Interest Rate Fluctuations and Equilibrium in the Housing Market*

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Job Market Paper

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Abstract

We study the general equilibrium of the housing market in an economy populated by overlapping generations of households. A contribution of the present paper is to solve for the housing market equilibrium in the presence of aggregate (interest rate) uncertainty with a realistic mortgage contract. In addition, households also face idiosyncratic uncertainty resulting from stochastic changes over the lifecycle in tastes (or need) for housing. In this environment, profit maximizing banks offer fixed-rate mortgage (FRM) contracts to home buyers. As seems plausible, each housing market transaction is subject to a fixed cost, which gives rise to S-s policy rules for housing transactions: existing home owners change the size of their houses only if there is a sufficiently large change in the state of the economy (i.e., in interest rates, in their taste for housing, etc.) A plausibly calibrated version of the model is consistent with three empirically documented features of the housing market: (i) highly volatile housing prices and transaction volume, (ii) a strong positive correlation between transaction volume and housing prices, and (iii) a significant negative relationship between interest rates and housing prices, which can rationalize a large part of the recent boom in housing prices in the U.S. and around the world.

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1 Introduction

In this paper we study the equilibrium of the housing market in an economy populated by overlapping generations of households. The particular model studied here combines two realistic features of the housing market: first, we allow for stochastic fluctuations in mortgage interest rates, and second, we model long-term mortgage contracts explicitly. The combination of these features distinguishes this paper from existing studies in the literature.

We use this model to study several important trends observed in the housing market in the last decades. The first one of these trends is the rise in housing prices observed in the United States since the late 1970’s, which turned into a substantial “boom” between 1995 and 2005. This trend was especially pronounced in large cities: for example, in places like Boston, New York, San Francisco, and Los Angeles real median housing prices have tripled, and in some cases quadrupled, between 1975 and 2004 (see figures 1 and 3). The upward spiraling in prices led to suggestions of a bubble in the housing market in the popular press. Although it seems hard to rule out the existence of a bubble definitively, it is important to note that similar trends in housing prices have been observed in many other countries: from 1997 to 2006, housing prices rose by 116 percent in Sweden, by 184 percent in Britain, by 173 percent in Spain, by 244 percent in Ireland, and by 315 percent in South Africa, exceeding the 98 percent rise observed in the United States. Overall, thirteen out of the eighteen countries in the survey saw price increases of 88 percent or more during this period (see the survey of the global housing market in the Economist magazine (2006b)).

This global nature of the housing price boom suggests that changes in some fundamentals common to these countries could be possible driving forces behind these trends. Clearly, the (mortgage) interest rate is one such variable with a global impact, which is also especially natural to think of in the context of housing market transactions. Figure 2 plots the real 30-year fixed rate mortgage rates in the U.S. from 1975 to 2005. Comparing the mortgage rate movements in this graph to the changes in real median housing prices shown earlier in figure 1 reveals a surprisingly strong (negative) relationship: housing prices were falling from 1979 to 1985 when real mortgage rates were high; and housing prices were “booming” in late 1990’s when mortgage rates were low (and falling). Therefore, figures 1 and 2 are at least qualitatively consistent with the hypothesis that the housing price boom of the last decade could be due to falling interest rates. Of course, the real question is how big the effect is quantitatively, which is the question we undertake in this paper.

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1 See for example, the article entitled “America’s house-price bubble” in the Economist magazine (2006a).
2 The data that we report from the Economist (2006b) is in nominal terms.
A parallel set of trends during this period have been observed in the transaction volume—which we define as the number of existing homes sold in a given time period. Transaction volume has increased steadily during this period (except for a brief but sharp fall from 1979 to 1982 after the sharp rise in mortgage rates) and almost tripled from 1982 to 2004 (figure 4). Furthermore, this trend in transaction volume appears to be closely associated with the trend in prices: the correlation between the two series is 0.90 in the national U.S. data (figure 5), and as shown by Genesove and Mayer (2001) the comovement is even more pronounced in local housing markets. Finally, the same comovement is also documented for Britain and France (Ortalo-Magne and Rady(2004); Stein (1995)).

Clearly, this paper is not the first to observe that (mortgage) interest rates are likely to have an important impact on housing prices and transaction volume. To give just one example, in his testimony to the Congress in 2005, the then-Chairman of the Federal Reserve Board Alan Greenspan said:

Historically, it has been rising real long-term interest rates that have restrained the pace of residential building and have suppressed existing home sales, high levels of which have been the major contributor to the home equity extraction that

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3The correlation between the two series is not entirely driven by the trend components, but they also appear at higher frequency (for example when the trends are removed using Hodrick-Prescott filter). See for example Rios-Rull and Sanchez-Marcos (2005).
arguably has financed a noticeable share of personal consumption expenditures and home modernization outlays.

Nevertheless, the difficulty of solving a model with interest rate uncertainty and realistic mortgage contracts (as well as some other key features of housing market transactions, such as fixed costs, which we consider here) has previously prevented a quantitative assessment of the channels explored here.4

The basic framework studied in this paper is a two-period overlapping-generations model.5 At the beginning of each period, the economy is hit by an exogenous interest rate shock. Households derive utility from housing and non-housing consumption and they are allowed to borrow and lend in the bond market. Households can also borrow in the mortgage market only for the purpose of financing their house purchase. Risk-neutral profit maximizing banks offer fixed-rate mortgages that are paid in two periods (end of periods 1 and 2). Because young households enter the economy with limited resources, they borrow in the mortgage market to finance the housing purchase. At the beginning of period 2 (old age) households receive an idiosyncratic shock to their taste for housing, which could be thought of as resulting from changes in family size, etc. Consequently, households would like to adjust the size of

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4To our knowledge, there has been no work on the role of the interest rate fluctuations on the transaction volume.

5We calibrate a richer version of the framework described here in quantitative analysis. In particular, we allow for individuals to live 30 years, but let them trade at 2 points during their lifecycles.
their houses. As noted earlier, housing market transactions are subject to a fixed cost, which implies that only households with a sufficiently large shock (in absolute value) will choose to adjust. These “movers,” will sell their homes, receive the proceeds, pay the balance of their first mortgage debt, buy a new house and get a new mortgage (which in this case is a one-period mortgage paid at the end of period 2). Non-movers simply pay their existing mortgage and enjoy their existing home and non-housing consumption. Before death, individuals sell their homes and derive utility from the sales price which can be thought of as the utility from leaving a bequest. For simplicity we assume that aggregate house supply is fixed and there is no rental market.

Using a plausibly calibrated version of the model we find that housing prices are quite sensitive to the interest rates: a 10 percent decline in interest rates (close to the decline observed in the US in the last 2 decades) causes a 50 percent increase in the real median housing prices. This quantity is large enough to explain almost entire real housing price appreciation in the US and around half of the appreciation in big cities observed during the last two decades (see figures 1 and 3). An interesting implication of the model is that current interest rates not only affect the current housing prices but also affect the next period’s housing prices. Higher interest rates in the current period induce young agents to save more, which increases the total wealth in the next period. As wealthier agents will demand more housing, prices will go up in the next period. Quantitatively, if current interest rates increase 1 percent, housing prices will increase around 4 percent in the next period.

In the model, transaction volume also varies significantly with interest rates. With low interest rates in the current period, homeowners are more willing to sell their houses since their mortgage contract they got in the previous period is relatively more costly to hold. With a similar intuition, if home buyers purchase their home with low interest rates in the current period, they will be less likely to sell their houses in the next period, because they will already be holding a good contract. The model predicts that, on average, a 1 percent decline in the current interest rates causes a 3 percent increase in transaction volume in the current period, and a 5 percent decrease in transaction volume in the next period. The combination of the two effects further implies that transaction volume will decline when interest rates rise, which is the observation of Alan Greenspan that we quoted earlier.

In the model, consistent with the US data, transaction volume and housing prices comove with a large positive correlation (around 0.8) as interest rates affect both variables similarly. Moreover, although we abstracted from the sources of dispersion other than interest rates, the dispersions of the housing prices and transaction volume—measured by the coefficient of variation—are around two thirds of the dispersions observed in the US data.
The model predicts that the welfare effects of housing price movements differ across agents. In case of a housing price increase, an old agent who downsizes his house will be better off since his return from the housing transaction will be higher (due to the difference of the sizes of houses). Old agents who move to bigger houses and young agents will be worse off. Finally, we look at the welfare implications of a possible tax reform which eliminates the tax deductibility of the mortgage interest payments. A comparison of the two steady states (before and after the tax reform) shows that the young agents are willing to give up around 1 percent of their consumption to stay with the current tax system. However, the cost of the tax reform is larger for the old agents. They are willing to give up around 5 percent of their consumption, because after the tax reform, they will not be getting the tax returns and the returns they expect to get by selling their houses in the next period will be lower.6

Himmelberg, Mayer and Sinai (2005) and Martin (2005) emphasize the importance of interest rates in explaining the recent increase in housing prices. Himmelberg, Mayer and Sinai (2005) develop a formula to account for the changes in the annual cost of owning a house. In their formula, interest rates affect the homeowner’s forgone earnings. Martin (2005) studies the baby boom’s impact on the US housing prices by using a simple Lucas asset pricing model. He argues that the baby boom affected the demand for housing as well as the interest rates. Due to lower interest rates, his model can predict the housing price increase in the last decade as opposed to Mankiw and Weil (1988). The current paper differs from the mentioned studies as they do not consider the effects of interest rate fluctuations on transaction volume. Moreover, our model has aggregate interest rate uncertainty with a realistic mortgage contract which do not exist in these studies.

There is also an extensive literature which tries to explain the comovement of housing prices and transaction volume. Stein (1995) and Ortalo-Magne and Rady (2004) explain the comovement by using credit constraints and large down payment requirements. Wheaton (1990) and Berkovec and Goodman (1996) use search models to explain the comovement. Using a data set for the Boston condominium market, Genesove and Mayer (2001) argue that loss aversion can explain price-volume correlation. Each of the mentioned studies relies on some frictions. In our model, it is the mortgage contracts and the interest rate movements that cause the price-volume comovement.

The paper is organized as follows. The next section documents some observations in the housing market. Section 3 describes the model. In Section 4 we give an outline of the algorithm that we use to solve the model. We do the calibration in Section 5. The main results of the paper are reported in Section 6. In Section 7 and Section 8 we look at the

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6Housing prices decrease around 20 percent after the tax reform.
effect of housing prices movements on both the macro economy and welfare of the consumers. We do a policy analysis where we eliminate the tax deductibility of the mortgage interest payments in Section 9. In Section 10 we conclude.

2 Some Features of the Housing Market

Office of Federal Housing Enterprise Oversight (OFHEO), National Association of Realtors (NAR) and Census Bureau are the major sources of housing price data in the US. McCarthy and Peach (2005) show that the Census constant quality index consistently predicts less housing price appreciation than the other two. The NAR index and the OFHEO index have very close housing price predictions. But the bottom line is that all of them predict that housing prices have been rising exceptionally, especially in the last 10 years. Here we present the housing prices index from NAR, after adjusting for the inflation. Figure 1 shows that the real median price of a house in the US increased more than 50 percent in the last 20 years. Housing prices have been increasing more than the national average for most of big cities. The median price in New York, San Francisco, Boston and Los Angeles almost doubled in last 10 years (figure 3).

Transaction volume (number of existing units sold) has been increasing steadily during the last three decades other than the sharp decline around 1980’s. After that decline, transaction volume more than tripled in the last two decades (figure 4). The data comes from The 2005
State of the Nation report prepared by the Joint Center of Housing of Harvard University. It includes existing detached single-family homes and townhomes, but does not include condos and co-ops.

An interesting and also puzzling feature of the housing market is the high positive correlation between transaction volume and housing prices (the correlation coefficient is around 0.9 for the national data). In figure 5 we plot the price and volume series together. To show the comovement clearer, we divide every number in both series with the mean of the series that they belong to (we do a rescaling). Another way of showing the comovement in the high frequency is using the Hodrick-Prescott filter to remove the trends. By using this method, Rios-Rull and Sanchez-Marcos (2006) show that the comovement of both series holds also for higher frequencies.

Housing prices and transaction volume have been highly volatile. Genesove and Mayer (2001) report that in a real estate cycle, from peak to trough, it is not unusual to see a 50 percent decline in transaction volume, especially in local markets. Rios-Rull and Sanchez-Marcos (2006) show that housing prices and transaction volume are much more volatile than the GDP both in Canada and the US. In table 1 we report the dispersion of some housing market variables measured by coefficient of variation. Housing prices in big cities are more dispersed than the national prices and transaction volume is more dispersed than housing prices.\(^7\)

\(^7\)Coefficient of variation of interest rates are calculated by using the net interest rates. If we use gross
Table 1: Housing Market Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV(real 15 year mortgage rate)</td>
<td>0.50</td>
</tr>
<tr>
<td>CV(housing price, national)</td>
<td>0.05</td>
</tr>
<tr>
<td>CV(housing price, big cities)</td>
<td>0.30</td>
</tr>
<tr>
<td>CV(transaction volume, national)</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Note.– CV is coefficient of variation and calculated as standard deviation divided by mean.
A house is the major asset of a typical household in the US. As can be seen from figure 6, from 40th percentile to 85th percentile, housing constitutes more than half of the wealth of the consumers. Only after 97th percentile corporate equities become more important in the portfolio. The large share of housing in the portfolios of the most of the households also makes them very undiversified, which then make the house price movements a risk factor for both the individuals and the aggregate economy.

3 The Model

The model is a two period overlapping generations model. At the beginning of each period, the economy is hit by an exogenous interest rate shock. A young agent enters into the economy with a limited amount of resources which makes him borrow in the mortgage market to finance his housing purchase. After he buys his house he receives his remaining income, saves in the bond market and consumes the non-housing consumption good. At the rates the coefficient of variation is around 0.03.

\[8\]In this section we present the two period version of the model for clarity. In section 5 we extend it to 30 periods to have a more realistic calibration.
end of the period he makes the first mortgage payment. Old agents receive idiosyncratic housing taste shocks, which can be interpreted as changes in family sizes or changes in need of housing, causing optimal housing to be different from the one obtained when young. Since there are fixed transaction costs, some of the agents will choose to sell their existing houses and some will choose to stay. If an old agent sells his house, he gets the return from selling the house and pays the remaining mortgage debt coming from the previous house. Then he borrows in the mortgage market to finance his new house. If he does not sell his house, he makes the second mortgage payment and uses the remaining income to purchase non-housing consumption. At the end of their lives, all agents sell their houses and leave the economy. There are risk-neutral profit-maximizing banks which offer mortgage contracts. At the equilibrium, banks are indifferent between lending in the bond market and the mortgage market. For simplicity we assume that aggregate house supply is fixed and there is no rental market.

### Mortgage Market

Risk-neutral profit-maximizing banks offer fixed-rate 30 year mortgage contracts. We have two conditions to find the mortgage rates and the corresponding payments. The first
condition is the *present value condition*, which means that the present value of the payments should be equal to the loan amount.

\[
1 = \frac{D_t}{1 + d_t} + \frac{D_t}{(1 + d_t)^2}
\]  

(1)

An agent should pay \(D_t\) if he borrows 1 dollar in the mortgage market when the mortgage rate is \(d_t\). The next condition is the *no-arbitrage condition*. At equilibrium, banks should be indifferent between lending in the bond market and the mortgage market.

\[
1 = \frac{D_t}{1 + r_t} + (1 - \pi_t)\frac{D_t}{(1 + r_t)E[(1 + r_{t+1})]} + \pi_t\frac{1 - D_t + d_t}{1 + r_t}
\]  

(2)

The lender will receive the first mortgage payment \(D_t\) at the end of the first period, and he will discount this with current interest rate, \(1 + r_t\).\(^9\) With probability \(1 - \pi_t\) the agent will not move and he will do his second mortgage payment, which is \(\frac{D_t}{(1 + r_t)E[(1 + r_{t+1})]}\). With \(\pi_t\) probability the agent will move and prepay his loan by the beginning of the second period. The discounted value of this payment is \(\frac{1 - D_t + d_t}{1 + r_t}\). In the solution of the model, we should also find \(D_t\), \(d_t\) and \(\pi_t\) as functions of state variables.

The old agents who buy a house get a 15 year mortgage contract, and the interest on the mortgage is the same as the interest rate on the bond.

**Old Agent’s Problem**

An old agent decides whether to move or not, and if he moves how big a house to purchase depending on the taste shock \(\theta_h\) he received and the state of the market. The maximization problem is,

\[
V_{old}(\theta_h, h_{t-1}; P_{t-1}, H_t, H_{t-1}, r_{t-1}, r_t) = \text{Max} \{V_{move}, V_{stay}\}
\]  

(3)

The aggregate state of the economy is summarized by \(P_{t-1}, H_t, H_{t-1}, r_{t-1}, r_t\). The price of housing, \(P_{t-1}\), the quantity of housing, \(H_t\), and the interest rate, \(r_{t-1}\) are all part of the aggregate state. We used \(t - 1\), because it is the time when they first bought their houses. The price, the quantity, and the interest rate at \(t - 1\) are all effective for the old agents’ decisions today. The aggregate amount of housing held by the oldest generation which is

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\(^9\)As the agents in the model can transact only after 15 years, the corresponding discount factor will be “5 to the power of yearly interest rate”.
leaving the economy is $H_{t-2}$. The current period interest rate is denoted as $r_t$. The quantity of housing of an old agent is $h_{t-1}$. The value of moving can be formulated as,

$$ V_{move}(\theta_h, h_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = \max \{ U(c_{t,o,m}, h_{t,o,m}) + \beta E[U(c_{t,o,l}, 0)] \} \quad (4) $$

In equation 4, $c_{t,o,m}$ is the consumption, $h_{t,o,m}$ is the housing choice of an old moving agent. We assume that when the agents leave the economy, they sell their houses and use the returns from the transaction to purchase non-housing consumption good (no housing for the leavers). The value of the houses to the agents when they leave the economy is $\beta E[U(c_{t,o,l}, 0)]$. The utility function is quadratic in both consumption and housing. By quadratic utility function we are able to find most of the decision rules and all of the value functions analytically which simplifies the computation significantly.

$$ U(c_{t,o,m}, h_{t,o,m}) = -\alpha_c/2(\theta_c - c_{t,o,m})^2 - \alpha_h/2(\theta_h - h_{t,o,m})^2 \quad (5) $$

The parameters of the utility function of an old agent are $\alpha_c, \theta_c, \alpha_h, \theta_h$. The taste shock that an old agent receives at the beginning of the period is $\theta_h$. We assumed a taste shock in this form because of its convenience.

$$ c_{t,o,m} = w_{old} + s_{t-1}(1 + r_{t-1}) - \lambda P_t h_{t,o,m} - \frac{(1 - \lambda)P_t h_{t,o,m}(1 + r_t)}{1 + r_t} + P_t h_{t-1} - (1 - \lambda)P_{t-1}h_{t-1}(1 - D_{t-1} + d_{t-1}) - \Delta \quad (6) $$

If an old agent chooses to move, first he will sell his old house, then pay off the remaining mortgage debt. The seller will receive $P_t h_{t-1}$ as the price. The remaining debt from the old mortgage is $(1 - \lambda)P_{t-1}h_{t-1}(1 - D_{t-1} + d_{t-1})$. Once he is done with the old house, he purchases a new house by borrowing in the mortgage market. His mortgage payment at the end of the period is $(1 - \lambda)P_t h_{t,o,m}(1 + r_t)$. We discount this payment with $1 + r_t$ to measure...
the real cost of mortgage payment to the agent. The buying process has a fixed transaction cost of $\Delta$ and agents should make a down payment equivalent to $\lambda P_t h_{t,o,m}$, where $\lambda$ is the down payment ratio. In equation 6, $w_{old}$ is old agent’s income and $s_{t-1}(1 + r_{t-1})$ is the return from the savings made when young. The consumption of an agent who leaves the economy is equal to his return from selling his house.

$$c_{t,o,l} = P_{t+1} h_{t,o,m}$$  \hspace{1cm} (7)$$

The optimal housing decision of an old agent is,

$$h_{t,o,m} = \frac{\alpha_c P_t (-\Delta + h_{t-1})[P_t + (1 + \lambda)P_{t-1}(1 - D_{t-1} + d_{t-1})] + s_{t-1}r_{t-1} - \theta_c}{\alpha_h + \alpha_c (P_t^2 + \beta E[P_{t+1}^2])} \hspace{1cm} (8)$$

Although equation 8 seems complicated there are intuitive comparative static results. For example, if the transaction cost $\Delta$ increases, $h_{t,o,m}$ will decrease. This is intuitive since higher transaction costs will leave the moving agents with less assets, and with less assets agents cannot afford larger houses. With a similar intuition, higher mortgage debt from the previous period decreases the optimal quantity of housing of the mover. If expected prices $E[P_{t+1}]$ increase, movers’ optimal housing will increase, since the return of holding a larger house becomes larger. Higher taste shock $\theta_h$ implies higher optimal quantity of housing. Keeping the expected price constant, if $E[P_{t+1}^2]$ increases, demand for housing decreases. Here, $E[P_{t+1}^2]$ can be thought of as a measure of variance of the price. As variance increases (if mean stays same) risk averse agents will buy less housing.

Because of the existence of the transaction costs, not all agents will transact. The value of staying in their old houses is,

$$V_{stay}(\theta_h, h_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = U(c_{t,o,s}, h_{t,o,s}) + \beta E[U(c_{t,o,l}, 0)] \hspace{1cm} (9)$$

In equation 9, $c_{t,o,s}$ and $h_{t,o,s}$ are consumption and housing of an old agent if he stays in his old house. Before leaving the economy old agents sell their houses and get $P_{t+1} h_{t,o,s}$, and use it to buy non-housing consumption good, $c_{t,o,l}$. 

14
$$c_{t,o,s} = w_{old} + s_{t-1}(1 + r_{t-1}) - \frac{(1 - \lambda)P_{t-1}h_{t-1}D_{t-1}}{1 + r_t} \quad (10)$$

If the agents stay they will pay the remaining mortgage of their house, which is $(1 - \lambda)P_{t-1}h_{t-1}D_{t-1}$ and consume the rest.

$$h_{t,o,s} = h_{t-1} \quad (11)$$

Their housing will be the same housing they lived in during the previous 15 periods, $h_{t-1}$.

Given the state of the economy there will be $\theta_{low}$ and $\theta_{high}$ such that the old agents who receive a taste shock $\theta$ in between, will choose to stay, while the others will move (an S-s rule).\textsuperscript{14} As the old agents with taste parameters $\theta_{low}$ and $\theta_{high}$ should be indifferent between moving and staying, the value of $\theta_{low}$ and $\theta_{high}$ can be found by equating the value of staying to value of moving\textsuperscript{15}.

$$\theta_{low}(h_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) \quad (12)$$

$$\theta_{high}(h_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) \quad (13)$$

Once the boundaries of inactivity region are found we can also find the transaction volume implied by the model.

**Young agent’s problem**

Young agents (no heterogeneity) enter into the economy with limited amount of resources. They borrow in the mortgage market to finance their housing purchases. Then they receive their income, save for the next period, consume and make their first mortgage payments. Young agents solve the following optimization problem,

\textsuperscript{14} The existence and uniqueness of the boundaries hold under very general conditions.

\textsuperscript{15} We have the boundaries in closed form, but because of their complexity we do not present them here.
\[
V_{young}(P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) =
\max_{s_{t,y}, h_{t,y}} \left\{ U_y(c_{t,y}, h_{t,y}) + \right. \\
+ \beta E \left. \left[ \int_{\theta_{\text{min}}}^{\theta_{\text{high}}} V_{\text{move}} f(\theta) d\theta + \int_{\theta_{\text{low}}}^{\theta_{\text{high}}} V_{\text{move}} f(\theta) d\theta \right] \right\} \\
+ \beta E \left[ \int_{\theta_{\text{low}}}^{\theta_{\text{high}}} V_{\text{stay}} f(\theta) d\theta \right]
\]
\begin{align}
cty &= w_{young} - s_{t,y} - \lambda P_t h_{t,y} - \frac{(1 - \lambda) P_t h_{t,y} D_t}{1 + r_t} - \Delta \tag{15}
\end{align}

In the utility function of the young agents \(ct_{t,y}\) is the consumption, \(ht_{t,y}\) is the housing, and \(st_{t,y}\) is the saving at time \(t\). Young agent’s income is \(w_{young}\). They have a quadratic utility function, similar to the old’s utility function, the difference being the value of \(\theta_h\). We assume,

\[
U_y(c_{t,y}, h_{t,y}) = -\alpha_c/2(\theta_c - ct_{t,y})^2 - \alpha_h/2(\theta_h - ht_{t,y})^2
\]

While making their choices, they take into consideration that it will affect their moving decisions when they are old. We assume that the taste shock is uniformly distributed. Here we know the boundaries in closed form, so we may find the integrals algebraically which allows us to write the first order condition of the young agent in closed form.

**Market Clearing Condition**

There are two market clearing conditions that should be satisfied. The sum of the quantity of housing held by each generation should be equal to aggregate house supply \((\overline{H_{tot}})\). The first market clearing condition can be written as,

\[
\overline{H_{tot}} = \sum_{i=0}^{1} \overline{H_{t-i}} \tag{17}
\]

The total amount of housing owned by age \(i\) generation is \(\overline{H_{t-i}}\).

\textsuperscript{16}The market clearing conditions that we use here are more general than we need for a two period setting. But these are necessary when we modify the model while doing the calibration.
The market for available houses should also be cleared, as in each period only limited amount housing will be available for trade. This is equivalent to,

$$H_{t-2} + H_{t-1} = H_{t,o,s} + H_{t,o,m} + H_{t,y} \tag{18}$$

The left hand side of the equation is the total available housing for trade. $H_{t-2}$ is the total housing owned by the leavers and $H_{t-1}$ is the housing owned by period $t$ old agents. In the right hand side of equation 18, $H_{t,o,s}$ is the amount of housing held by the old generation who did not move, $H_{t,o,m}$ is the demand of the movers. The young agents’ housing consumption which we compute by using their optimization problem is $H_{t,y}$.

$$H_{t,o,s} = \int_{\theta_{low}}^{\theta_{high}} h_{t-1} f(\theta) d\theta \tag{19}$$

$$H_{t,o,m} = \int_{\theta_{min}}^{\theta_{low}} h_{t,o,m} f(\theta) d\theta + \beta \int_{\theta_{low}}^{\theta_{high}} h_{t,o,m} f(\theta) d\theta \tag{20}$$

4 Computational Algorithm

The exogenous process in the model is the interest rates. In a given period we expect that the endogenous variables are mostly affected by current interest rates. Normally, if we knew all the history of interest rates then we could (theoretically) know the current prices and choices, and we would not need any other state variables. However, it would be computationally impossible to solve. Here we assume that the most important part of the interest rate history is the part which is close to the current date.

In the model’s formulation, current period price should be formulated as,

$$P_t = F(P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) \tag{21}$$

As $P_{t-1}, H_{t-1}$ are also endogenous variables they would mostly be affected from interest rates. This reasoning will go until time 0. At that point current price would be a function of the history of interest rates.

$$P_t = F(r_0, \ldots, r_{t-1}, r_t) \tag{22}$$
Now suppose that agents are boundedly rational and they consider only the recent history of the interest rates while making their decisions. We find that $r_{t-1}, r_t$ gives a very accurate solution. We do not make any functional form guesses for any function. We find the values of the functions at those states only.

$$P_t = F(r_{t-1}, r_t) \quad (23)$$

$$H_{t,y} = G(r_{t-1}, r_t) \quad (24)$$

To form our objective function we simulate an interest rate history. At all points in the interest rate history we algebraically find the first order condition and market clearing condition.\(^{17}\) Our objective function is the sum of squared errors of first order conditions and market clearing conditions across time. Then we use a minimization technique (we used both local and global techniques) to solve for prices and allocations jointly that minimize the errors. This algorithm is much faster than the weighted residuals method, which seemed to be the best method (compared to the other available algorithms) for the particular model, and gives more accurate results. We demonstrate the errors in the appendix. We made some robustness analysis by adding more history to the agent’s decision. Additional histories gave similar results with higher accuracy. To check the possibility of multiple equilibria we solved the model also with global optimization techniques. We could not find any other equilibria.

## 5 Calibration

Although we have most of the decision rules in closed form, we cannot solve housing prices and the young agent’s housing decision analytically. For those, we need to calibrate the model to have a numerical solution comparable to the US data. If available, we take the parameter values directly from the U.S. data. For the other ones, we pick some target statistics from the US data, and try to match them with the model’s predictions by choosing the free parameter values.

To make the calibration more realistic we assume that the model period is a year. We allow agents to transact only twice in their life time to make the computation easier. First transaction is done when they are young (30 years old) and the second transaction can be

\(^{17}\)For the algorithm to work the algebraic first order conditions and market clearing conditions are necessary.
done (they may choose not to transact) when they are old (45 years old). We assume that agents leave the economy at age 60 after selling their house. This set-up implies that, given that the model period is a year, there will be 30 generations living together in a period, but only three of them (young, movers and leavers) will be transacting. In addition to their housing transaction decision, we allow agents to save only when they are 30 years old. Specifically, this extension helps us to calibrate the interest rates better. Extending the model in this particular way, helps us to solve the model with the computational algorithm that we described in the previous section.

In the next step, we are planning to improve the calibration by assuming each year consumers make a mortgage payment (still transacting in only 15 years). In this case the mortgage market will be more realistic. Also, it will be easier to calibrate the income growth of the households by age.

**Normalizations**

Housing supply in the model is set to 30 to allow each generation to hold 1 unit of housing on average. We set $\alpha_h = 2$.

**Parameters taken from the literature**

The yearly discount factor, $\beta$ is 0.96.

The down payment ratio, $\lambda$ is 0.2 (Campbell and Cocco (2003))

The fixed transaction cost, $\Delta$ is set to 0.015. This value corresponds to approximately 8 percent of the value of the house purchased, which is in the range of reported values (Smith, Rosen, and Fallis (1988)).

We assume that each generation has 1 unit of income ($\frac{w_{young} + w_{old}}{2} = 1$). In a two period set-up it is hard to calibrate the income growth of the households. An increase in income in the range of 50 percent seems plausible (US Census Bureau median household income by age of the householder).

**Statistics used for calibration**

We use the following 5 target statistics to calibrate the remaining 5 parameters; $\theta_y$, $\theta_c$, $\alpha_c$, $\theta_{\text{min}}$, $\theta_{\text{max}}$

1-The coefficient of risk aversion in consumption is on average 1.
2-The national average of housing price to income is around 2.6 (Joint Center for Housing Studies of Harvard University, table A-1). Since we have 15 years of income, our adjusted target price to income ratio is 0.22 (We use the discounted sum of payments).

3-Approximately, the aggregate housing wealth of people between ages 45-54 is 25 percent higher than aggregate housing wealth of people between ages 35-44 (Hurst, Luoh and Stafford (1998), table 1.2).

4-We use percentage of people who transact in 15 years to calibrate variance of the taste shock \( \theta \). PSID data shows that 5.44 percent of the respondents moved during the last year (Cocco (2000)). The corresponding number for 15 years would be 56 percent.

5-Hanushek and Quigley (1980) shows that price elasticity of housing is around \(-0.5\).

**Interest Rates**

We assume the stochastic process in the model is the real interest rate on a 15 year fixed-rate mortgage. This fits into our model, because when old agents borrow in the market we assumed they could get loans with the period interest rate. In reality it will correspond to a 15 year mortgage rate. To find the real rate on a 15 year fixed-rate mortgage, we simply subtract the inflation in that year from the nominal rate.\(^{18}\) The average of real 15 year fixed-rate mortgage during 1975-2005 period is 5 percent. Then we estimate an AR(1) process of these real rates which gives the autocorrelation coefficient as 0.88. Finally, we use Hussey and Tauchen (1991) method to approximate the AR(1) process with a 5 state Markov chain. The coefficient of variation of real 15 year fixed-rate mortgage is around 0.5. Because of our computational algorithm\(^{19}\), we cannot exactly match both the mean and the coefficient of variation at the same time. In the next sections we report the results for 6 percent average interest rate with 0.43 coefficient of variation. We also solved for different combinations of mean and coefficient of variation and none of the results depend on the specific choice we made here.

6 **Results**

Before going into more through analysis of the model’s findings we will demonstrate the model’s response to an interest rate history similar to the history observed in the US. Because

\(^{18}\)This is equivalent to assuming current period’s inflation is the best predictor of the future inflation.

\(^{19}\)If we increase the coefficient of variation to 0.5 then we have boundary solutions. Even though the algorithm can handle boundary solutions for most of the states, for a small number of states it cannot, which decreases the accuracy of the solution.
of computational reasons we assume the maximum value of the interest rate is 10 percent and the minimum value is 2 percent, whereas in the US, interest rates were as high as 15 percent at early 80’s and as low as 1.5 percent at 2005. With these values we can approximate the 1982-2005 period. In figure 8 we plot housing prices and transaction volume implied by the model. To make it comparable to the U.S. data, we rescale the value of housing price and the transaction volume to match their values in 1982. The average median housing price in big cities was approximately $200,000 and the transaction volume (number of units sold) was 2,000,000.

The median housing price in the model increases around 50 percent. We should remind that the decline in the interest rates that we feed into the model is smaller than the one observed in the US. If we had the original history the model would predict around 70 percent increase in the housing prices. The percentage increase that the model predicts can explain almost all of the housing price appreciation in the national level and about half of the price appreciation in big cities. Interestingly, the model predicts a price decline in the 90’s, the decline being more in the ”Unit Housing Price” measure. This prediction is also consistent with the U.S. data. In the 90’s, most of big cities as well as the US median housing prices declined.\footnote{We explain the reason of this prediction in the next section.} Transaction volume increases by around 25 percent in the model which is lower than the observed increase in the housing transactions which we presented in figure 4. The increase in the transaction volume in figure 4, includes the new home buyers who switch from renting because of lower down payment and attractiveness of the housing market. which we do not have in the model. If we had more than one transaction choice in the model, the model would be able to predict a larger increase in the transaction volume. Similar to the housing prices, the model predicts the decline in the transaction volume in the 90’s. Finally, even in this short time period, we see that housing prices and transaction volume move together.

6.1 Housing Prices

In this section, we show the functional relation between housing prices and interest rates. Figure 9 depicts housing price as a function of current interest rates. To obtain the figure, we took the average of housing prices across different possible histories. On the $y$ axis we have $Log$ of the price and on the $x$ axis we have $Log$ of the current interest rate. Unit prices are higher than the median prices because the size of the median house is almost always smaller than 1.

\footnote{Price of a unit size of house.}
The slope of the graph gives the interest elasticity of housing prices. The elasticity is around $-5$, which means a 1 percentage point decrease in interest rates will cause prices to increase by 5 percent. The real interest rates decreased more than 10 percentage points during last two decades.\footnote{The real interest rates in the model corresponds to real interest rates on a 15 year FRM contract.} In this case, the model predicts housing prices would appreciate approximately 50 percent. This amount of housing price appreciation explains a large part of the housing price appreciation in big cities (it explains almost all of the increase in the national median housing prices).

There are other factors that affect the current period housing prices. The biggest effect comes from last period’s interest rate, as it affects the savings of the agents in the last period.
If the interest rate during the last period was high it means that the old agents will have more assets in the current period, which increases their willingness to pay higher housing prices. Looking at the history of the interest rates we see that it was very high in the 80’s (more than 10 percent) and very low in the last decade (2 – 3 percent). Because of this reasoning, the interest (last period) elasticity of unit housing prices is 4.8. For the median housing prices the elasticity is smaller, 2.5. This is due to the change in the size of the median house as response to increase in prices. If housing prices increase because of the last period’s interest rate, the young agent’s mortgage payments increase linearly with the prices, which decreases the housing demand of the young agent. As in the model, the median house is always bought by a young agent, the size of the median house will decrease, pushing down the median housing price.

It is this last period’s interest rate effect that caused the model to match the decrease in housing prices in 1990 in figure 8. In our approximate interest rate history, during the 1982-1985 period, interest rates are 10 percent. During this period, because of the high interest rates, agents in the model save more, bringing extra assets to the next period (1988-1992). In 1988, interest rate decreases to 6 percent. The low interest rates together with extra assets
of old agents cause housing prices to increase. As this period is a low interest rate period, young agents in this period save less compared to the young agents of 1985. When the next period (1991-1994) comes although interest rates are still 6 percent, in this period there will be less asset available, which will cause a price decline.

In the model, we have only a risk free asset that agents can use as an investment tool as an alternative to housing. In reality there is also the stock market which had booms and busts in the past. Similar reasoning would hold if we had stock market instead of the bond market. The large amount of wealth accumulated during the stock market boom can be transferred to the housing market as the boom ends, pushing the housing prices up.

In literature, while questioning the existence of a housing price bubble, one of the measures that researchers used is the housing price-income ratio. Especially for the high housing price appreciation cities, such as New York and San Francisco, looking at the housing price-income ratio they find evidence of a housing price bubble, since that ratio became unusually high compared to its historical values. In the model, we did not have any income change. The average housing price to income ratio is 2.6. As housing prices fluctuate because of interest rates, the ratio becomes more than 3.5 when housing price reaches its maximum, and it becomes less than 1.5 when housing prices reaches its minimum. Our conclusion is, housing price-income ratio cannot be a real measure of the real cost of housing since that measure does not take into account the effect of interest rates on the housing prices.\footnote{Although we did not have rental market in the model, a similar conclusion will hold for housing price to rent ratio. The key thing is, interest rates affect housing prices, as it determines the cost of borrowing, while it would not affect the price of rent that much.}
Predictability of the Housing Prices

The last period interest rate has predictive power on the current period's price. If the last period’s interest rate was high, people expect (on average) higher prices. This is because young agents in the last period saved more, making them more wealthy in the current period which then increases their willingness to pay higher prices. In figure 10 we showed the relation between housing prices and the last period’s interest rate. The elasticity of prices with respect to last period interest rate is approximately $4$.

To show the importance of savings, we solve the model with the same parameters, but we assume agents cannot save. Figure 11 shows that there is almost no predictability anymore and the new elasticity is approximately $-0.5$.

One may think that the predictability can be coming from fixed transaction costs. To check that claim we solve the model with no transaction cost ($\Delta = 0$). The new interest rate (last period) elasticity of housing is around $5$. So the transaction cost does not cause the predictability in the model.

Is There a Bubble in the Housing Market?

The unusually high housing price appreciations in the housing market raised questions about a possible housing bubble. If it is a bubble, it will be the biggest bubble in history, since it is a global phenomenon. Before going into our findings, we write the definition of the housing price bubble written in Case and Shiller (2003).
The term bubble is widely used but rarely defined very closely. We believe that in its widespread use the term refers to a situation in which excessive public expectations for future price increases cause prices to be temporarily elevated. During a house price bubble, home buyers think that a home that would normally be considered too expensive for them is now an acceptable purchase because they will be compensated by significant further price increases.

In the previous section we showed that housing prices are predictable in the model. Think about an old agent in a high interest rate period. He expects high housing prices next period (because of reasons explained in the previous section). Since in the model setup we assumed the agents who leave the economy sell their houses before leaving the economy, high expected prices in the next period increases housing demand of the old agents.

\[ h_{t,o,m} = \Lambda + \frac{\beta \alpha_c \theta_c E[P_{t+1}]}{\alpha_h + \alpha_c (P_t^2 + \beta E[P_{t+1}^2])} \]

The effect of price expectations on the housing demand of an old agent is, \( \frac{E[P_{t+1}]}{\alpha_h + \alpha_c (P_t^2 + \beta E[P_{t+1}^2])} \) (\( \Lambda \) is used for the other effects). Given the bubble definition of Case and Shiller (2003), the model predicts housing price bubbles when interest rates are high. In other words, there will be housing price bubbles when prices are low. On the other hand, when housing prices are high (or interest rates are low), current housing prices will be lower than the case where the prices were not predictable. This is because people expect low housing prices in the next period and low price expectations in the next period will decrease the prices today.

### 6.2 Transaction Volume

Stein (95), Ortalo-Magne and Rady (2004) use the credit constraints and large down payments to explain the positive relation. Wheaton (90) and Goodman and Berkovec (96) use information imperfections to explain the comovement. Genesove and Mayer (2001) claims that transaction volume-price correlation is consistent with the loss aversion of the sellers.

The scatter plot in figure 12 shows that housing prices and transaction volume are positively related in our model as in the U.S. data. But the mechanism which generates the positive relation is different from the mechanisms used in Stein (95), Ortalo-Magne, Rady
To check the effect of down payment on transaction volume we solve the model with no down payment requirement. The results look very similar, meaning the down payment requirement is not an important part of the mechanism of the model that generates the comovement of prices and transaction volume (see appendix figure 23).

Figure 13 shows the effect of the current and the last period’s interest rate on the transaction volume. To obtain the graph in the left panel, we average across different histories. For the right panel we average across different possible transaction volumes in the current period. The graph in the right panel of Figure 13 shows that the current period interest rate and transaction volume are negatively related. The interest (current) elasticity of transaction volume is around $-3$, which implies a 10 percent decrease in interest rates, similar to the U.S. data, will cause a 30 percent increase in the transaction volume, explaining a significant part of the increase in the transaction volume. In the left panel of figure 13, we plot transaction volume as a function of last period’s interest rate. The last period interest elasticity of transaction volume is around 3. Both effects combined helps the model to predict the large increase in the transaction volume observed in the U.S. data.

To see the intuition clearly we write the costs and returns of a transaction. To make it

\[ \text{(2004)}^{24}. \] The mechanism is obviously different from Wheaton (90) and Goodman and Berkovec (96), as the model does not have any information frictions.
simpler we assume that the down payment requirement, \( \lambda \), is 0 (We already showed that it is not crucial in the model). If an agent sells his house the return from the transaction is

\[
P_t h_{t-1} - P_t h_{t,0,m} - P_t h_{t-1} (1 - D_{t-1} + d_{t-1})
\]

The return the agent gets when he sells his old house is \( P_t h_{t-1} \). The agent will pay \( P_t h_{t,0,m} \) for his new house. The last term, \( P_t h_{t-1} (1 - D_{t-1} + d_{t-1}) \), is the payment that the agent has to make to the lender for the debt coming from the old house he sold. When the agents stays in his house his cost of housing will be,

\[
\frac{P_t h_{t-1} D_{t-1}}{1 + r_t}
\]

We discount the second mortgage payment \( P_t h_{t-1} D_{t-1} \) with \( 1 + r_t \) to find the discounted value of the payment.

Suppose that the current interest rate is low. In this case the second mortgage payment will be more expensive (compared to a higher interest rate state) for an agent who borrowed in the last period since the denominator in \( \frac{P_t h_{t-1} D_{t-1}}{1 + r_t} \) will be smaller. As the cost of the second mortgage payment increases more people find it optimal to move, which increases the transaction volume. We also showed in the previous section that low interest rates cause high prices. If we combine both of the findings, it is the interest rates which cause the comovement of the housing prices and transaction volume. But this is only half of the story. We showed in figure 13 and figure 10 that last period’s interest rate positively affect the transaction volume and housing prices. If the last period’s
interest rate was high, it means that the agents who borrowed in the mortgage market during the last period have higher interest payments in the current period. If the cost of staying is high, then more people choose to move. This is the other channel that causes the comovement of the housing prices and transaction volume.

### 6.3 The Dispersion of Interest Rates, Housing Prices and Transaction Volume

The coefficient of variation of housing prices is 0.05 for the national data and 0.3 for big cities. The model predicts a coefficient of variation of 0.14 for housing prices. This is approximately half of the observed value. The model predicts a coefficient of variation of 0.16 for the transaction volume, which is more than half of the observed value. It appears that interest rates can be a major factor in explaining the volatilities in the housing market.

Rios-Rull and Sanchez-Marcos (2006) document that housing prices and transaction volume are more volatile than the GDP. Since we do not have income uncertainty in the model all of the volatility in the housing market comes from the fluctuations in the housing market. If we had income uncertainty in the model, this would further increase the volatilities in the housing market and this would make them closer to the U.S. data.

### 6.4 Housing and Saving

We have two assets in our model, housing and a risk free bond. In addition to serving as an asset, housing also gives utility to the owner. Furthermore, interest rates affect both the prices and the returns in the housing market, making the relation between them more sophisticated. In this section we analyze the housing and saving choices of the agents in our model.

In figure 14 (upper graphs) we plot the young agent’s housing as a function of the current and the last period’s interest rates implied by the model. If the last period interest rate is
Figure 14: Young’s Housing and Saving. For the effect of current interest rates averages over history is taken. For the effect of past interest rates averages over possible interest rates in the current period are taken.

If current interest rates are high, the old agents in the current period will have more wealth compared to a low interest rate history which causes higher prices in this period, and higher prices cause higher mortgage payments. In this case mortgage payments will increase linearly with prices. Higher mortgage payments will decrease the quantity of housing demanded by the young agents. On the other hand, if current interest rates increase, housing becomes less attractive as an asset which then causes a decline in the young agent’s housing choice.

Saving is the other option for a young agent which he can use to accumulate wealth and transfer resources from one period to the other. In the model we assume that young agents can save and borrow at the same rate. We present our findings in figure 14 (lower graphs). The first and more apparent (and intuitive) conclusion from the graph is that savings and interest rates are positively related. More interestingly, young agents in the model have negative liquid wealth for more than half of the states. They start to save only when the real
returns exceed 8 percent. The maximum saving amount is 5 percent of their income and it is seen when the interest rates reach 10 percent. When the interest rate are low, young agents choose to borrow, which for the lowest interest rates exceed 25 percent of their income.

6.5 The Importance of the Mortgage Market

The existence of the mortgage market in the model made the computation much more intensive. Then it is fair to ask whether the mortgage market is important in our findings. To eliminate mortgage market from the model we assume that the down payment requirement, \( \lambda \), is 1. In this case the agents will have to pay all the price of their purchase in cash, so they will not borrow in the mortgage market. We calibrate the model using the same targets that we used to calibrate the baseline model. Our results show that: A model without a mortgage contract has a similar interest elasticity of housing prices (about -5). Without the mortgage contract the model is incapable of explaining the negative correlation between transaction volume and interest rates. Transaction volume has a much smaller coefficient of variation (around 0.03).

These findings show that mortgage market is a key component of the baseline model. (See figures 20, 21 and 22 in the appendix)

7 Macroeconomic Effects of Housing Price Movements

One of the reason why housing is important for macroeconomics is, housing price changes may affect the consumption of the agents, as it is the biggest part of the wealth for most of the households. The large price movements will also cause large movements in wealth of the majority of the society, which can cause movements in consumption pushing economies to booms or recessions. A major concern in the last couple of years was, whether in the case of a housing price decline the economy will go into a recession.

In this section we analyze the relation between housing prices and consumption. First we report the results where we allow agents to save (our baseline model), then we report the results where we shot down the saving channel. The no-saving part is also important since most of the consumers in the US do not save. Solving for no-saving case also helps us to decompose the effect of saving on consumption.
7.1 With Saving

Our computational algorithm relied on the assumption that only the recent history of interest rates matter. The results of the solution showed us going back only 1 period, provided us very accurate results. After this point, we will report our results, mostly, as a function of current and last period interest rates.

Figure 15 shows that aggregate consumption is decreasing with the current interest rate. But, it is hard to tell why it is so by just looking at this graph (from savings or housing?). To further analyze the case, we decompose the aggregate consumption into young agents’ and old agents’ consumption. Old agents’ consumption is an increasing function of current interest rates. As old agents do not have saving choice, all of the increase comes from the housing price movements. Quantitatively, if interest rates increase from 2 percent to 3 percent, old agents’ consumption will increase from 0.86 to 0.87 which is equivalent to 0.5 percent of the total income\(^{25}\). The consumption of the old agents increase, because as interest rates increase housing prices decrease. Lower housing price is good for the agents who move to bigger houses but bad for the ones who move to smaller houses. As most of the old agents move to bigger houses, on average, old agents are better off with lower housing prices. Young agents’ consumption decreases from 0.75 to 0.45, but as the young agents have the saving choice, the decrease comes mainly from the increase in the savings because of the higher interest rates.

To isolate the effect of housing price movements on consumption, we look at the housing expenditures. If housing expenditure decreases, the amount of decrease will be available for consumption (for old agents it will be consumption, but for young agents some part may be used for saving). For the young agents housing expenditure is the down payment, the first mortgage payment, and the transaction cost.

\[
\lambda P_t h_{t,y} + \frac{(1 - \lambda)P_t h_{t,y}D}{1 + r_t} + \Delta
\]  

(26)

For the old moving agents in addition to the mortgage payment we also include gains from selling their old houses, their prepayment amount and the transaction cost\(^{26}\)

\[
P_t h_{t,o,m} - P_t h_{t-1} + (1 - \lambda)P_{t-1}h_{t-1}(1 - D_{t-1} + d_{t-1}) + \Delta
\]  

(27)

\(^{25}\)To calculate the aggregate output we weigh each generations’ consumption with 0.5, their population weight.

\(^{26}\)We include gains from selling their old house because it is another channel which housing can affect consumption.
For the old agents who do not move, only housing expenditure is their second mortgage payment. Since we have fixed rate mortgage, their discounted cost of second mortgage payment is,

\[ P_{t-1} h_{t-1} D_{t-1} \frac{1}{1 + r_t} \]

We demonstrate the effect of interest rates on housing expenditure in figure 16. With higher interest rates housing prices decrease which decreases the housing expenditure of the young agents. Also for the old agents housing expenditure decreases. As most of the old agents want to upgrade their houses lower prices decrease their housing expenditure. For the old agents who move to smaller houses, their housing expenditure increases with higher interest rates. Overall, housing expenditure decreases if interest rates increase.

### 7.2 Without Saving

The consumers in our baseline model have the saving choice. An the equilibrium model predicts that when interest rates are 10 percent, consumers save around 5 percent of their income. At the other extreme, when interest rates are 2 percent, the model predicts 25 percent borrowing. These predictions do not come very close to a real consumer in the US.
Moreover, we want to see the pure effect of housing prices on the consumption, not the one combined with savings.

Figure 17 summarizes our findings with the saving channel closed. As with the saving case the aggregate consumption increases with higher interest rates. This time, as we closed the saving channel, young agents’ consumption almost does not change. Similar to the saving case the increase in consumption comes from the increase in the old agent’s consumption. If interest rates increase from 2 percent to 3 percent, a possible scenario for the US, aggregate consumption increases around 0.5 percent.

The main result of this section is that higher interest rates will increase the consumption. The underlying reason behind this result is that in the model most of the agents who are moving are moving to larger houses. As they will pay for the extra size of the house, lower prices will increase their consumption and decrease the housing expenditure. There may be other scenarios which we do not have in the model that can reverse the implications of the model. For example, if the economy is hit by a low income shock, most of the agents will decrease their housing consumption. In this case, aggregate consumption as well as moving agents’ consumption will be decreasing with higher interest rates. In fact, in the next section we show that welfare of the old agents crucially depend on their taste shock. If they receive a low taste shock which causes them to move to a smaller house they are better
of with low interest rates. Another important assumption which we made and which can affect the results is that old agents will get a 15 year mortgage contract. If we had extra periods in the model and allowed the old agents to borrow in the 30 year mortgage market then old agents would potentially use some part of the returns from selling their houses as consumption. Then with lower interest rates we would see higher transaction volume and higher consumption.

8 A Welfare Analysis

In this section we analyze the effect of housing price movements on welfare (we use value function as a measure of welfare). Although we say the effect of “housing price movements”, what we will do is, analyze the welfare as a function of interest rates, both of which together closely approximate the economy. The conclusion of the analysis can easily be extended to infer the “effect of housing price movements on welfare”, as we already know the price-interest relationship from the previous analysis.

We first look at the welfare of the each generation (figure 18). As young agents borrow most of the time, rising interest rates worsens their welfare. Only when the interest rates are
around 10, percent they start to save and higher interest rates start to be welfare improving for them. For the old agents rising interest rates are welfare improving as most of the agents upgrade their houses. With higher interest rates they pay less for their higher quantity of housing that they purchase.

We go further and ask whether all moving old agents are affected in the same way from interest rates. In this part we separate the old agents into two. First we plot the welfare of the old agents who receive a low taste shock (low $\theta_h$) and then we plot the welfare of the old agents who receive a high taste shock (high $\theta_h$). We find that, while the old agents with low taste shock are better off if the economy is hit by low interest rate shock causing high housing prices, the old agents with high taste shock are worse off. The increase in the young agents’ welfare in the left panel when interest rate is 10 percent is due to positive savings.

9 Tax Reform

The tax deductibility of the mortgage interest payments gives additional incentives to the consumers to be homeowners, which then affects both prices and allocations. Here we ask what would happen to the economy, if there is a tax reform which eliminates this incentive. This question is also motivated by the current tax reform proposal in the US.

In our baseline economy we did not have any taxes. To study the effect of the tax reform we introduce taxes into the model. We assume each homeowner will receive 20 percent of his
mortgage interest payment as a tax return.\textsuperscript{27} We solve the model with the same parameters that we used for the baseline. Then we implement the tax reform by assuming agents cannot get any tax returns anymore. We solve the model again with the same parameters. We report our results in table 3.

As a result of the tax reform, median housing prices decline around 25 percent. Young agents’ non-housing consumption increases around 6 percent because of the lower housing prices. Old agents non-housing consumption decreases around 1 percent. The tax reform affect young and old agents differently because they pay different amount of interests on their mortgages. As old agents get a 15 year mortgage contract and as they buy larger houses the amount of interest payment is larger for them which makes the tax return more important. This is consistent with the intuition of Nicolas Retsinas, director of the Joint Center for Housing Studies at Harvard University. He says, “The people who have the biggest homes, who make the most money are the greatest beneficiaries of this tax subsidy”. The housing consumption of both agents remains almost same after the tax reform.

To further compare both policies we ask what percent of their consumption are the consumers willing give to stay with the current policy. This is an exercise similar to the one performed by Lucas (1987). It appears that both young and old are worse off with the tax reform. Young agents would be willing to give up their 1.3 percent of their consumption to avoid a tax reform. Old agents are willing to give around 6 percent of their consumption, more than the young agents. Despite the increase in the consumption of the both agents

\textsuperscript{27}We make this assumption because of computational reasons. Higher values gives less accurate results. We also assume that government finances the tax return from other resources, not from the homeowners.
Table 3: The Effect of a Tax Reform

<table>
<thead>
<tr>
<th></th>
<th>Before Tax Reform</th>
<th>After Tax Reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median Price</td>
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<td>0.200</td>
</tr>
<tr>
<td>Young’s Consumption</td>
<td>0.699</td>
<td>0.737</td>
</tr>
<tr>
<td>Old’s Consumption</td>
<td>0.986</td>
<td>0.979</td>
</tr>
<tr>
<td>Young’s Housing</td>
<td>0.857</td>
<td>0.859</td>
</tr>
<tr>
<td>Old’s Housing</td>
<td>1.143</td>
<td>1.141</td>
</tr>
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</table>

because of a tax reform, they do not like it, because it will decrease the value of their assets.

The welfare exercise that we performed only compares the steady states before and after the tax reform. If we could include the transition, the costs of a tax reform which eliminates the tax deductibility of the mortgage interest payments will be higher. Moreover, we only take into account the consumers. The costs will be even higher if we considered the effect on the house production and investment. As a conclusion, the welfare costs of tax reform that we report here can be seen as a lower bound.

10 Conclusion

In this paper, we have examined the role of interest rate movements in determining the fluctuations in housing prices and in transaction volume. First, we found that the substantial decrease in interest rates in the last two decades can account for a large part of the increases in real housing prices. In the next step, we analyzed the model’s implication about transaction volume. The model can account for both the comovement of housing prices and transaction volume and the increase in the level of the transaction volume. Moreover, the model implies that housing prices and transaction volume have high volatilities.

The present model abstracts from some important aspects of the housing market to isolate the effect of interest rates. First of all, we assumed that income does not fluctuate. It is straightforward to write and solve a very similar model with income uncertainty instead of interest rate uncertainty. It is also possible to solve a model with both income and interest rate uncertainty.\textsuperscript{28} On the other hand there are some ingredients in the model that mimic the behavior of income such as savings. We showed that the effect of interest rates on the

\textsuperscript{28}In this case we should use 2 or 3 state Markov processes for both income and interest rates.
housing prices are similar to the effect of savings on the housing prices. We also assumed that housing supply is fixed. This assumption makes the model work better for places where land is scarce or zoning restrictions are more effective. It would be interesting to include home production into the model and see the effect of zoning restrictions on the housing market.

We assumed that there are only FRM contracts available. We solved (but did not report) a very similar model for adjustable-rate mortgage (ARM) contracts. Housing price and transaction volume movements implied by ARM contracts are close to the movements implied by the FRM contracts. But it is our interest to see in a future research the implications of type of mortgages on the macro economy.

For computational reasons, we assumed that the agents can transact only twice during their life time. Adding extra transaction choices to the model would bring large computational burden. Another computationally costly extension is adding mortgage choice to the model. We solved a case where we had mortgage choice, but in that case we had to assume that interest rates were following a two state Markov process. We are still looking for ways by which we can incorporate these extensions without much computational burden.

References


APPENDIX

The Effect of Mortgage Contract

Without a mortgage contract we would not be able to have a negative relation of interest rates and transaction volume. As could be seen from figure 20 interest rates have a very small effect on the transaction volume. As a result, the coefficient of variation of transaction volume is very small, around 0.03 (one tenth of the observed value).

Figure 20: Transaction Volume without the Mortgage Contract.

The housing price implications of a model without mortgage contracts are similar to the one with the mortgage contract (figure 21).

Figure 22 shows that the model without the mortgage contract fails to predict the increase in the transaction volume.
Figure 21: Housing Prices without the Mortgage Contract

Figure 22: The Response of a Model without the Mortgage Market to an Approximate Interest Rate History
The Effect of Downpayment Requirement

Several papers used large downpayment requirements to explain the comovement of the transaction volume and housing prices. In figure 23 we show that the existence of the downpayments is not the driving force in the model.

Figure 23: Comovement of the Transaction Volume and Housing Prices. No Downpayment

First Order Condition and Market Clearing Errors