which parties move sequentially.

**Sequential version of the game.** Let us consider the game in which parties play sequentially. The party that plays first, say \( A \), is the 'first mover'. The 'follower', \( B \), observes the strategy chosen by \( A \) before choosing its own time speech: its strategy is a reaction function. There is no bad connotation in the term 'follower'. In political life, the incumbent is likely to be the follower. In
fact, recall that, in two players zero-sum game, it is never a (strict) advantage to move first.

An important property is that an equilibrium in pure strategies remains an equilibrium in the sequential version. This implies that the order of play has no consequence when an equilibrium in pure strategies exists. It remains to study the situation where there is no pure equilibrium. Under the linear assumption, this happens whenever $\Pi(1/2) < \min(\Pi(0), \Pi(1))$ or $\Pi(1/2) > \max(\Pi(0), \Pi(1))$. An equilibrium is 'computed' by backward induction.

Proposition 3 Let precisions $H_J(t)$ be linear with respect to $t$ for each party. At an equilibrium in the sequential game with $A$ as first mover,

1. A necessary condition for having both parties talk is that $\Pi(1/2) > \max(\Pi(0), \Pi(1))$. In that case, $A$ chooses the value $t_A \in [0, 1]$ that makes $B$ indifferent between not talking at all and talking full time.

2. Whenever this condition does not hold, one party talks full time whereas the other remains silent.

The proof is presented in the appendix.

4.3 Welfare analysis

The analysis in the previous subsections showed that the campaign technology (resumed by the parameter $\rho$) has important consequences regarding the qualitative properties of parties’ communication strategies. We now briefly discuss these consequences in terms of voters’ welfare. We do not provide a full welfare analysis here, but simply want to underline some a priori counter-intuitive properties of an electoral campaign. A simple example shows that although parties convey unbiased information, electoral campaign may prove to be detrimental to voters, in the sense that voters’ welfare would be higher with no information at all, than with the electoral conveyed at equilibrium during the campaign.

4.3.1 Definition of voters’ welfare

We use an ex ante utilitarian criterion to assess welfare. Let $p_J(x, t)$ denote the probability that $J$ wins the election, given true platforms $x = (x_A, x_B)$ and the emphasis $t = (t_A, t_B)$.

What is the ex ante voters’ welfare when the emphasis strategies are $t = (t_A, t_B)$ and campaign technology $\rho$? The expected utility of voter $i$ is

$$p_A(x, t)u_i(x_A) + p_B(x, t)u_i(x_B)$$

$^6$This is due to the fact that the minmax is equal to the maxmin at an equilibrium. More precisely a pure equilibrium $(t_A^*, t_B^*)$ is characterized by $\pi(t_A^*, t_B) \geq \pi(t_A^*, t_B^) \geq \pi(t_A, t_B^*)$ for any strategies $t_A, t_B$. By choosing $t_A^*$. $A$ obtains $\pi(t_A^*, t_B^*)$ since $t_B^*$ is a best response to $t_A^*$. By deviating to another strategy $t_A$, player $B$ is sure to be as well off by sticking to $t_B^*$ (and may possibly be better off).
Hence the ex ante welfare criteria is
\[
W(t) = E_{x_A, x_B}[p_A(x, t)(- (x - x_A)^2 + (x - x_B)^2) - \int_x (x - x_B)^2 f(x)dx]
\]

We suppose that the probability that A wins the election is an affine function of party A’s expected vote share \( \pi(t) \): \( p_A = \gamma + \beta \pi \). For example, following Persson and Tabellini (2000), we assume that an additive uniformly distributed macro random shock occurs after parties have decided their emphasis strategies. Hence the ex ante welfare criterium is
\[
W(t) = W(0) + \beta E_{x_A, x_B} \left[ (2A^2h_A^2(t) + 2B^2h_B^2(t)) \left( (x - x_B)^2 - (x - x_A)^2 \right) \right].
\]

4.3.2 The positive value of unconditional information

Consider first as a benchmark the case where the communication strategies are independent of the true positions \( x_A, x_B \), that is, the communication by the parties \( t_A, t_B \) are fixed ex ante. In that case, some simple computation yields:
\[
W(t) = W(0) + 2\beta \sum_{J=A,B} \left[ 2s_J^2(\pi - m_J)^2h_J(t) + s_J^4h_J^2(t) \right].
\]

Therefore the welfare is increasing and convex in \( h_A \) and \( h_B \). When the precision conveyed is independent from the positions \( x_A, x_B \), more precision is always valuable.

Now, to assess welfare in the electoral campaign game, one needs to replace in the expression (11) above the emphasis vector \( t \) by the equilibrium strategies computed in the previous subsections.

4.3.3 An example of welfare reducing campaigning

We show in a very simple example that an electoral campaign may be detrimental to voters’ welfare. Consider the special case where both parties’ a priori positions coincide with the average bliss point in the electorate (normalize \( \pi = 0 \)), and precisions are linear:
\[
m_A = m_B = \pi = 0, h_A = \frac{t_A + \rho t_B}{1 + \rho}, h_B = \frac{t_B + \rho t_A}{1 + \rho}.
\]

We restrict attention to the two polar cases \( \rho = 0 \) and \( \rho = 1 \). When \( \rho = 0 \), party \( J \) talks full time when \( x^2_J \leq s^2_J \), and remains silent in all other cases. When \( \rho = 1 \), there is always an equilibrium in pure strategies, where one party talks full time and the other remains silent (as seen in subsection 4.2.).

It can be shown in that case that when \( \rho = 0 \),
\[
W = W(0) + \beta \left[ 2s_A^2s_B^2 (C_0 - C_2) - (s_A^4 + s_B^4) (C_2 - C_4) \right],
\]
where $C_n = \int_{y^2 < 1} y^n f(y) dy$; and whereas when $\rho = 1$,

$$W = W(0) + \frac{\beta}{2} (s_A^4 + s_B^4)$$

Comments:

(1) Note first that an electoral campaign may be detrimental to the voters’ welfare when $\rho = 0$. For example, if $s_A$ is small enough (given a value for $s_B$), voters would be better off with no campaign. It might sound counter-intuitive, since during the campaign, only unbiased information is conveyed to the electorate. The reason is that the candidate who has the more incentives to talk - and benefits the more from the campaign - might not be the one who is the better for the electorate (that is, whose position is closer to the average voter).

(2) Note that when $\rho = 1$, the campaign always improves welfare, whatever the variances $s_A, s_B$. It stems from the fact that at equilibrium, one and only one candidate talks full-time, whatever the positions $(x_A, x_B)$. In that case, the quantity of information on each party’s position is independent of the positions $(x_A, x_B)$, which is necessarily beneficial, as has been seen in the previous subsection.

(3) For any $s_A, s_B$, voters are better off when $\rho = 1$ than when $\rho = 0$.

5 Multi-issue electoral campaign

We have so far considered a single issue. In the case of multiple issues, one interesting question is not only whether a candidate benefits from speaking on an issue but also which issue he selects, especially when he faces time constraints.

5.1 The model

There are $K$ relevant issues a priori. The one-dimensional model extends to a multidimensional policy space, $X = R^K$, as follows. The timing is as before. Before the electoral campaign starts, parties have fixed their platforms and voters share the same a priori beliefs on these platforms. Parties allocate their time across issues, which modifies the prior of the voters. Voting take place and vote shares are determined by a probabilistic model. Let us describe the assumptions and notation.

Denote by $x_A = (x_{A1}, x_{A2}, ..., x_{AK})$ party $A$’s platform and similarly for $B$. Voters’ a priori on $x_A$ and $x_B$ are independent across parties and across issues. Voter’s a priori on $x_{Ak}$ follows a normal distribution $\mathcal{N}(m_{Ak}, (s_{Ak})^2)$ and similarly $x_{Bk} \sim \mathcal{N}(m_{Bk}, (s_{Bk})^2)$.

\footnote{Indeed, one may show that since $C_2 = C_0 - \sqrt{2 \pi}$ and $C_4 = 3C_2 - \sqrt{2 \pi} = 3C_0 - 4 \sqrt{2 \pi}$, the welfare when $\rho = 0$ also writes: $W = W(0) + \beta \left[2C_2 (s_A^4 + s_B^4) - \sqrt{2 \pi} (s_A^4 - s_B^4)^2\right]$, with $2C_2 < 1/2$.}
Voter $i$’s utility is represented by (the opposite of) a weighted distance to a bliss point: if policy $x = (x^1, x^2, ..., x^K) \in X$ is implemented is

$$u_i(x) = -\sum_k \alpha_k^i (x^k - x^k_i)^2,$$

where $x_i = (x^1_i, x^2_i, ..., x^K_i)$ is voter $i$’s bliss point in $X$ and $\alpha_k^i \geq 0$ is the salience of issue $k$ for voter $i$. The parameters $(\alpha^k, x^k)$ are distributed with density $f_k(\alpha^k, x^k)$.

The strategy set. Each party chooses the importance it wants to devote to each issue: $t = (t^1, t^2, ..., t^K) \in T$. In the absence of a global time constraint, the strategy space for a party writes $T = [0,1]^K$, up to a normalization.

When parties are limited by a global time constraint, the strategy space is instead $T = \{ (t^1, t^2, ..., t^K) \in [0, T]^K \text{ and } \sum_k t_k \leq T \}$, where $T$ is the total time available.

Denote by $t$ the pair of strategies: $t = (t_A, t_B)$. Signals on a particular issue are described as for a single issue: they are noisy, unbiased, with a precision that depends on the time spent by both parties on that issue. For $t_j \in T$, when party $J$ spends time $t^k_j$ speaking on issue $k$, voter $i$ receives an imperfect signal on party $A$’s true position on this issue $y^k_{i,A}$, and an imperfect signal on party $B$’s true position $y^k_{i,B}$. A single parameter $\rho$ summarizes the leakage in the speeches on the opponents’ strategies. Signals $y^k_{i,A}, y^k_{i,B}$ normally distributed, with a variance that depends on the time spent:

$$y^k_{i,A} \sim N\left(x^k_A, (\sigma^k_A)^2 \left(\frac{t^k_A + \rho t^k_B}{1 + \rho}\right)\right), y^k_{i,B} \sim N\left(x^k_B, (\sigma^k_B)^2 \left(\frac{t^k_B + \rho t^k_A}{1 + \rho}\right)\right)$$

where for each issue $k$:

$$\sigma^k_J(0) = +\infty, \text{ and } (\sigma^k_J)'(t) \leq 0 \text{ for all } t \in [0, T].$$

The campaign modifies the information and uncertainty on each issue. Let $h^k_A$ denote for each issue $k$ the measure of the gain in the precision on $A$’s position on issue $k$. The measure depends on the strategies $t$ and the parameter $\rho$ as in (5). Defining

$$H_A(t) = \frac{s^k_A^2}{s^k_A^2 + (\sigma^k_A)^2(t)}, t \in [0, T]$$

and similarly for $B$, we have

$$h^k_A(t) = H^k_A \left(\frac{t_A^k + \rho t_B^k}{1 + \rho}\right) \text{ and } h^k_B(t) = H^k_B \left(\frac{\rho t_A^k + t_B^k}{1 + \rho}\right).$$

Observe that $h^k_A$ only depends on the time spent on issue $k$. Hence, assumptions bearing on the behavior of $\sigma^k_J$ and $h^k_J$ with respect to the effective time spent
on issue $k$ can be stated for each issue $k$ in an analogous manner as for a single issue. In particular the linearity assumption can be stated issue by issue.

Expected shares  Similar computation as in the single issue case yields that the average increase in the expected utility for $A$ being elected that is due to the campaign is given by

$$U_A(t) - U_A(0) = \sum_k \alpha_k^A h_k^A(t) \left[ 2P_k^A + Q_k^A h_k^A(t) \right]$$

where

$$P_k^A = (x_k^A - m_k^A) (\bar{x} - m_k^A); \quad Q_k^A = (s_k^A)^2 - (x_k^A - m_k^A)^2$$

$$\alpha_k^A = \int_\alpha \int_x \alpha^k f_k(\alpha^k, x^k) d\alpha^k dx^k; \quad \bar{x} = \int_\alpha \int_x \frac{\alpha^k x^k}{\alpha^k} f_k(\alpha^k, x^k) d\alpha^k dx^k.$$

The electoral campaign modifies the relative salience of the issues, which is a function of the precisions. Function $U_A$ is separable across issues. Hence changing the precision on a particular issue can be analyzed through the "average position effect" and the "reduced variance effect" as in the previous section. In particular, a position is said favorable when whenever it is on the same side of the median relative to the prior so that $P_k^A$ is positive ($x_k^A$ and $\bar{x}$ are located on the same side of $m_k^A$). Also the marginal benefit from increasing the precision on the party’s position on that particular issue depends on two opposite effects, and is increasing when the position is standard ($Q_k^A$ is positive) and is decreasing when it is non standard ($Q_k^A$ is negative).

Finally, the expected vote share for party $A$ is

$$\frac{1}{2} + \phi [U_A(t) - U_B(t)]$$

and we shall denote by $\pi$ the difference $U_A - U_B$ that defines our zero sum game.

Comments  Observe that the objective $\pi$ is separable across issues, as $U_A$ and $U_B$ are. This immediately implies that in the absence of a global time constraint, that is, when the strategy space for a party writes $T = [0, 1]^K$ (up to a normalization), the game can be analyzed issue by issue. All the previous results extend. This assumption about the strategy space is certainly the right assumption to make in the case $\rho = 0$ when we interpret the strategies of the parties in terms of precision. In that case, we may assume that a party decides the precision it wishes to reach on each issue, with no global constraint.

Hence we concentrate from now on on the situation in which parties face a global time constraint and have to allocate their overall time across issues. Furthermore, the amount $T$ will be assumed small enough (relative to the noise in the signals transmitted) so that full precision cannot be reached on an issue. We shall say that the time constraint is $\ddot{O}it$ tight.

5.2 Equilibria for $\rho = 0$

Without leakage, each party controls, and has an impact on, the utility expected by the electorate if it becomes elected. That is, $J$ controls $U_J$ which is its
objective, and the game is degenerate. Consider $A$ for example. Precision index $h^k_A(t)$ depends only on $t^k_A$. Party $A$ maximizes

$$\sum_k \alpha^k H^k_A(t^k) \left[ 2P^k_A + Q^k_A H^k_A(t^k) \right] \quad \text{under the time constraint } \sum_k t^k \leq T.$$ 

The optimal solution is determined not only by the values of $P^k_A$ and $Q^k_A$, in particular by their sign, but also by the sensitivity of the precision indices with respect to the strategies. The marginal benefit derived by a marginal increase in addressing issue $k$ writes

$$\frac{\partial U_A}{\partial t^k_A}(t) = 2\alpha^k (H^k_A)'(t^k) \left[ P^k_A + Q^k_A H^k_A(t^k) \right] \quad (13)$$

To go further, we shall introduce an assumption of linearity of the precision indices. Precision $H^k_A$ is linear with respect to $t^k_A$ if $H^k_A(t^k) = a^k t^k_A$ for some scalar $a^k$. Under the linearity of each index, we have

$$U_A(t) - U_A(0) = \sum_k \alpha^k a^k t^k_A \left[ 2P^k_A + a^k Q^k_A t^k_A \right]$$

Under this assumption, the marginal benefit from an additional speech on an issue is increasing or decreasing depending on the sign of $Q^k_A$, that is on whether the issue is in standard position. The next proposition shows that the distinction between standard and non standard issues turn out to be useful.

**Proposition 4** When there is no leakage,

1. A party speaks on an issue that is in non standard position only if it is in favorable position.

2. Assume the linearity of the precision measures $H^k_J$ for party $J$, and for each issue. Under a global tight time constraint, a party speaks on one issue in standard position at most.

Point 1 is straightforward. An issue that is in non-standard and unfavorable position, is one with both terms $P^k_J$ and $Q^k_J$ negative. Hence speaking on such issue is strictly dominated because the marginal benefit is always negative as can be seen from (13). As for point 2, we present the basic intuition (the formal proof is in the appendix). Let the party speak about two issues in standard position, say $k$ and $\ell$. Under the tight time constraint, the marginal benefits are equalized between these issues and both positive. The objective is convex with respect to the time spent on each issue (because they are in standard position). Hence increasing the time spent on one issue at the expense of the other is surely beneficial, which gives the contradiction.

A simple consequence of Proposition 4 can be drawn if all issues are in standard position. Then, the party concentrates on a single issue. If there are both kinds of issues, the time spent on those with decreasing marginal benefit is
allocated by equalizing the marginal benefit on these issues and the remaining
time is concentrated on a single issue in standard position.

Both the values for the position terms and the concavity terms are on average
null. The probability for an issue to be in standard position is equal to 0.68 .
Hence with many issues at stake, it is likely for a party to have issues both in
standard and non standard positions. The proposition gives an upper bound of
0.16 per cent on the number of issues that are addressed on average: 0.68 %
of the issues are not addressed (those in standard position but one) and half of
those in non standard position are not addressed (because their position terms
are negative).

The proposition says nothing about the issues that are effectively addressed.
We shall argue that the probability of addressing an issue is larger the more
extreme it is (prior far from the median) when there are potential many issues.
Consider an issue \( k \) and make it more extreme a priori by moving the prior
away from the median. Considering \( B \) for instance, increase \( m = m^k_B \), \( m \geq 0 \)
(with the normalization \( x^k_B = 0 \)). Other issues are unchanged. The question we
want to address is whether the issue has more chance to be addressed when it
becomes more extreme.

The law of the deviation to the mean \( \delta^k_B = m^k_B - x^k_B \) is unchanged. For
each possible value for the deviation \( \delta^k_B \), we shall translate the position as the
prior changes: \( x^k_B = m^k_B - \delta^k_B \). Observe that the value for \( Q^k_B \) is not affected
by this translation and that \( P^k_B \) writes as \( \delta^k_B m \). Thus, the marginal benefit
for \( B \) increases with \( m \) without ambiguity when the position is more favorable
to \( B \) initially, which occurs when \( \delta^k_B \) is positive, and decreases in the opposite
situation. Now consider \( \delta^k_B \) for which the issue is addressed by \( B \). From the
remark just above it is surely still addressed by \( B \) as \( m \) increases when the
position is more favorable to \( B \) (\( \delta^k_B \geq 0 \)). We know that this is surely the case if
the issue is in non standard position. Thus, the only situation where an increase
in \( m \) may deter \( B \) from addressing the issue occurs when the issue is in standard
position but unfavorable. However, this case occurs with small probability. An
heuristic argument is as follows. Consider a symmetric framework in which the
parameters \( \alpha^k \) and \( a^k \) are identical, normalized to 1. Assume also identical
variances \( s^k_B \). We know that at most one issue in standard position is addressed
by \( B \). Let \( t \) be the time spent on this issue. The issue in standard position which
is addressed is surely the one for which the value \( 2P^k_B + Q^k_B t \) is maximum,
or using our notation for which \( 2\delta^k_B m^k_B + (s)^2 - (\delta^k_B)^2 t \) is maximum. The
chances that this maximum is reached for an unfavorable position, that is one
for which \( \delta^k_B \) is negative, are small.

What can be said without assuming the linearity of the precision parameters
\( H^k_B \) with respect to time? First, if for each issue, the objective is either concave
or convex with respect to time, the same properties as stated above hold. This
is likely to occur if the convexity or concavity in the precision parameters is
moderate, which is likely under our assumption of tight time constraint. Other-
wise, the concavity of the precision parameter seems to be the more plausible
assumption. In that case, the objective is more concave with respect to the
allocation of time than with respect to precision. In particular it is concave for each non-standard issue.

5.3 Equilibria for $\rho = 1$

The criterion of the game now depends on the strategies through the total time spent on each issue. Function $U_A - U_B$ depends on $t = (t_A + t_B)/2$ only, and writes as $\Pi(t)$ where $t = (t_A + t_B)/2$ is the vector of total times.

The same argument as used for a single issue extends and implies that parties do not address the same issues at an equilibrium in pure strategies, ($\Pi$ is locally non constant in any direction thanks to our assumption). In that case, parties never engage in dialogue, as defined by Simon: because “no themes can work to the advantage of both candidates, they will never allocate resources to the same theme. Dialogue is defined as candidates discussing (spending money on) the same dimension, so rational candidates should never and will never dialogue” (Simon 2002, 64).

The argument made by Simon is different from ours: he proposes an electoral campaign game in which parties’ positions are fixed and perfectly known by the voters, but in which the time parties spend discussing an issue increases the salience of this issue (the $\alpha^k$ with our notation). When this salience only depends on the total time spent by both parties discussing the issue ($\alpha^k = \overline{\alpha}^k (t_A^k + t_B^k)$), talking about an issue cannot work to the advantage of both candidates simultaneously. Amoros and Puy (2007) further explore this model (in particular solving for equilibria in mixed strategies when an equilibrium in pure strategies fails to exist).

To say more, we assume the linearity of the precision indices with respect to the time spent, for each issue $k$ and each party:

$$H^k_A(t^k) = a^k t^k$$

$$H^k_B(t^k) = b^k t^k$$

Write $\Pi(t)$ as the sum $\sum_k \Pi^k(t^k)$, where $\Pi^k$ is the component stemming from issue $k$:

$$\Pi^k(t^k) = \overline{\alpha}^k t^k \left[ 2(a^k P^k_A - b^k P^k_B) + ((a^k)^2 Q^k_A - (b^k)^2 Q^k_B) t^k \right]$$

Let us say that when $a^k P^k_A < b^k P^k_B$ (resp. $a^k P^k_A > b^k P^k_B$), candidate $B$ (resp. $A$) has a relative position advantage on this issue. For such an issue, the first unit of precision induces the perceived position by the electorate to be closer on average to the median position than for its opponent.

Observe that each $\Pi^k$ is either concave or convex in $t^k$. Let us divide the issues into two disjoint sets: $DR$ are the issues for which an additional speech decreases the relative marginal benefit for $A$: $(a^k)^2 Q^k_A < (b^k)^2 Q^k_B$, or equivalently for which the function $\Pi^k$ is concave with respect to the time spent speaking on them and $IR$ are those for which the converse holds (we neglect the case where $(a^k)^2 Q^k_A = (b^k)^2 Q^k_B$ is null). Clearly, taking the point of view of party $B$ the issues in $DR$ (resp. $IR$) are those for which an additional speech increases (resp. decreases) the relative marginal benefit for $B$. 

As for the case without leakage, there are strictly dominated strategies that can be eliminated, whether parties play simultaneously or sequentially. Speaking on an issue which is in relative position disadvantage and with decreasing returns (in DR) is strictly dominated for A.

Besides, under linearity of the precision parameters, it cannot be the case that party A spent some time discussing two issues in IR at a pure equilibrium. Indeed, observe that the concavity or convexity of $\Pi^k$ is not affected by the opponent’s strategy when it is a pure strategy. It suffices to apply the same arguments as in proposition 4, point 2.

Proposition 5 sums up these properties.

**Proposition 5** Consider $\rho = 1$ and a tight global time constraint. At a pure equilibrium (assuming it exists)

1. Parties do not address the same issues.
2. Furthermore, under linear precision indices $H^k$ for each issue $k$ each party,
   2a. party A speaks on an issue in DR only if it has a relative position advantage on this issue,
   2b. party A speaks on at most one issue in IR and party B speaks on at most one issue in DR.

As for a single issue, an equilibrium in pure strategies may not exist and we consider the sequential version of the game.

**Sequential version of the game** With multidimensional strategies, it is still true that an equilibrium in pure strategies in the simultaneous game gives rise to an equilibrium in the sequential game and that the order of play has no consequence. In general, an equilibrium is ‘computed’ by backward induction.

A strategy for the follower may depend on the observed strategy chosen by the first mover: it is a reaction function. Consider for each $t_A$ all best responses for $B$ to $t_A$, i.e. all $t_B$ that solve $\inf_{t_B} \pi(t_A, t_B)$. Thanks to the continuity of $\pi$ and the compactness of the set of strategies, the inf is reached and can be replaced by min. The ‘value’ of the game is defined as $\max_{t_A} \min_{t_B} \pi(t_A, t_B)$, and $A$ chooses $t^*_A$ that solves $\max_{t_A} \min_{t_B} \pi(t_A, t_B)$.

A key point is that $B$ may have multiple best responses at $t^*_A$ and this acts as a threat to $A$: if $A$ deviates a little bit, $B$ will adjust its reaction in one direction or another. In the case of a single issue for instance, $A$ may choose the value $t^*_A$ that makes $B$ indifferent between not talking at all and talking full time (Proposition 3). If $A$ was to choose a lower value, $B$ would not speak for sure and if it was to choose a higher value, $B$ would speak full time for sure. With multiple issues, a party has many options. Instead of not speaking at all, it may switch and address another issue. It will surely do so if time has value, that is if the party is time-constrained. This principle extends as described by the first point in the following proposition.
Proposition 6  Consider $\rho = 1$ and a tight global time constraint. Let precisions $H^k_J$ be linear for each issue $k$, each party $J$. At an equilibrium in the sequential game with $A$ as first mover,

1. Let party $A$ speak on an (undominated) issue $k$ in $DR$. If $B$ also speaks on $k$ at a best response, then $B$ does not speak on any other issue in $DR$ at this best response and furthermore $B$ has another best response at which it does not speak on $k$.

2. Party $A$ speaks on at most one issue in $IR$. Furthermore if it addresses one, then party $B$ does not speak on this issue.

The proof of the proposition is presented in the appendix. According to this proposition, parties end up addressing at most one issue in common. Such an issue is necessarily an issue with a relative position advantage and a decreasing marginal advantage for the first mover. Note however that when this occurs, ex ante, there is some indeterminacy about which issue will be addressed in common: the follower is indifferent among several issues, and this acts as a threat to the first mover. The following table summarizes the proposition.

<table>
<thead>
<tr>
<th>k in DR: $(a^k)^2 Q_A^k &lt; (b^k)^2 Q_B^k$</th>
<th>k in IR: $(a^k)^2 Q_A^k &gt; (b^k)^2 Q_B^k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_k P_A^k &gt; b_k P_B^k$</td>
<td>at a best response for $B$, $t_B^k = 0$ dominating</td>
</tr>
<tr>
<td>$a_k P_A^k &lt; b_k P_B^k$</td>
<td>$t_A^k = 0$ dominating</td>
</tr>
</tbody>
</table>

Example  This example illustrates the fact that even though $B$ may address several issues a priori, $B$ will end up addressing a single one.

Let us consider a two-issue case, and assume them to be both in $DR$ with a relative position advantage for $A$. Furthermore let them be symmetrical, i.e. $\Pi^1$ and $\Pi^2$ are identical functions, and take $T = 1$. There is a pure equilibrium in which $A$ splits equally its total time between the two issues if $\Pi^1(1/4) \leq \Pi^1(3/4)$. If not, $A$ speaks equally a lapse $t^*$ smaller than $1/2$ on each of the issues, and $B$ concentrates on either issue: The value $t^*$ satisfies $\Pi^1\left(\frac{t^*}{2}\right) = \Pi^1\left(\frac{t^*}{2} + \frac{1}{2}\right)$. A best response for $B$ is to concentrate on a single issue, that is to choose either $t_B = (1,0)$ or $t_B = (0,1)$.

The example easily extends to $n$ issues in $DR$ all with a position advantage for $A$ and symmetrical. A pure strategy in which $A$ splits equally its total time between the $n$ issues if $\Pi^1\left(\frac{1}{2n}\right) \leq \Pi^1\left(\frac{1}{2n} + \frac{1}{2}\right)$. Otherwise, $A$ chooses the value $t^*$, which is smaller than $1/n$, that satisfies $\Pi^1\left(\frac{t^*}{n}\right) = \Pi^1\left(\frac{t^*}{n} + \frac{1}{2}\right)$. For this value, it is optimal for $B$ to concentrate on a single issue and a $B$’s best response is of the form $t_B^k = 0$ but one issue for which time is equal to 1.

When some issues are not in $DR$, the total time allocated by $A$ and by $B$ on the issues in $DR$ (with a relative position advantage for $A$) is endogenous.
Finally, to say more about the issues that are effectively addressed, one can consider symmetric issues around the median, and make them more divisive by increasing the distance of the priors to the median. An argument similar to the one used for $\rho = 0$ when an issue is made more extreme shows that the probability for an issue to be addressed is larger the more divisive it is (assuming a large enough number of issues).

6 Discussion

6.1 Truthful parties and naive voters

Our analysis relies on two important assumptions. The first assumption bears on voters, who may be considered as ‘naive’ (although they are bayesian). Consider $A$’s strategy as a function of position $x_A$, as depicted in Figure 2. A ‘sophisticated’ or strategic voter who knows this function is able to infer more than what we have assumed on voters. In particular, a voter can infer that a candidate who does not address an issue has a position below the threshold $c_1$. Similarly, by observing a positive precision below 1, she can infer the position since this occurs only for a position above $c_2$ and $A$’s precision is one-to one for these positions. This changes the voter’s behaviour. Knowing this, a candidate changes in turn his strategy. But the impact depends on the assumed number of strategic voters. In what follows, we shall assumed all voters to be strategic.

The second assumption bears on candidates, who are ‘sincere’ or in other words are committed somewhat to their announcements.

We investigate how equilibria are modified when these assumptions are relaxed. Combining the assumptions naive versus sophisticated, and sincere versus no commitment, there are three cases to consider. We conduct the analysis in the single issue model so that it is equivalent to argue in terms of precision or time. With commitment, a strategy specifies a precision as a function of the position. In the absence of commitment, a strategy specifies a mean and a precision, $z_A$ and $h_A$ respectively for $A$ as a function of the position. To simplify, we shall assume that maximal precision ($h = 1$) is available to the candidate. Furthermore, a candidate does not speak about its opponent. (Dropping the sincerity assumption, parties are likely to send false messages on their opponents and not to be listened to, or at least there are equilibria for which this must be true.)

We are in the situation where $\rho$ is equal to zero, and candidates interact only because their payoffs both depend on the median position. There is a ‘game’ between each candidate and voters. A candidate sends signals in order to influence voters’ decisions.

Denote by $\bar{x}$ the ideal point of the representative voter (taking all the $\alpha_i$ identical to 1), and by $\hat{x}_A$ and $h_A$ the expected mean and precision on $A$’s position induced by $A$’s strategy. The expected utility $U_A$ for the representative voter for $A$ being elected is given by

$$- [\hat{x}_A - \bar{x}]^2 - [s_A^2 (1 - h_A^2)]$$

(14)
(this formula is justified for normal variables only. We neglect this point here).

Observe that A’s true position influences A’s payoff only through the posterior \( \hat{x}_A \). When signals are unbiased, as we have considered so far, A’s true position influences the posterior \( \hat{x}_A \), hence the payoff. In contrast, in the absence of any commitment or cost incurred by the winner in case of deviation from an announcement, A’s payoff is completely independent of its true position.

**Naive voters and no commitment.** With naive voters, an action \((z_A, h_A)\) determines the posterior assigned to A position as \( \hat{x}_A = m_A + h_A(z_A - m_A) \). The expected utility derived by the strategy is \[-[m_A + h_A(z_A - m_A) - \bar{x}]^2 - \left[s_A^2 (1 - h_A^2) \right]\] By choosing the maximal precision (equal to 1) and an announcement that is equal to position \( \bar{x} \) the utility is null, which is the maximum possible value: announcing the average position without any ambiguity is a dominant strategy. (Without full precision, the result extends straightforwardly). Of course both candidates will do the same. Hence, with naive voters, the standard convergence result applies (in announcements): both announce the position of the representative voter. No information is transmitted.

**Sophisticated voters and no commitment.** As we have seen, in the absence of commitment, A’s payoff does not depend on its known position. Assume that, at equilibrium, A strategy could induce two different pairs of (posterior, precision) that generate distinct values for \( U_A \), that is distinct payoffs to A. Then A would always choose the one with the largest value. Sophisticated voters know this. Hence all actions taken by A at a strategy can only induce all the same payoff to A (note that the argument is valid for pure or mixed strategy as well). No information is transmitted.

**Sophisticated voters and commitment.** Consider the strategy where A sends perfect information on its position \( (h_A = 1) \). It is an equilibrium strategy. To show this, the voters’ behavior ‘out of equilibrium’ must be specified. Assume that when voters observe imprecise messages they vote for the opponent, which is supported by the belief that A’s position is extreme. This is where the sophisticated behavior comes into play. With naive voters, there are always extreme values far enough from the mean for which A benefits from being imprecise. For example by not talking the candidate secures itself the value \( U_A(0) \) with naive voters. Thus with sophisticated voters and commitment all information can be revealed at an equilibrium.

**Remark.** Voting for the opponent in case of imprecise messages is the only behavior supporting the strategy as an equilibrium for an infinite support for positions: otherwise, there are always extreme values for \( x_A \) for which the candidate is better off by deviating. These values are however implausible. If one concentrates on a compact support, say around two standard errors from the mean, it suffices to say that in case of a deviation (i.e. imprecise messages), voters assign for sure the worst position, i.e. the extreme value that is the further away from the average electorate position \( \bar{x} \).
6.2 Strategic leakage

Our modeling on the speeches on the opponent’s position through a single parameter $\rho$ is parsimonious. One could consider instead that candidates freely choose to spend time on their opponents. A strategy for $A$’s candidate for example specifies for each issue $k$ the amount of time that she spends on speaking on her own position on this issue ($t_{AA}^k$) and on her opponent’s position ($t_{AB}^k$).

We have still a zero-sum game with the criterion given by $\pi$ the difference $U_A - U_B$. The key point is that, due to unbiased signals, a speech by either candidate on $A$’s position on an issue moves $\pi$ in the same direction, either increase the plurality shares of $A$ or decrease it. Formally, this writes as $\frac{\partial \pi}{\partial t_{AA}}(t_A, t_B)$ and $\frac{\partial \pi}{\partial t_{BA}}(t_A, t_B)$ are of the same sign, and when they are null the second derivative is not null.

Arguing as previously when $\pi$ depends only on the total time spent on each issue (for $\rho$ equal to 1), one obtains that at an equilibrium in pure strategies, parties do not both speak on the position of $A$ on issue $k$ (or both on the position of $B$ on issue $k$).

7 Conclusion

We proposed a simple model designed to capture the incentives candidates may face for transmitting to voters information regarding their platforms. The analysis reveals that the equilibrium communication strategies are very different depending on the campaign technology, resumed here by a single parameter $\rho$ describing how much information about his opponent a candidate is constrained to involuntarily transmit when he tackles an issue. We focus on two polar cases: $\rho = 0$ means that a candidate only transmits information about his own platform, whereas $\rho = 1$ means that a candidate is constrained to transmit the same quality of information on both his and his opponent’s platforms when he decide to talk about an issue.

The type of issues that are addressed is affected both by the technology and the type of constraints that parties face. When $\rho = 0$, both parties may address the same issue -they can engage in dialogue, as defined by Simon (2002)- but even if they face no constraint, some issues may very well not be addressed. When $\rho = 1$, in the multi-issue case parties address one issue at most in common under time constraint, so that dialogue will be the exception rather than the rule. Furthermore, with no global constraint, all issues will be addressed by at least one party: there is always a party that has benefits from improving information either on itself or on its opponent.

8 Appendix

Proof of Proposition 1

Assume wlog that $m_A \leq \pi$. Party $A$ can choose any precision $h_A$ in $[0,a]$, $0 < a \leq 1$. 
Consider first the case where \( U_A \) is convex in \( h_A \). \( U_A \) convex in \( h_A \) iff \( Q_A \geq 0 \). In that case, the optimal precision for \( A \) is either 0 or \( a \). The precision \( h_A = a \) is optimal iff \( 2P_A + aQ_A \geq 0 \).

Consider now the case where \( U_A \) is concave in \( h_A \). \( U_A \) convex in \( h_A \) iff \( Q_A \leq 0 \). In that case, \( U_A \) reaches its maximum in \( h_A^* = -\frac{P_A}{Q_A} \). If \( h_A^* \leq 0 \), the optimal precision for \( A \) is 0, if \( 0 \leq h_A^* \leq a \), the optimal precision for \( A \) is \( h_A^* \), and if \( a \leq h_A^* \), the optimal precision for \( A \) is \( a \).

These conditions are written in terms of \( P_A \) and \( Q_A \). It remains to write them down in terms of the initial parameters of the model. Straightforward computation gives the expressions on the table.

**Proof of Proposition 2** Assume that \( \Pi(t) \) is not locally constant on \([0, 1]\). Note that for all \( t \in [0, 1]^2 \), for any integer \( k \geq 1 \)

\[
\frac{\partial^k \pi}{\partial t^k_A} (t) = \frac{\partial^k \pi}{\partial t^k_B} (t) = \Pi^{(k)} \left( \frac{t_A + t_B}{2} \right).
\]

Assume by contradiction that \( (t_A^*, t_B^*) \) is an equilibrium with \( t_A^* \in [0, 1] \). In that case, necessarily, \( \Pi \) reaches a strict local maximum in \( \frac{t_A^* + t_B^*}{2} \). But this implies that candidate \( B \) would be strictly better off both by increasing his time speech (if possible) and by decreasing his time speech (if possible) his time speech. Since at least one of these options (increasing or decreasing his time speech) is available for candidate \( B \), this contradicts the fact that \( t_B^* \) is a best response against \( t_A^* \).

Assume now by contradiction that \( (0, 0) \) is an equilibrium. Since not speaking is a best response for candidate \( A \) against zero time speech by candidate \( B \), it must be the case that there exists some \( \varepsilon > 0 \) such that \( \Pi \) is strictly decreasing on \([0, \varepsilon]\). But symetrically, since not speaking is a best response for candidate \( B \) against zero time speech by candidate \( B \), it must be the case that there exists some \( \varepsilon' > 0 \) such that \( \Pi \) is strictly increasing on \([0, \varepsilon']\). These conditions cannot simultaneously hold. Similarly, it cannot be the case that \( t_A = 1 \) and \( t_B = 1 \).

**Proof of Proposition 3.** **Point 2.** Consider first the case \( \Pi(1/2) < \min(\Pi(0), \Pi(1)) \). Then it must be the case that \( \Pi \) is convex \((a^2Q_A - b^2Q_B > 0)\) and reaches its minimum in some \( t \in \left[ \frac{1}{4}, \frac{3}{4} \right] \). Consider an issue \( k \) in \( IR \). The function \( \Pi \) is convex with respect to \( t \). Let \( t \) be the time at which the minimum is reached. Note that \( \Pi \) decreases on \([0, t]\). It is a dominated strategy for \( A \) to choose \( t_A/2 \) in \([0, t]\) : since either \( B \) does not speak or speaks so as to decrease \( \Pi \) at most up to the point where \( t \) is reached, \( A \) is always better to choose 0. Hence, either \( A \) does not speak, or \( A \) chooses a time speech with \( t_A/2 \geq t \). If \( A \) does not speak, \( B \) speaks full time (since by assumption \( \Pi(1/2) < \Pi(0) \)), and if \( A \) chooses a time speech with \( t_A/2 \geq t \), \( B \) faces an increasing \( \Pi \) (since \( \Pi \) is increasing for any \( t \) larger than \( t_A \)) and \( B \) does not speak.

**Point 1.** Consider now the case \( \Pi(1/2) > \max(\Pi(0), \Pi(1)) \). Then it must be the case that \( \Pi \) is concave. For any \( t_A \), since \( B \)'s payoff function is convex in \( t_B \), it chooses either \( t_B = 0 \) or \( t_B = 1 \). Therefore, when \( A \) chooses time speech \( t_A \), it obtains as a payoff \( \min \left( \Pi \left( \frac{t_A}{4} \right), \Pi \left( \frac{t_A}{4} + \frac{1}{2} \right) \right) \). Let us denote
δ(t_A) = Π\left(\frac{t_A + 1}{2}\right) - Π\left(\frac{t_A}{2}\right). By assumption, δ is affine, with δ(0) > 0 and δ(1) < 0. Therefore there exists a unique t^*_A ∈ ]0,1[ such that δ(t^*_A) = 0\#; besides, for t_A ≤ t^*_A , Π\left(\frac{t_A}{2}\right) < Π\left(\frac{t_A + 1}{2}\right) and for t_A ≥ t^*_A , Π\left(\frac{t_A + 1}{2}\right) < Π\left(\frac{t_A}{2}\right).

This shows that min\left(Π\left(\frac{t_A}{2}\right), Π\left(\frac{t_A + 1}{2}\right)\right) reaches its (unique) maximum in t^*_A , which concludes the proof of the proposition.

**Proof of Proposition 4.** Let us write the first order conditions associated with the maximization of \(U_A\) under a tight time constraint. There is λ nonnegative such that
\[
\frac{∂U_A}{∂t^k_A}(t) ≤ λ \text{ with an equality if } t^k_A > 0
\]
and \(\sum_k t^k_A ≤ \bar{T}\) with an equality if \(λ > 0\), where
\[
\frac{∂U_A}{∂t^k_A}(t) = 2\pi\left(H^k_A\right)'(t^k_A) [P^k_A + Q^k_A H^k_A(t^k_A)]
\]
Under linearity, \(H^k_A(t^k_A) = a^k t^k_A\), \((H^k_A)\)' is constant and equal to \(a^k\). Property 1 is trivial because speaking on an issue in non standard and non favorable position is dominated. Property 2 follows from the second order condition. At a solution where both \(t^k_A\) and \(t^k_B\) are positive the second order condition writes as
\[
\pi\left(a^k\right)^2 Q^k_A + \pi\left(a^k\right)^2 Q^k_B ≤ 0
\]
which cannot be satisfied if both \(k\) and \(\ell\) are in standard position, i.e. if both \(Q^k_A\) and \(Q^k_B\) are positive.

**Proof of Proposition 6** Point 1. Given \(t^*_A, B\) chooses its best response so as to minimize \(π\) under the time constraint. Under the additional assumption that the time constraint is binding, the derivatives of Π\(^k\) with respect to \(t\) for \(k\) with \(t^k > 0\) are all equalized and strictly negative at a best response.

Point 1. Using that the concavity of Π\(^k\) is not affected by \(t_A\), any \(B\)'s best response addresses at most one issue in \(DR\). Now assume that \(B\) addresses an issue \(k\) in \(DR\) that is also addressed by \(A\). Let \(\bar{t}\) be the time at which the maximum of Π\(^k\) is reached. Surely we have \(t^*_A t^k_B ≤ t^k ≤ (t^*_A t^k_B)/2\) at a best response of \(B\) that addresses \(k\). By contradiction, assume that \(B\) addresses \(k\) at any other best response. Then Π\(^k\) is strictly decreasing at \((t^*_A t^k_B)/2\) for any best response. This implies that \(A\) benefits from lowering the time spent on \(k\) by \(ε > 0\): the value of Π\(^k\) will increase except if \(B\) increases its time spent on \(k\) by at least the same \(ε\). But, if \(B\) does that, it has to decrease its speech on other issues (since \(B\) is time constrained), which is harmful for \(B\) hence benefits to \(A\).

Point 2. Consider an issue \(k\) in \(IR\). The function Π\(^k\) is convex with respect to \(t\). Let \(t^k\) be the time at which the minimum is reached. Note that Π\(^k\) decreases on \([0, t^k]\). As for a single issue, it is a dominated strategy for \(A\) to choose \(t_A/2\) in \([0, t^k]\): since either \(B\) does not speak or speaks so as to decrease

\[t^*_A = -2\frac{a^A b^B}{\pi (a^A b^B - a^B b^A) - \frac{1}{2}}.\]
Π^k at most up to the point where \( t_k^A \) is reached, A is always better to choose 0. Hence, either A does not speak, or A chooses a time speech with \( t_k^A / 2 \geq t_k^k \) so that B faces an increasing Π^k (since Π^k is increasing for any \( t_k^k \) larger than larger \( t_k^k \)) and B does not speak.

This implies that party A addresses at most one issue in IR. By contradiction, let A address two issues in IR. For each one, as we have just seen A must choose a time speech so that B faces an increasing Π^k : \( t_k^A / 2 \geq t_k^k \). If both inequalities are strict, A can change its allocation of time between both issues at the margin in such a way that it is still dominated for B not to speak. Hence, we can argue as in the case of \( \rho = 0 \): since the marginal benefit for A is increasing on each issue, A benefits to increase the time spent on one issue and decrease it on the other one. This is true until the constraint \( t_k^A / 2 \geq t_k^k \) binds for one issue. But since the minimum of Π^k is reached for \( t_k^A / 2 = t_k^k \), A can only be better off by not addressing this issue at all and using its time on another issue.

References


