Investment Banks as Information Providers in IPOs

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Abstract

Financial intermediaries are known to have access to privileged information on firm value, potentially providing important services by revealing it to uninformed investors. An important issue that arises is whether Investment Banks have an incentive to manipulate prices by communicating biased information on the firms they are underwriting. Reputation acquisition may mitigate this problem as intermediaries may lose credibility by incorrectly forecasting the profitability of firms. In this paper we argue that the introduction of reputation may not suffice to eliminate all scope for manipulation, allowing less talented intermediaries to profit from not revealing their private information to the market.

1 Introduction

The role of information providers in reducing the informational asymmetries in financial markets has received considerable attention. In this paper we consider the incentives of Investment Banks (IBs) as information providers
in the underwriting process of IPOs. We show that these agents have both the ability and the incentives to manipulate asset prices through strategically distorted announcements.

We address this question in a signalling game with three classes of players: Investment Banks, Firms and Investors. Firms sell shares in an equity market with asymmetric information either directly or through investment banks. Investment banks have better information on firm profitability than the market (although still incomplete) and interact with the equity market, evaluate entrepreneurs’ projects and report to investors in return for a fee. We assume that whenever an IB underwrites a firm’s equity, the underwriting comes together with a report of the evaluation performed by the IB about the state of the firm.

IBs differ for having a different "evaluation technology". By "evaluation technology" we mean the ability of the IBs in acquiring accurate information about the true state of the firm whose equities are eventually underwritten. We assume that the information technology is exogenously determined by nature and cannot be changed by the IB. The belief held by the market about the ability (i.e. the type of evaluation technology) of an IB represents its reputation and we assume that it affects the IB’s payoffs.

In such a framework, we investigate the firm’s decision to go public either through an IB or directly, and the IBs’ decisions to underwrite and report their private information to the market.

Since the compensation the IB gets from the firm for the underwriting activity is typically related to the success of the IPO, the IB faces a strong incentive to inflate the price of the firm’s stock through distorted reports in order to enhance the short term profits from the underwriting fee. In fact, these incentives are limited by the IB’s concerns about its own reputation. Indeed, since the IB interacts with the market repeatedly, biased reports may cause a loss of reputation, which may in turn lead to a loss of future profits.

We show that in some cases reputation is not enough to induce all IBs to
truthfully report their private evaluation. We define such equilibria as informa-
tionally inefficient because investment banks manage to influence the price of shares without transmitting their private information on firm profitabil-
ity. In particular, less talented IBs that have a worst evaluation technology endogenously assign less weight to future compensations based on reputation. Therefore, they strategically distort their private information in order to enhance short-run profits. Furthermore, investment banks with the worst technology will have a greater incentive to disregard their signal pretending to be more informed, the greater is the market perception of investment bank ability (initial reputation).

On a theoretical ground, the role of IBs’ reputation in IPOs has been explicitly considered by Fulghieri and Chemmanur (1994). However, they fo-
cus on the impact that reputation has on the IBs’ incentives to improve their evaluation technology, and do not address the issue of information transmis-
sion, which is the main object of our analysis.

In this respect, our model is closely related to the burgeoning literature on reputational cheap talk and in particular to the papers by Benabou and Laroque (1992) and Ottaviani and Sorensen (2006). In Benabou and Laroque (1992), insiders perform the joint actions of speculating and spreading information at no intrinsic cost, managing to manipulate prices repeatedly, without being fully discovered. Insiders do not differ in their forecasting abilities (i.e., they all receive an equally informative signal), but rather in their degree of honesty in reporting private information. In particular, some types of insiders are constrained to provide truthful reports, while others are allowed to act strategically. In our model, IBs are characterized by different forecasting abilities and the reporting strategies of all types of IBs are determined endogenously.

In a very general setup, Ottaviani and Sorensen (2006) study information transmission by a privately informed expert concerned about being perceived to have accurate information. They characterize the expert’s incentives to
deviate from truthtelling in a setup in which the expert is solely concerned with the receivers’ perception of his forecasting ability. We draw upon their model and adapt the information transmission analysis to the context of the IPO underwriting activity. The institutional setup we consider allows us to analyze the issue of information reporting in an economic setup in which the expert is not exclusively concerned about his reputation. Indeed, the expert’s concern for being perceived to have accurate information is entwined with the concern for the impact that his report has on the decision of the firm and eventually on the price of the firm’s stock.

Trueman (1994) considers a model where analysts with different forecasting abilities are concerned about building a good reputation for their forecasting accuracy. He finds that analysts display herding behavior, whereby they disregard their private information and release forecasts similar to those previously announced by other analysts in order to maximize their expected reputation. His finding is in line with Sharfsein and Stein (1990) where managers exhibit herding behavior in a framework in which the expert has to make an investment decision as opposed to reporting his private information to a third party. In these papers, experts choose their actions sequentially and, as in Ottaviani and Sorensen (2006), are solely concerned about their reputation.

Our work is also related to the recent literature on analysts’ conflict of interest. On an empirical ground, Michaely and Womak (1999) show that underwriters’ analysts tend to release over-optimistic recommendations in the attempt to inflate the stock price of the firm taken public by their IB. Morgan and Stocken (2003) present a theoretical model that analyzes the informational content of stock reports when investors are uncertain about an analyst’s incentives. Analyst incentives may be aligned with those of investors or misaligned. They find that any investor uncertainty about incentives makes full revelation of information impossible. Under certain conditions, analysts with aligned incentives can credibly convey unfavorable information, but can
never credibly convey favorable information. The first difference with respect to our work is that in their model analysts do not differ in the degree informativeness of their signals, but in the degree of divergence of their preferences with respect to those of investors. Basically, as in Benabou and Laroque (1992), the analyst is not concerned about being perceived as having accurate information, but about being perceived as honest. Furthermore, though our model is suited to study analysts’ conflict of interests, it is nevertheless thought to address the issue of information production and transmission in the period preceding the offer date, when the IB’s role of information producer and provider affects not only the decisions of the investors, but also those of the firm candidate to go public.

The main departure of our paper from the previous literature comes from recognizing that poorly informed intermediaries may incorrectly discount the reputation ”punishment” precisely because of their scarcely informative signals. This may lead them to overvalue the immediate benefit of communicating biased messages to the market, both in the direction of overstating and understating the firm’s value. This effect may imply that reputation acquisition is not sufficient to induce all intermediaries to report their private information.

The paper is organized as follows. In section 2, we introduce the general setup of the model by defining what we intend for value of a firm and reputation, where the information structure is such that the bad IBs have less informative private signals. In section 3, we analyze the both condition under which truthtelling by IBs is possible and the incentives that IBs have to deviate from truthtelling. We characterize a family of ”partial pooling” equilibria where talented IBs transmit truthful evaluations while untalented IBs transmit untruthful evaluations to the market and manage to influence prices of firms. In section 4 we compare the relative informational efficiency of different market scenarios as defined by the relevant parameters. Section 5 concludes.
2 The Model

We consider a financial market populated by a large pool of firms that want to go public, a large pool of investment banks (IBs) that possibly underwrite their shares and a large pool of investors interested in buying the firms’ shares. Firms differ in their fundamental values. IBs differ in their ability to recover information about the true value of the firm that they possibly underwrite.

Suppose that there is only one period $t$. At the beginning of $t$, a firm and an IB are randomly selected from their respective pools and matched.\textsuperscript{1} The IB privately evaluates the firm and proposes its underwriting conditions, which consist of: 1) an evaluation of the firm to be publicly communicated to the market; 2) a fee that the firm must pay to the IB. The firms observes both the fee and the proposed evaluation and chooses either to be underwritten by the investment bank or to go public directly.

If the IPO occurs through an IB’s service, the IB’s evaluation reaches the market and, based on this evaluation, investors determine the price of the firm’s shares. If the firm goes public directly, investors determine the value of its shares based on the observation that the firm has refused to use the IB.

At the end of $t$, the true value of the firm is revealed and observed by all market participants. Thus, every player in the market can compare the true value of the firm with the actions taken by the IB and the firm and accordingly form his own belief about the IB’s ability of recovering information on the true value of a firm that is about to go public. We interpret this belief as the IB’s reputation about its ability and assume that all IBs care about their own reputation.

The rest of the section is devoted to explain in more detail the model just described.

\textsuperscript{1}The analysis of how the firm chooses the IB is out of the scope of the present paper.
2.1 Agents

2.1.1 Firms

There are two types of firms. High profits firms, whose true value is 1 and low profits firms, whose value is 0. Let $F$ denote the value of a firm operating at $t$ and assume that $F \in \{0, 1\}$. Let $\theta$ be the fraction of high type firms and $1 - \theta$ the complementary fraction of low type firms. Notice that $\theta$ can be interpreted either as the prior probability at time $t$ that the firm is worth 1, or as the probability at time $t$ that the firm is worth 1 given the past history up to $t$, that we denote with $\Omega_{t-1}$. Formally, $\theta = \Pr(F = 1 | \Omega_{t-1})$. Let us assume that $\theta$ is common knowledge and that firms do not know their own type.\(^2\)

A firm can choose either to accept ($A$) or refuse ($R$) to be underwritten by an IB. This choice is taken after the IB has assessed the quality of the firm and revealed the firm the evaluation that will be sent to the market in case the firm accept to go public. If the firm were to refuse to be underwritten, it has the outside option of going public directly.

2.1.2 Investment Banks (IBs)

Although IBs do not know firms’ types, they receive a private signal about the true type of the firm. This signal is binary and can be either high or low. Let $S_h$ and $S_l$ denote the events that IB receives a high or low signal in period $t$ respectively.

We assume that there are two types of IBs, good ($G$) and bad ($B$). Let $IB$ denote a generic Investment bank active at $t$, so that $IB \in \{G, B\}$. Good IBs receive a more informative signal\(^3\) about the true state of the firm than

\(^2\)This seemingly implausible assumption is without loss of generality. Furthermore, notice that in reality, most of the firms that aim to go public do not have accurate information about the way the market is going to react to the IPO. One reason to hire an IB for the IPO is exactly that of getting some help in determining how the market perceive the offer.

\(^3\)We assume that signals are private and non-verifiableAccordingly, a court cannot
bad IBs, as described by the following probability distributions:

\[
\begin{align*}
Pr(S_h \mid IB = G, F = 1) &= Pr(S_l \mid IB = G, F = 0) = p, \quad p \in \left[\frac{1}{2}, 1\right] \quad (1) \\
Pr(S_h \mid IB = B, F = 1) &= Pr(S_l \mid IB = B, F = 0) = z, \quad z \in \left[\frac{1}{2}, p\right] \quad (2)
\end{align*}
\]

This information structure allows each type of IB receiving a signal to update the prior on firm’s state and thus form its own belief about the fact that the firm is good. In particular, we have that

\[
\begin{align*}
Pr(F = 1 \mid IB = G, S_h) &= \frac{\theta p}{\theta p + (1 - \theta)(1 - p)} \\
Pr(F = 1 \mid IB = G, S_l) &= \frac{\theta(1 - p)}{\theta(1 - p) + (1 - \theta)p} \\
Pr(F = 1 \mid IB = B, S_h) &= \frac{\theta z}{\theta z + (1 - \theta)(1 - z)} \\
Pr(F = 1 \mid IB = B, S_l) &= \frac{\theta(1 - z)}{\theta(1 - z) + (1 - \theta)z}
\end{align*}
\]

Let \(\alpha\) be the fraction of good IBs, while \((1 - \alpha)\) the complementary fraction of bad IBs. Also for IBs, \(\alpha\) can be interpreted either as the prior probability of being good, or as the probability at time \(t\) of being good given all the history up to \(t\), \(\Omega_{t-1}\), that is \(\alpha = Pr(IB_t = G \mid \Omega_{t-1})\). We assume that \(\alpha\) is common knowledge and that each IB knows its own type.

Once the signal is received, the IB chooses which evaluation to publicly release in the form of a binary message \(s \in \{s_h, s_l\}\).\(^4\) The evaluation is distinguish whether the analyst received the high or low signal. This prevents a contract from being written with payment contingent on the analyst truthfully reporting the private signal.

\(^4\)Notice that we use \(S\) for the private signal received by the IB and \(s\) for the message sent by the IB. While the first is determined exogenously by the IB’s type, the latter is a
observed by the firm, that decides whether to accept or refuse to be underwritten. The IB’s message reaches the market if and only if the firm accepts to be underwritten. Let $\sigma_{IB}$ denote a behavioral strategy for the IB of type $IB$.

**IBs’ reputation.** We assume that at the end of period $t$ the true value of the firm $F$ can be observed. Thus, every player in the market can compare it with the observable actions taken by the IB and the firm and accordingly update his own belief about the ability of the IB. We interpret the updated belief about the IB’s ability as the new level of reputation acquired by the IB at the end of period $t$ and we denote it with $\tilde{\alpha}$. Formally, in the case in which the firm accepted to be underwritten and evaluation $s_j$ has eventually reached the market, we have that:

$$\tilde{\alpha} = \Pr(IB = G \mid F, s_j)$$

where $F \in \{0, 1\}$ and $s_j \in \{s_l, s_h\}$. On the other hand, in the case in which the firm refused to be underwritten and thus no evaluation has eventually reached the market\(^5\), we have that

$$\tilde{\alpha} = \Pr(IB = G \mid F, R)$$

Clearly, the value of $\tilde{\alpha}$ is endogenous, since it depends on the equilibrium strategies of firms and IBs and on the equilibrium beliefs held by investors, in a way that will be clear soon.

To ease notation, let us define

$$\tilde{\alpha}_{F,s_j} \equiv \Pr(IB = G \mid F, s_j) \text{ in case the firm accepts}$$

$$\tilde{\alpha}_{F,R} \equiv \Pr(IB = G \mid F, R) \text{ in case the firm refuses}$$

\(^5\)See appendix 1 for a better insight about IBs’ updated reputation and for relative computations.
2.1.3 Investors

There is a large pool of risk neutral investors interested in buying the shares of the firm that goes public. We assume that investors observe the IB’s report and then bid à la Bertrand in order to get the firm’s shares. This implies that the stock price of the firm, \( v \), is set equal to its expected value given all publicly available information, \( \Omega_t \).\(^6\) Hence, if the firm accepts to be underwritten, the IB’s message reaches the market and \( \Omega_t = \{\Omega_{t-1}, s\} \) and

\[
v = \Pr(F = 1 \mid \Omega_{t-1}, s)
\]

On the other hand, if the firm refuses to be underwritten, no IB’s message reaches the market. The only information available to the market is the refuse of the firm and

\[
v = \Pr(F = 1 \mid \Omega_{t-1}, R)
\]

To ease notation, let \( V(s_j, \alpha) \equiv \Pr(F = 1 \mid s_j, \alpha) \), with \( j \in \{h, l\} \) and \( d \equiv \Pr(F = 1 \mid R) \). In words, \( V(s_j, \alpha) \) is the value of the firm when underwritten by an IB with prior reputation \( \alpha \) sending an evaluation \( s_j \). On the other hand, \( d \) denotes the value that the market assigns to a firm that chooses to go public directly (refusing to be underwritten by an IB).

It is important to stress that both \( V(s_j, \alpha) \) and \( d \) will be determined endogenously in equilibrium, that is, they will depend on the equilibrium strategies of IBs and on the equilibrium (and out of equilibrium) beliefs of the investors.\(^7\)

2.2 The fee

A firm that goes public has to pay an underwriting fee to the IB. We assume that the firm pays a fee equal to a fraction \( k \) of value \( V(s_j, \alpha) \) that the IB assures to the firm by underwriting its shares. Formally, the fee is given by

\(^6\)That is, the market for the IPO is semi-strong efficient
\(^7\)Firm values are derived in appendix 1.
$kV(s_j, \alpha)$. We assume that $k \in (0, 1)$ and that it is determined exogenously. It is important to bear in mind that the fee is paid only by firms that go public (i.e., by firms that accepts to be underwritten by an IB).\(^8\)

### 2.3 Payoffs

**Firms** The payoff of a firm that goes public at time $t$ is assumed to be given by the following function:

$$
\pi^F = \begin{cases} 
(1 - k)V(s_j, \alpha) & \text{if the firm accepts} \\
\alpha & \text{if the firm refuses}
\end{cases} \tag{3}
$$

**Investment Banks.** The underwriting activity is typically characterized by the presence of both explicit and implicit incentives. The explicit incentives are those related to the direct compensation that the IB gets for assisting the firm along the IPO process, that is, the underwriting fee. The implicit incentives are those related to the reputation that the IB acquires about its ability and honesty to provide correct information to the market. Usually, these incentives works in opposite directions. Indeed, since the compensation the IB gets from the firm is usually proportional to the success of the IPO, the IB has the incentive to inflate the value of the firm. On the other hand, this incentive is mitigated by the fear of building up a bad reputation. Indeed, a bad reputation would translate into a loss of market shares in the market of the underwriting activity (and hence in a loss of future fees), since no firm would use an IB with a bad reputation. Accordingly, we assume that by sending a message $s_j$, with $j \in \{h, l\}$, an investment bank $IB \in \{G, B\}$ gets:

$$
\pi^{IB} = \begin{cases} 
\alpha + kV(s_j) + \hat{\alpha}_{F,s_j} & \text{if the firm accepts} \\
\hat{\alpha}_{F,R} & \text{if the firm refuses}
\end{cases} \tag{4}
$$

\(^8\)In a very simplified way, this linear fee structure bears some of the essential features present in the contractual arrangements used in practice (see Chemmanur Fulgheri, 1994).
where $kV(s_j, \alpha)$ represents the part of the IB’s payoff related to the IB’s compensation for the service provided during the IPO, and $\hat{\alpha}_F$ represents the reputational component of the IB’s payoff.\footnote{Notice that we are here assuming for simplicity that the two payoff components (i.e. the fee-related and the reputational component) are equally weighted. In a more general framework, we should consider a payoff structure given by:}

$$\pi^{IB} = \beta k(V(s_j, \alpha) + (1 - \beta)\hat{\alpha}$$

with $\beta \in [0, 1]\footnote{Notice that from the point of view of an IB, $V(s_j, \alpha)$ is a non-stochastic value. Indeed, $V(s_j, \alpha)$ depends on $s_j$, which is decided by the IB, and on $\alpha$, which is a given value at the beginning of the period in which the IB makes its evaluation. On the other hand, $\hat{\alpha}_F$ is a stochastic value, since at the moment in which evaluation $s_j$ is proposed (and eventually sent to the market), the IB does not know $F$.}

The previous reduced form is meant to represent the trade-off that an IB typically faces while producing and reporting information for the market along the IPO process, where the effects of the evaluation activity persist well beyond the immediate benefits of providing untruthful information to the market.\footnote{This reduced form is widely adopted in many papers that deals with experts or managers’ reputation or career concerns (see for example Holmstrom 1982, Shafstein and Stein 1990, Dasgupta and Prat 2004 and Jackson, 2005)}\footnote{From a technical point of view, since the payoff of the IB (the sender) depends on the belief of investors (the receivers), our game belongs to the class of psychological games. See Battigalli and Dufwenberg (2005) for an analysis of extensive-form psychological games.}

### 3 Equilibrium Analysis

For the sake of simplicity and w.l.o.g, from now on, let us assume that the good IB has a completely informative signal (i.e., $z < p = 1$). We will focus on equilibria in pure strategies as mixed strategies would complicate computations without significantly altering the intuition behind the results. Within this framework, we first analyze the conditions under which there exist truth-telling equilibria where both IBs honestly report the signal they have received. We will show that truth-telling by both IBs is not always guaranteed and that there exist both partial pooling equilibria where the good...
IB truthfully reports its signal, while the bad IB always pools around the high evaluation, and *partial pooling equilibria* where the good IB truthfully reports its signal, while the bad IB always pools around the low evaluation.

### 3.1 Truth telling equilibria

We start analyzing whether the presence of a reputational component in the IB’s payoffs may induce IBs to truth tell.

In a truth telling equilibrium, the strategies of the good and bad IBs are such that both types of IBs report the signal received (formally, for every \(j \in \{h, l\}\) and \(IB = \{G, B\}\) we have that \(\sigma_{IB}(s_j | S_j) = 1\)), conditional on firm accepting to be underwritten. We will focus on (putative) equilibria in which the firm accepts after \(s_h\) and refuses after \(s_l\).\(^{13}\) It follows that in any equilibrium in which IBs truth tell, only evaluation \(s_h\) reaches the market. On the other hand, in a truth telling equilibrium in which the firm refuses after \(s_l\), a refuse by the firm is interpreted as the IB proposing \(s_l\), so that \(\hat{\alpha}_{F,R} \equiv \hat{\alpha}_{F,s_l}\). Accordingly, \(\hat{\alpha}\) can assume only the following two values *in equilibrium*\(^{14}\):

\[
\hat{\alpha}_{\text{min}} = \hat{\alpha}_{0,s_h} = \hat{\alpha}_{1,R}
\]
\[
\hat{\alpha}_{\text{max}} = \hat{\alpha}_{1,s_h} = \hat{\alpha}_{0,R}
\]

where \(\hat{\alpha}_{\text{max}} > \alpha > \hat{\alpha}_{\text{min}}\).

A first consequence of our assumption that the good IB receives com-

\(^{13}\)It can be shown that in terms of information transmission by IBs, these equilibria have the same qualitative properties of those equilibria in which the firm accepts to be underwritten both after a low and a high evaluation.

\(^{14}\)See appendix 1
pletely informative signals $^{15}$ is that:

\begin{align*}
\tilde{\alpha}_{\text{min}} &= 0 \\
\tilde{\alpha}_{\text{max}} &= \frac{\alpha}{\alpha + z(1 - \alpha)}
\end{align*}

Let us consider the values the firm can take on in this equilibrium (on and off the equilibrium path). In equilibrium, the firm accepts after $s_h$ and refuses after $s_l$. Accordingly, let $V^{TT}(s_h, \alpha)$ denote the value of the firm in this truthtelling equilibrium when the firm accepts and evaluation $s_h$ reaches the market. It is easy to show $^{16}$ that:

$$V^{TT}(s_h, \alpha) \equiv \Pr(F = 1 \mid s_h, \alpha) = \frac{\theta [\alpha + (1 - \alpha)z]}{\theta [\alpha + (1 - \alpha)z] + (1 - \theta)(1 - \alpha)(1 - z)}$$

Accepting after $s_l$ is an out of equilibrium action by the firm. In this case, $s_l$ would reach the market. Given the equilibrium strategies of the IBs that prescribes truthtelling, the market would evaluate the firm accordingly. In particular, let $V^{TT}(s_l, \alpha)$ denote the (out of equilibrium) value of the firm in this truthtelling equilibrium when the firm has been reported a low signal. In this case, we have that:

$$V^{TT}(s_l, \alpha) \equiv \Pr(F = 1 \mid s_l, \alpha) = \frac{\theta(1 - \alpha)(1 - z)}{\theta(1 - \alpha)(1 - z) + (1 - \theta)[\alpha + (1 - \alpha)z]}$$

Finally, in equilibrium, the firm refuse after $s_l$. Since a refuse follows after $s_l$ has been proposed, we have that:

$$d \equiv \Pr(F = 1 \mid R) = \Pr(F = 1 \mid s_l, \alpha) \equiv V^{TT}(s_l, \alpha)$$

We are ready to check under which conditions there exists an equilibrium in which both IB’s type truthtell and the firm accepts only after a good

$^{15}$ Again, see appendix 1 and remember that here we are assuming $p = 1$.

$^{16}$ See appendix 1 for the computation of $V^{TT}(s_h, \alpha)$ and $V^{TT}(s_l, \alpha)$. 
evaluation is proposed to be reported.

**IBs’ problem.** An investment bank of type IB receiving private signal $S_j$ will truthtell if and only if the expected profits from reporting the private signal is greater than that obtained when reporting an evaluation that differs from the private signal. Notice that given the assumed equilibrium strategy of the firm (accept if $s_h$ and refuse if $s_l$), the IB gets no underwriting fee when proposing $s_l$. Hence, truthtelling is an equilibrium if for every $IB \in \{G, B\}$, the following conditions holds:

\[ kV^{TT}(s_h, \alpha) + E(\hat{\alpha}_{F,sh} \mid IB, S_h) \geq E(\hat{\alpha}_{F,R} \mid IB, S_h) \]  
\[ E(\hat{\alpha}_{F,R} \mid IB, S_l) \geq kV^{TT}(s_h, \alpha) + E(\hat{\alpha}_{F,sh} \mid IB, S_l) \]  

As mentioned previously, $\hat{\alpha}_F \cdot$ is the investors’ belief about IB’s ability at the end of period $t$. This can be interpreted as the new level of reputation acquired by the IB at the end of $t$, when the true value of the firm has been observed and used to assess the ability of the IB. However, by the time the IB proposes its evaluation, the value of the firm is not known. Therefore, an IB computes the expected reputation from truthfully reporting (or misreporting) the signal it has received, that is $E(\hat{\alpha}_{F,\cdot} \mid \cdot, \cdot)$.

Notice that conditions (5) and (6) can be written as follows:

\[ kV^{TT}(s_h, \alpha) \geq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,sh} \mid IB, S_h) \]  
\[ kV^{TT}(s_h, \alpha) \leq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,sh} \mid IB, S_l) \]

In words, $kV^{TT}(s_h, \alpha)$ represents the gain from the underwriting fee (that the IB gets by providing a high evaluation instead of a low evaluation, since the low evaluation would be follow by a refuse of the firm). The RHS of (7) is the net expected reputational payoff of misreporting the signal received when the signal is $S_h$. The RHS of (8) represents the net expected reputational payoff of correctly reporting the signal received when the signal is $S_l$. 

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Lemma 1 A sufficient (and necessary) condition for there to be a truthtelling equilibrium is that the truthtelling conditions for the bad IB are satisfied. (proof in appendix 2)

This result follows directly from the fact that the good IB has a more informative signal and therefore it assigns greater weight to the expected reputation loss of providing an incorrect evaluation. In other words, if a bad IB has the incentive to truthtell, then, a fortiori, this must be true for a good IB too. The lemma above allows us to focus on the truthtelling conditions of the bad IB to determine the truthtelling equilibrium. Thus, we just have to prove that the two following conditions are satisfied:

\[ kV_{TT}^T(s_h, \alpha) \geq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} | B, S_h) \quad (9) \]
\[ kV_{TT}^T(s_h, \alpha) \leq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} | B, S_l) \quad (10) \]

Lemma 2 For any values of \( k \in (0, 1), \alpha \in (0, 1) \) and \( z \in (\frac{1}{2}, 1) \), there always exist a \( \theta_{TT}^T \in (0, 1) \) and a \( \theta_{TT}^T \in (0, 1) \) with \( \theta_{TT}^T > \theta_{TT}^T \) such that for any \( \theta_{TT}^T \leq \theta \leq \theta_{TT}^T \) conditions (9) and (10) are satisfied and IBs truthtell in equilibrium. (proof in appendix 2)

For an intuition of the previous result, focus on the case of a bad IB. A bad IB receives an informative but imprecise signal. When the values of the prior on firm profitability \( \theta \) are relatively extreme (so that it is ex ante very likely that the actual value of the firm is 0 or 1), a bad IB gets less confident about a private signal that is contrarian to what indicated by the prior. Consider for example the case in which \( \theta \) is close to 1 and the bad IB receives \( S_l \). When \( \theta \) is close to 1, the gain from fees of reporting a high evaluation instead of a low evaluation are close to zero. Indeed, since the prior is very high, the market expects the firm to be valuable, whatever the evaluation sent. Basically, the LHS of (8) gets close to zero. On the other hand, when \( \theta \) is close to 1, the bad IB (which cannot count on a very precise signal), expects that the high state is more likely than the low one (even if the
signal received is $s_l$). Accordingly, the bad IB expects that it will be more likely to be correct and to improve its reputation when sending $s_h$ instead of $s_l$.

We are left with proving that given the truth-telling strategies of both types of IB, when $\theta^{TT} \leq \theta \leq \bar{\theta}^{TT}$, it is in fact optimal for the firm to follow the strategy of accepting after $s_h$ and refusing after $s_l$. We will show that this in fact case whenever the underwriting fee is not too high. Indeed, it is apparent that if the fee were very high, all the benefits from using an IB in order to reduce the information asymmetries in the market would be ripped off by the cost of the IB’s service. In particular, we impose that the fee satisfies the following condition:

$$k < \bar{k}^{TT} \equiv \frac{V^{TT}(s_h, \alpha) - V^{TT}(s_l, \alpha)}{V^{TT}(s_h, \alpha)} \quad (11)$$

**The firm’s problem.** If the firm is proposed $s_l$ and according to its equilibrium strategy refuses, it gets $d = \Pr(F = 1 \mid s_l) = V^{TT}(s_l, \alpha)$. If the firm deviates by accepting after $s_l$, then it is valued $V^{TT}(s_l, \alpha)$ and gets $(1 - k)V^{TT}(s_l, \alpha)$. Thus, for the firm is always optimal to refuse after $s_l$.

If the firm is proposed $s_h$ and accepts, it gets $(1 - k)V^{TT}(s_h, \alpha)$. If it refuses, the market believes that $s_l$ has been proposed and the firm gets $V^{TT}(s_l, \alpha)$. Thus, for the firm is optimal to accept after $s_h$ as long as $(1 - k)V^{TT}(s_h, \alpha) \geq V^{TT}(s_l, \alpha)$. Notice that under condition (11), this inequality is always satisfied.

We can thus summarize the previous results in the following proposition.

**Proposition 1** Let $\bar{k}^{TT}$ be defined by condition (11). Then, for any given value of $\alpha \in (0, 1)$, $z \in (\frac{1}{2}, 1)$ and $k \in (0, \bar{k}^{TT})$, there always exist a $\bar{\theta}^{TT} \in (0, 1)$ and a $\bar{\theta}^{TT} \in (0, 1)$ with $\bar{\theta}^{TT} > \bar{\theta}^{TT}$, such that for any $\theta \in \left[\bar{\theta}^{TT}, \bar{\theta}^{TT}\right]$ there exists an equilibrium in which both good and bad IBs propose to truthfully report their private information, and the firm accepts to be underwritten only when a good evaluation is proposed.
In such equilibria, only good evaluations reaches the market (because it is never the case that a firm goes public with a low evaluation), and these evaluations are credible. However, we have shown that a truthtelling equilibrium exists only for intermediate values of \( \theta \). Whenever the prior on firm’s value gets too extreme, truthtelling is destroyed by the incentives of the bad IB to report the signal that is more likely to reveal correct ex-post (that is, once the value of the firm has realized). In words, if for example common sense (represented by the prior) suggests that a firm is very likely to be highly profitable and the IB is not too confident about its low signal, then the IB does not contradict common sense. This result can be interpreted as a sort of conservative and conformist behavior by the bad IB when the prior on firm profitability is too high (or too low) relative to its signal precision. The important conclusion is that this result is driven by exactly those incentives that allow truthtelling to be sustainable in a region of the parameters space, that is the reputational concerns of the IB. When the prior on firm’s value is relatively high, the bad IB fails at perceiving the trade off between the temptation to inflate the firm’s value and ex-post reputation. In fact, when \( \theta \) is very high, for the bad IB it is true that ex-ante, the incentives related to the underwriting fee and those related to expected reputation are aligned. On the other hand, when the prior on firm’s value is relatively low, the fear to incur in a reputational punishment is much stronger than the temptation to inflate the firm’s stocks.

This result allows us to formulate the conjecture that for extreme values of \( \theta \), there exist equilibria in which good IBs will truthtell and bad IBs will always provide a negative evaluation when \( \theta \) is low and always provide a positive evaluation when \( \theta \) is high. These equilibria are informationally inefficient as the bad IB’s private information on firm profitability never reaches the market and is never incorporated in firm values.
### 3.2 Partial pooling

We refer to Partial Pooling (PP) as an equilibrium in which the bad IB always gives a unique evaluation independently from the signal actually received, while the good IB always truthfully reports its signals. Notice that a partial pooling equilibrium is informationally inefficient, because we are assuming that the signal received (but disregarded) by the bad IB contains some information \( z > \frac{1}{2} \). Again, we will focus on equilibria in which the firm accepts after \( s_h \) and refuses after \( s_l \).

We first analyze whether there exists a PP equilibrium in which the bad IB always proposes the high evaluation (independently from the private signal actually received) and the good IB always truthfully reports. In such equilibrium, both evaluations \( s_l \) and \( s_h \) are sent in equilibrium. Thus, given the equilibrium strategies of the firm and of the IBs, we have that ex-post reputation \( \hat{\alpha} \) assumes the following values in equilibrium:

\[
\hat{\alpha}_{1,s_h} = \alpha \\
\hat{\alpha}_{0,R} = \hat{\alpha}_{0,s_l} = 1 \\
\hat{\alpha}_{0,s_h} = 0
\]

Indeed, in this equilibrium the pair \((F = 1, s_h)\) is non informative about IB’s ability, while pairs \((F = 0, R)\) and \((F = 0, s_l)\) reveal completely the IB’s type. Notice that the case in which a refused by the firm is followed by a high state of the firm cannot arise on the equilibrium path, that is, the event \((F = 1, R)\) has zero probability to occur in the putative equilibrium that we are considering. Indeed, low evaluations are sent in equilibrium only by good IBs, which in turn have complete information about the true value of the firm and thus do not make mistakes. Accordingly \( \hat{\alpha}_{1,s_l} \) cannot be computed via Bayes rule. However, it is reasonable to assume that when the market observes the outcome \((F = 1, R)\) it believes that the message has been sent
by a bad IB. Thus, we assume:

\[ \hat{\alpha}_{1,R} = 0 \]

As of the possible values that the firm can take in a partial pooling equilibrium, notice that if the firm refuses, then it is valued \( d = 0 \). Indeed, when the firm refuses, the market infers that \( s_l \) has been proposed. Furthermore, given the equilibrium strategies of good and bad IBs, the market also infers that \( s_l \) has been proposed by a good IB that has received \( S_l \) (remember that the good IB has complete information about the firms value). On the other hand, if the firm accepts after \( s_h \), then the IB’s evaluation reaches the market and the value of the firm is computed accordingly. In particular, let \( V_{PP}^{H}(s_h, \alpha) \) denote the value of the firm in this partial pooling equilibrium when \( s_h \) reaches the market. It is possible to show that\(^{17}\)

\[ V_{PP}^{H}(s_h, \alpha) \equiv \Pr(F = 1 \mid s_h, \alpha) = \frac{\theta}{\theta + (1 - \theta)(1 - \alpha)} \]

Accepting after \( s_l \) is an out of equilibrium action by the firm. Notice that in this case, the low evaluation reaches the market. Let \( V_{PP}^{H}(s_l, \alpha) \) denote the (out of equilibrium) value of the firm in the case in which the low evaluation reaches the market. Given the equilibrium strategies of the IBs, the market sets \( V_{PP}^{H}(s_l, \alpha) = 0 \).

**Lemma 3** In an equilibrium in which the firm accepts only if a high evaluation is proposed and the bad IB always proposes to report a high evaluation, the good IB truthfully reports its private information

**Proof.** Given the equilibrium strategies of the firm and of bad IBs, a

\(^{17}\)See appendix 1
good IB truthtells if:

\[ kV_H^{PP}(s_h, \alpha) + E(\hat{\alpha}_{F,s_h} \mid G, S_h) \geq E(\hat{\alpha}_{F,R} \mid G, S_h) \]

\[ E(\hat{\alpha}_{F,R} \mid G, S_l) \geq kV_H^{PP}(s_h, \alpha) + E(\hat{\alpha}_{F,s_h} \mid G, S_l) \]

Since the good IB has a completely informative signal, it is easy to show that the previous conditions can be written as:

\[ kV_H^{PP}(s_h, \alpha) + \alpha \geq 0 \]

\[ 1 \geq kV_H^{PP}(s_h, \alpha) \]

Since \( 0 < V_H^{PP}(s_h, \alpha) < 1 \), the previous conditions are always satisfied for any values of \( \theta, \alpha, k \in (0, 1) \). □

This result is a consequence of two facts. First, given the features of the equilibrium at hand, proposing \( s_l \) delivers a very high reputational reward if this turns out to be the correct forecast on firm’s state \( (\hat{\alpha}_{0,R} = 1) \). Second, since the good IB has a perfectly informative signal, it is sure that \( F = 0 \) whenever it receives \( S_l \). Thus, a good IB receiving \( S_l \) is certain to get the high reputational reward by proposing \( s_l \). Hence, (whatever the value of the prior on firm profitability) the good investment bank will always trust its private signal more than public information. This allows us to focus on the bad IB’s conditions to determine the equilibrium,

**Lemma 4** In an equilibrium in which the firm accepts only when a high evaluation is proposed and the good IB truthtells, the bad IB will always propose to report a high evaluation (independently form its private information), provided that the prior on firm’s profitability is not too low.

**Proof.** Given the equilibrium strategies of the firm and of good IBs, a
bad IB always sends $s_h$ if

\[
k^V_H(s_h, \alpha) + E(\tilde{\alpha}_{F,s_h} \mid B, S_h) \geq E(\tilde{\alpha}_{F,R} \mid B, S_h)
\]

Since the signal received by the bad IB is informative, the last inequality is stricter than the first one. Thus, it is sufficient to show that the last inequality holds. Notice that the last inequality can be written as:

\[
k^V_H(s_h, \alpha) \geq E(\tilde{\alpha}_{F,R} - \tilde{\alpha}_{F,s_h} \mid B, S_l) \tag{12}
\]

which reads as follows\textsuperscript{18}:

\[
k \frac{\theta}{\theta + (1 - \theta)(1 - \alpha)} \geq \Pr(F = 0 \mid S_l, B) - \alpha \Pr(F = 1 \mid S_l, B)
\]

At $\theta = 0$ the LHS=0 and the RHS=1. At $\theta = 1$, the LHS=$k > 0$ and the RHS=$-\alpha < 0$. Since for every $\theta, \alpha \in (0, 1)$ and $z \in (\frac{1}{2}, 1)$ the LHS is continuous and strictly increasing in $\theta$ while the RHS is continuous and strictly decreasing in $\theta$, there always exists a unique value $\theta^*_H \in (0, 1)$ such that LHS=RHS and such that for every $\theta > \theta^*_H$ the previous condition holds and it is optimal for the bad IB to pool around $s_h$. \)

The intuition behind this result lies in the fact that the bad IB’s signal is not perfectly precise (though informative). Accordingly, a bad IB receiving $S_l$ is not certain to get the high reputational reward by proposing $s_l$. At the same time, proposing $s_l$ implies to give in the underwriting fee. The

\textsuperscript{18}Using the results in appendix 1, notice that

\[
E(\tilde{\alpha}_{F,R} \mid B, S_l) = \tilde{\alpha}_{0,s_l} \Pr(F = 0 \mid S_l, B) + \tilde{\alpha}_{1,s_l} \Pr \Pr(F = 1 \mid S_l, B) =\Pr(F = 0 \mid S_l, B)
\]

\[
E(\tilde{\alpha}_{F,s_l} \mid B, S_l) = \tilde{\alpha}_{0,s_l} \Pr(F = 0 \mid S_l, B) + \tilde{\alpha}_{1,s_l} \Pr \Pr(F = 1 \mid S_l, B) = \alpha \Pr \Pr(F = 1 \mid S_l, B)
\]
incentives to always propose a high evaluation vanishes when the prior on firm’s profitability is low. In this case, the probability that $F = 0$ is high and the bad IB gets more confident about its low signal and about the fact that proposing a low evaluation is the correct choice.

At this point, we are left with proving that given the strategies of the bad and good IBs, it is indeed optimal for the firm to accept after $s_h$ and to refuse after $s_l$.

**The firm’s problem.** Accepting after $s_h$ gives the firm a payoff equal to $(1 - k)V^{PP}_H(s_h, \alpha)$. Deviating to a refuse after $s_h$ gives a payoff of $d$. Since $d = 0$, for the firm is optimal to accept after $s_h$ as long as $(1 - k)V^{PP}_H(s_h, \alpha) \geq 0$, which is always satisfied. What about the case in which $s_l$ is proposed? If the firm follows its equilibrium strategy and refuses after $s_l$ it gets $d = 0$. If the firm deviates and accepts after $s_l$, the market observes $s_l$, which is an out of equilibrium outcome. Again, given the equilibrium strategies of good and bad IBs, the market believes that $s_l$ comes from a good firm that has received $S_l$ and accordingly sets the firm’s value equal to 0. In fact, the firm is indifferent between accepting and refusing after $s_l$ since both actions give a payoff of zero.\(^{19}\)

We can summarize the previous analysis in the following proposition.

**Proposition 2** for every $k \in (0,1)$, $\alpha \in (0,1)$ and $z \in [1/2,1)$, there always exists a $\theta^{PP}_H \in (0,1)$ such that for any $\theta > \theta^{PP}_H$ a Partial Pooling equilibrium exists in which the good IB truthfully reports its information, the bad IB always reports a positive evaluation $s_h$ independently from the signal received, and the firm accepts to be underwritten after a good evaluation and refuses after a bad evaluation.

What happens when the prior on firm’s value $\theta$ is low? It is possible to show that when $\theta$ is below a given threshold, there exist a (partial pooling)\(^{19}\)Notice that assuming a small fixed cost to be paid to the IB for the underwriting service would break the indifference in favour of a refuse by the firm.
equilibrium in which the good IB always truthtells and the bad IB always proposes to report a low evaluation, even though the firm accepts to be underwritten only when a high evaluation is proposed. We refer to this equilibrium as $PP_L$. We formalize this result in the following proposition.

**Proposition 3** Let $\bar{k}^{PP}$ be defined by condition (22). Then, for every value of $z \in (\frac{1}{2}, 1)$ and for every values of $k \in (0, \bar{k}^{PP})$ and $\alpha \in (0, 1)$ such that $\alpha > k$, there always exists a $\theta_L^{pp} \in (0, 1)$ such that for any $\theta \in (0, \theta_L^{pp})$ there exists an equilibrium in which the good IB truthfully reports its information, the bad IB pools by always reporting a negative evaluation $s_1$ independently from the signal received, and the firm accepts after a good evaluation and refuses after a low evaluation. (proof in appendix 3)

We will relegate the proof of the previous result in the appendix, since the logic to follow in order to prove it is the same as for the equilibrium $PP_H$. At an intuitive level, proposition (5) suggests that when the prior on firm profitability is very low, the bad IB is better off disregarding its private information and conforming to public information on $\theta$. In this case the loss in terms of fees that the IB incurs is less than the expected reputation loss that it suffers by not following the prior. In other words, when private information is not complete, the underwriter will tend to attribute less weight to its signal for extreme values of public information.

### 4 Comparative Statics

We have shown that a unique equilibrium that is invariant with respect to $\theta$ does not exist. An equilibrium is thus defined based on the threshold values $\theta_L^{pp}, \theta_H^{pp}, \theta_T^{TT},$ and $\tilde{\theta}_T^{TT}$ that determine the existence of the three different types of pure strategy equilibria $PP_L, PP_H$ and $TT$.

In the next section we consider how each of the parameters $\alpha$ and $k$ affect the threshold values of $\theta$ that respectively guarantee the existence of the
inefficient partial pooling equilibria, and the efficient truthtelling equilibrium. The purpose of this section is to identify how variations in the exogenous parameters can lead to more or less efficient equilibria. We thus identify more or less efficient equilibria based on these threshold parameters in the following way. Whenever a variation in a given parameter reduces the parameter space over $\theta$ for which the inefficient partial pooling equilibria are satisfied, without reducing the space for which the truthtelling equilibrium is satisfied, we have an improvement in informational efficiency.

4.1 Variations in prior reputation ($\alpha$) and in fees ($k$)

The threshold values mentioned above are formally defined as the values of $\theta$ that make the necessary conditions of each type of equilibrium binding.

**Partial pooling equilibria.** In the partial pooling equilibria in which bad IBs always report a high evaluation, $\theta^{PP}_H$ is defined as the value of $\theta$ for which condition (12) is binding.

Notice that condition (12) can be written as follows:

$$P P_H \geq \frac{(1 - \theta)z - \alpha \theta (1 - z)}{(1 - \theta)z + \theta (1 - z)}$$

(13)

Remember that this partial pooling is defined for every value of $\alpha \in (0, 1)$, $z \in (\frac{1}{2}, 1)$ and $k \in (0, 1)$ (see proposition 2). In this parameters space, the LHS of (13) is increasing in $\theta$ while the RHS is decreasing in $\theta$. Thus, as $k$ increases, condition (13) is relaxed and $\theta^{PP}_H$ decreases. This means that an increase of the underwriting fee makes the partial pooling around the high evaluation more easily sustainable. Notice now that the LHS of (13) is increasing in $\alpha$, while the the RHS is decreasing in $\alpha$. Hence, as $\alpha$ increases, condition (13) is relaxed and $\theta^{PP}_H$ has to decrease to keep it binding. Therefore, also an increase in the level of initial reputation makes the the partial pooling around the high evaluation more easily to be sustained. The intuition behind this result is that when $\alpha$ is high, a positive evaluation is
trusted by the market. Thus, sending a positive report increases the firm’s value and consequently the underwriting fee (the LHS is increasing in $\alpha$). On the other hand, the higher is the initial reputation the lower is the net reputational gain that the IB enjoys by correctly reporting its low signal (the RHS is decreasing in $\alpha$). These findings are represented in in figure 1.

![Figure 1: $\theta_{PP}^H$ plotted against $\alpha$; $k = 0.1$ dashed, $k = 0.5$ solid](image)

Consider now the partial pooling equilibrium in which the bad IB always propose a low evaluation. Remember that $\theta_{LP}^{PP}$ is the value of $\theta$ for which condition (21) is binding. Notice that condition (21) reads as follows:

$$ \frac{\alpha(1-\theta)(1-z) - \theta z}{(1-\theta)(1-z) + \theta z} \geq k $$

(14)

The partial pooling equilibrium in the low signal was defined for $\alpha \in (0,1)$, $z \in \left(\frac{1}{2},1\right)$ and $k \in \left(0,\bar{k}_{PP}\right)$, with $\alpha > k$ (see proposition 3). Notice that the RHS of (14) is a constant while it is easy to show that in the parameters space for which the equilibrium is defined, the LHS is always decreasing in $\theta$. Therefore, as $k$ increases and the RHS increases too, $\theta_{LP}^{PP}$ has to decrease to
keep (14) binding. As expected, an increase in $k$ makes the partial pooling in which the bad IB always proposes a negative report more difficult to be met.

How does $\theta_{LP}^{PP}$ varies with $\alpha$? Notice that the LHS of (14) is increasing in $\alpha$. The reason is that in the equilibrium at hand, the initial level of reputation $\alpha$ is exactly the reward the bad IB gets if it reports a negative evaluation. On the other hand, the reward for a deviation to a positive evaluation is fixed equal to 1. Thus, the higher the initial level of reputation, the more appealing (in expectation, of course) it gets for the IB to follow the equilibrium strategy and report a negative evaluation. As $\alpha$ increases, the LHS increases and $\theta_{LP}^{PP}$ has to increase as well to keep condition (14) binding. Figure 2 summarizes the previous findings (remember that the equilibrium $PP_L$ holds as long as $\alpha > k$).

![Figure 2: $\theta_{LP}^{PP}$ plotted against $\alpha$; $k = 0.1$ dashed, $k = 0.5$ solid](image)

**Remark 1** An increase in prior reputation $\alpha$ has a perverse effect, since it makes the parameters space for which the inefficient partial pooling equilibria are sustained larger.

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Thruthtelling equilibrium

In the truth-telling equilibrium outlined in section 4.1, we defined \( \bar{\theta}^{TT} \) and \( \tilde{\theta}^{TT} \) as the values of \( \theta \) for which respectively conditions (9) and (10) were satisfied with equality. Let us write again conditions (9) and (10):

\[
\begin{align*}
kV^{TT}(s_h, \alpha) &\geq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} | B, S_h) \quad (15) \\
kV^{TT}(s_h, \alpha) &\leq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} | B, S_l) \quad (16)
\end{align*}
\]

Remember that the truth-telling equilibrium at hand was defined for \( \alpha \in (0, 1), z \in (\frac{1}{2}, 1) \) and \( k \in (0, \bar{k}^{TT}) \) (see proposition 1). It is immediate to see that in this parameters space, \( kV^{TT}(s_h, \alpha) \) is increasing in \( \theta \), while both RHSs of (15) and (16) are decreasing in \( \theta \).

Therefore, as \( k \) increases, both \( \bar{\theta}^{TT} \) and \( \tilde{\theta}^{TT} \) have to decrease to keep (15) and (16) binding. It is possible to show that \( \bar{\theta}^{TT} \) decreases less rapidly than \( \tilde{\theta}^{TT} \), so that we can conclude that as \( k \) increases, the range \((\bar{\theta}^{TT}, \tilde{\theta}^{TT})\) in which truth-telling is sustainable shifts downwards and shrinks. The intuition for this expected result is that as the underwriting fee increases, the incentives to provide a positive evaluation increase as well. In order for truth-telling to be an equilibrium, the prior on firm profitability must be low so that the reputation punishment of misreporting a low signal is sufficiently high to offset the gain from pooling around a positive evaluation.

How does \( \theta^{TT} \) and \( \bar{\theta}^{TT} \) vary with \( \alpha \)? Notice that the signs of the derivatives of \( \theta^{TT} \) and \( \bar{\theta}^{TT} \) with respect to \( \alpha \) depend on the values of \( z \). However, as shown in figure 3, it is possible to show that for any given value of \( z \in (\frac{1}{2}, 1) \) and \( k \in (0, \bar{k}^{TT}) \), the range \((\theta^{TT}, \bar{\theta}^{TT})\) shifts upwards and gets larger as \( \alpha \) increases.
Figure 3: $\tilde{\theta}^{TT}$ and $\overline{\theta}^{TT}$ against $\alpha$; $k = 0.1$ dashed, $k = 0.5$ solid

The previous analysis leads to the following remarks:

**Remark 2** As prior IB reputation $\alpha$ rises, there is a larger parameter space on prior firm profitability $\theta$ for which both partial pooling equilibria are sustained. On the other hand, also the range of $\theta$ for which truthtelling can be sustained gets larger.

Thus, in this case nothing can be concluded in terms of informational efficiency.

**Remark 3** An increase in the IB’s underwriting fee $k$ shrinks the range of parameters on firm profitability for which truthtelling is sustained and shifts it downward (both $\tilde{\theta}^{TT}$ and $\overline{\theta}^{TT}$ decrease, with $\tilde{\theta}^{TT}$ decreasing at a lower rate than $\overline{\theta}^{TT}$), increases that for which partial pooling $PP_H$ is sustained ($\theta_H^{PP}$ decreases) and reduces that for which $PP_L$ is sustained ( $\theta_L^{PP}$ decreases).

As mentioned above, increases in fees have an asymmetric effect on the partial pooling equilibria. Larger fees tend to reduce the chances of having
equilibria in which bad IBs pools around low evaluation, but increase the incentive to pool around the good evaluations. Furthermore, there is no doubt that an increase in fees reduces the parameters space where truthtelling is sustainable. Thus, focusing on values of \( \theta \) relatively high, we can state that increases in fees lead to a worsening in informational efficiency.

5 Conclusions

In IPOs, investment banks typically have privileged information on the profitability of firms they are underwriting. They are therefore in a position to reduce the informational asymmetries between firms that are going public and investors, acting as information providers for the market. We introduce reputation to take into account of the fact that providing incorrect evaluations may hinder future profits of the underwriters by reducing their credibility. It turns out however, that in many cases IBs misreport their private information and actually profit from doing so.

Misreporting takes the form of a conformist behavior where IBs tend to disregard their private information once the public signal is extreme. Thus when investors have an ex-ante perception that firm profitability is either very high or very low, underwriters will tend to conform to the prior. In particular when prior reputation is higher underwriters attribute less weight to reputation acquisition increasing the incentives to misreport. Paradoxically, reputation may actually exacerbate the informational inefficiency.

High evaluations tend to inflate the price of firms on which fees are based. We assume that the fee structure is exogenously given and equal to a fraction of the difference between the value of the firm after being underwritten, and the value of the firm that would prevail in case the firm went public by itself. Raising the investment bank fees will lead underwriters to more frequently provide positive evaluations independently from the signal they receive.

The underwriting process is a complex phenomenon. Here we highlight
some aspects that have been left out of this paper, and that may be addressed in the future, in a similar framework.

First of all, we assume that the fee structure is exogenously given. An interesting extension would be to endogenously derive underwriting fees as a contract between firms and investment banks.

Furthermore in this model we have concentrated on the strategic information transmission problem faced by investment banks. Another aspect concerns the incentives firms face in disclosing information to the underwriters. Combining these aspects may provide a more complete theory of information disclosure in the IPO underwriting process.

6 Appendix 1: Values

6.1 Firm’s value

The value of a firm that operates at period \( t \) and is underwritten with an evaluation \( s_j \) by an IB with (prior) reputation \( \alpha \) reads:

\[
V(s_j, \alpha) = \Pr(F = 1 \mid s_j, \alpha) = \frac{\Pr(F = 1, s_j \mid \alpha)}{\Pr(F = 1, s_j \mid \alpha) + \Pr(F = 0, s_j \mid \alpha)}, \text{ with } j = h, l
\]

where

\[
\Pr(F = 1, s_j \mid \alpha) = \theta[\sigma_{IB}(s_j \mid S_{h,t}, G, \alpha) \Pr(S_h \mid F = 1, G) + \sigma_{IB}(s_j \mid S_{l,t}, B, \alpha) \Pr(S_h \mid F = 1, B)(1 - \alpha)]
+ \sigma_{IB}(s_j \mid S_{l,t}, G, \alpha) \Pr(S_l \mid F = 1, G) \alpha + \sigma_{IB}(s_j \mid S_{h,t}, B, \alpha) \Pr(S_h \mid F = 1, B)(1 - \alpha)
+ \sigma_{IB}(s_j \mid S_{l,t}, B, \alpha) \Pr(S_l \mid F = 1, B)(1 - \alpha)
\]

and
\[
\Pr(F = 0, u, s_j \mid \alpha) = (1 - \theta)\sigma_{IB}(s_j \mid S_{h,t}, G, \alpha)\Pr(S_h \mid F = 0, G)\alpha + \\
\sigma_{IB}(s_j \mid S_{l,t}, G, \alpha)\Pr(S_l \mid F = 0, G)\alpha + \sigma_{IB}(s_j \mid S_{h,t}, B, \alpha)\Pr(S_h \mid F = 0, B)(1 - \alpha) + \\
\sigma_{IB}(s_j \mid S_{l,t}, B, \alpha)\Pr(S_l \mid F = 0, B)(1 - \alpha)
\]

In particular, under our information structure (1) and (2), we have that

\[
\Pr(F = 1, s_j \mid \alpha) = \theta[\sigma_{IB}(s_j \mid S_{h,t}, G, \alpha)p\alpha + \\
\sigma_{IB}(s_j \mid S_{l,t}, G, \alpha)(1 - p)\alpha + \sigma_{IB}(s_j \mid S_{h,t}, B, \alpha)z(1 - \alpha) + \\
\sigma_{IB}(s_j \mid S_{l,t}, B, \alpha)(1 - z)(1 - \alpha)]
\]

and

\[
\Pr(F = 0, u, s_j \mid \alpha) = (1 - \theta)[\sigma_{IB}(s_j \mid S_{h,t}, G, \alpha)(1 - p)\alpha + \\
\sigma_{IB}(s_j \mid S_{l,t}, G, \alpha)p\alpha + \sigma_{IB}(s_j \mid S_{h,t}, B, \alpha)z(1 - \alpha) + \\
\sigma_{IB}(s_j \mid S_{l,t}, B, \alpha)(1 - z)(1 - \alpha)]
\]

On the other hand, the value of a firm in period \(t\) upon not being underwritten reads:

\[
d = \Pr(F = 1 \mid R)
\]

Two things are important to bear in mind.

First, the value of the firm upon underwriting, \(V(s_j, \alpha)\), is a function of the Investment Bank’s reputation (the higher IB’s reputation, the more credible is the evaluation and eventually the more the value of the firm aligns with the evaluation).

Second, as for the case of IB’s reputation, the equilibrium value of a firm depends on the equilibrium strategies of both the IB and the firm. Therefore,
besides the evaluation itself, the value of a firm depends on the type of
equilibrium in which this evaluation occurs.

6.2 IB’s reputation

Suppose that at the end of period $t$ the true state of the firm is revealed to
be $F = 1$. Then, the reputation of an IB that in period $t$ has sent message $j$
reads:

$$\hat{\alpha}_{1,s_j} = \Pr(\text{IB} = G \mid F = 1, s_j) = \frac{\Pr(\text{IB} = G, F = 1, s_j)}{\Pr(\text{IB} = G, F = 1, s_j) + \Pr(\text{IB} = B, F = 1, s_j)},$$

with $j = h, l$

where:

$$\Pr(\text{IB} = G, F = 1, s_j) = \left[ \sigma_{IB}(s_j \mid S_h, G)p \alpha + 
\sigma_{IB}(s_j \mid S_l, G)(1 - p) \alpha \right]$$

and

$$\Pr(\text{IB} = B, F = 1, s_j) = \left[ \sigma_{IB}(s_j \mid S_h, B)z(1 - \alpha) + 
\sigma_{IB}(s_j \mid S_l, B)(1 - z)(1 - \alpha) \right]$$

Analogously, at the end of period $t$, if the true state of the firm has revealed
to be $F = 0$, the reputation of an IB that in period $t$ has sent message $j = h, l$
reads:
\[ \hat{\alpha}_{0,s_j} = \Pr(IB = G \mid F = 0, s_j) = \frac{\Pr(IB = G, F = 0, s_j)}{\Pr(IB = G, F = 0, s_j) + \Pr(IB = B, F = 0, s_j)}, \]

with \( j = h, l \)

where

\[ \Pr(IB = G, F = 0, s_j) = [\sigma_{IB}(s_j \mid S_h, G)(1 - p)\alpha + \sigma_{IB}(s_j \mid S_l, G)p\alpha] \]

and

\[ \Pr(IB = B, F = 0, s_j) = [\sigma_{IB}(s_j \mid S_h, B)(1 - z)(1 - \alpha) + \sigma_{IB}(s_j \mid S_l, B)z(1 - \alpha)] \]

Basically, in each period, an IB’s reputation is the Bayesian update on the IB’s previous period reputation (starting from the prior \( \alpha_0 = \alpha \)) given the message sent by the IB in \( t \) and the observed true state of the firm revealed at the end of \( t \).

It is important to notice that here we focus on Markov Perfect equilibria where we assume that \( \alpha \) is a summary statistic of the \( t - 1 \) period history.

The previous formulas hold whenever the respective denominators are positive. When they are not, \( \alpha_{1,s_j} \) and \( \alpha_{0,s_j} \) are arbitrary, in the sense that they are not determined via Bayes rule (although they still enter the perfect Bayesian equilibrium conditions and hence they are co-determined by these conditions together with the behavior strategies \( \sigma \)).

**IB’S REPUTATION IN A POOLING EQUILIBRIUM**

Consider an equilibrium where both good and bad IB pools around \( s_h \). The previous formulas give us (as it is intuitive):
\[ \hat{\alpha}_{1,s_h} = \hat{\alpha}_{0,s_h} = \frac{p\alpha + (1 - p)\alpha}{p\alpha + (1 - p)\alpha + z(1 - \alpha) + (1 - z)(1 - \alpha)} = \alpha \]

In words, when the equilibrium is pooling, the market does not care about the fact that the evaluation is correct, basically assuming that if this occurs, it is just a coincidence.

**IB’S REPUTATION IN A TRUTHTELLING EQUILIBRIUM**

Consider an equilibrium where both IBs truthtell, when the true state is \( F = 1 \). If the IB makes a correct evaluation (i.e. has sent \( s_h \)) we will have:

\[ \hat{\alpha}_{1,s_h} = \hat{\alpha}_{0,s_h} = \frac{p\alpha}{p\alpha + z(1 - \alpha)} = \hat{\alpha}_{\text{max}} \]

If instead the IB makes an incorrect evaluation (i.e. has sent \( s_l \)) we will have

\[ \hat{\alpha}_{1,s_l} = \hat{\alpha}_{0,s_h} = \frac{(1 - p)\alpha}{(1 - p)\alpha + (1 - z)(1 - \alpha)} = \hat{\alpha}_{\text{min}} \]

In words, in a truthtelling equilibrium, making a correct evaluation increases reputation above the prior, while making a mistake decreases it below the prior.

**IB’S REPUTATION IN A PARTIAL POOLING EQUILIBRIUM**

Finally, consider an equilibrium in which the good IB always truthtells while the bad IB always pools around \( s_h \) (Partial pooling \( PP_H \)). Notice that when the true state is \( F = 1 \), making a correct evaluation gives:

\[ \hat{\alpha}_{1,s_h} = \frac{p\alpha}{p\alpha + (1 - \alpha)} = \hat{\alpha}_{h,\text{correct}}^{PP} \]

while making a mistake (transmitting \( s_l \)) gives:

\[ \hat{\alpha}_{1,s_l} = \frac{(1 - p)\alpha}{(1 - p)\alpha} = 1 = \hat{\alpha}_{l}^{PP} \]
When the true state is $F = 0$, making a mistake gives:

$$\hat{\alpha}_{0,s_h} = \frac{(1 - p)\alpha}{(1 - p)\alpha + (1 - \alpha)} = \hat{\alpha}_{h,\text{incorrect}}$$

while making a correct evaluation gives:

$$\hat{\alpha}_{0,s_l} = \frac{p\alpha}{p\alpha} = 1 = \hat{\alpha}_{l}^{PP}$$

In words, in this kind of equilibrium, it does not matter whether a correct or incorrect evaluation is made if the IB makes a low evaluation, since transmitting $s_l$ immediately identifies the IB as being good. When instead a high evaluation is made, being correct or incorrect makes a difference, since $\hat{\alpha}_{h,\text{incorrect}}^{PP} < \hat{\alpha}_{h,\text{correct}}^{PP}$. Note that when $p = 1$, $\hat{\alpha}_{1,s_l}$ is an out of equilibrium value since only good IBs send low evaluations and $p = 1$ implies that good IBs have complete information and never make mistakes.

The previous result also tells us that in this partial pooling equilibrium if the IB sends the high message and makes a mistake, it will be highly penalized. Furthermore, sending the high message correctly forecasting the true state is not even enough to increase the IB’s reputation above the prior. This is because the market heavily weighs the presence of the bad IBs that pool around the high message.

7 Appendix 2

7.1 Proof of Lemma 1

**Proof.** Consider conditions (7) and (8) for the existence of a truth-telling equilibrium. They can be spelled out as follows:
Lemma 1 implies that (20) and (19) are necessary and sufficient conditions for the existence of a truth-telling equilibrium. Notice that the good IB has more informative signal than the bad IB. Hence, the following inequalities hold:

\[ kV^{TT}(s_h, \alpha) \geq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid G, S_h) \]  \hspace{1cm} (17)

\[ kV^{TT}(s_h, \alpha) \leq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid G, S_l) \]  \hspace{1cm} (18)

and

\[ kV^{TT}(s_h, \alpha) \geq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid B, S_h) \]  \hspace{1cm} (19)

\[ kV^{TT}(s_h, \alpha) \leq E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid B, S_l) \]  \hspace{1cm} (20)

This implies that conditions (18) and (17) are satisfied whenever (20) and (19) are satisfied.

7.2 Proof of Lemma 2

Proof. Consider conditions (20) and (19). First, consider condition (20). To ease notation, let \( M^{TT} \equiv kV^{TT}(s_h, \alpha) \) and \( R^{TT}(B, s_l, S_l) \equiv E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid B, S_l) \), so that we can write this condition as follows:

\[ M^{TT} \leq R^{TT}(B, s_l, S_l) \]
Using the results appendix 1, it is possible to show that:

\[ M^{TT} = k \frac{\theta [\alpha + (1 - \alpha)z]}{\theta [\alpha + (1 - \alpha)z] + (1 - \theta)(1 - \alpha)(1 - z)} \]

and\(^{20}\)

\[ R^{TT}(B, s_l, S_l) = \frac{\alpha}{\alpha + z(1 - \alpha)} \left[ \frac{z - \theta}{\theta - 2\theta z + z} \right] \]

where \( \frac{\alpha}{\alpha + z(1 - \alpha)} \equiv \alpha_{\text{max}} \).

For every \( k, \alpha \in (0, 1) \) and \( z \in \left( \frac{1}{2}, 1 \right) \), the following properties are satisfied:

(i) at \( \theta = 0 \), \( M^{TT} = 0 \) and \( R^{TT}(B, s_l, S_l) = \alpha_{\text{max}} \). Thus, at \( \theta = 0 \), \( M^{TT} < R^{TT}(B, s_l, S_l) \).

(ii) at \( \theta = 1 \), \( M^{TT} = k \) and \( R^{TT}(B, s_l, S_l) = -\alpha_{\text{max}} \). Thus, at \( \theta = 1 \), \( M^{TT} > R^{TT}(B, s_l, S_l) \).

(iii) for \( \theta \in (0, 1) \), \( M^{TT} \) is a continuous and strictly increasing function of \( \theta \), while \( R^{TT}(B, s_l, S_l) \) is a continuous and strictly decreasing function of \( \theta \).

(i), (ii), guarantee that there exists a value \( \theta = \bar{\theta}^{TT} \in (0, 1) \) such that \( M^{TT} = R^{TT}(B, s_l, S_l) \). (iii) guarantees that \( \bar{\theta}^{TT} \) is unique.

Now consider condition (19). Again, let \( M^{TT} \equiv kV^{TT}(s_h, \alpha) \) and \( R^{TT}(B, s_h, S_l) \equiv \)

\(^{20}\)Notice that

\[ E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid B, S_l) = E(\hat{\alpha}_{F,R} \mid B, S_l) - E(\hat{\alpha}_{F,s_h} \mid B, S_l) \]

with

\[ E(\hat{\alpha}_{F,R} \mid B, S_l) = \hat{\alpha}_{\text{max}} \Pr(F = 0 \mid B, S_l) + \hat{\alpha}_{\text{min}} \Pr(F = 1 \mid B, S_l) = \]

\[ = \frac{\alpha}{\alpha + z(1 - \alpha)} \frac{1 - \theta}{(1 - \theta) z} \]

\[ E(\hat{\alpha}_{F,s_h} \mid B, S_l) = \hat{\alpha}_{\text{max}} \Pr(F = 1 \mid B, S_l) + \hat{\alpha}_{\text{min}} \Pr(F = 0 \mid B, S_l) = \]

\[ = \frac{\alpha}{\alpha + z(1 - \alpha)} \frac{\theta}{(1 - \theta) z} \]

the results follows from the fact that in a truth telling equilibrium \( \hat{\alpha}_{\text{max}} = \frac{\alpha}{\alpha + z(1 - \alpha)} \) and \( \hat{\alpha}_{\text{min}} = 0 \) (see appendix 1)
\( E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid B, S_h) \), so that we can write this condition as follows:

\[
M^{TT} \geq R^{TT}(B, s_l, S_h)
\]

where

\[
M^{TT} = k \frac{\theta [\alpha + (1 - \alpha)z]}{\theta \left[ \alpha + (1 - \alpha)z \right] + (1 - \theta)(1 - \alpha)(1 - z)}
\]

and

\[
R^{TT}(B, s_l, S_h) = \alpha_{\text{max}} \left[ \frac{1 - z - \theta}{1 - z - \theta + 2\theta z} \right]
\]

For every \( k, \alpha \in (0, 1) \) and \( z \in \left( \frac{1}{2}, 1 \right) \), the following properties are satisfied:

(iv) for \( \theta = 0 \), \( M^{TT} = 0 \) and \( R^{TT}(B, s_l, S_h) = \alpha_{\text{max}} \). Thus, at \( \theta = 0 \), \( M^{TT} < R^{TT}(B, s_l, S_h) \).

(v) for \( \theta = 1 \), \( M^{TT} = k \) and \( R^{TT}(B, s_l, S_h) = -\alpha_{\text{max}} \). Thus, at \( \theta = 1 \), \( M^{TT} > R^{TT}(B, s_l, S_h) \).

(vi) for \( \theta \in (0, 1) \), \( M^{TT} \) is a continuous and strictly increasing function of \( \theta \); \( R^{TT}(B, s_l, S_h) \) is a continuous and strictly decreasing function of \( \theta \).

(vi), (v), guarantee that there exists a unique value \( \theta = \hat{\theta}^{TT} \in (0, 1) \) such that \( M^{TT} = 0 \) and \( R^{TT}(B, s_l, S_h) \). (vi) guarantees that \( \hat{\theta}^{TT} \) is unique.

In order to complete the proof we must show that \( \hat{\theta}^{TT} < \hat{\theta}^{TT} \).

This can easily be seen by observing that for \( \theta \in (0, 1) \), for every \( k, \alpha \in (0, 1) \) and \( z \in \left( \frac{1}{2}, 1 \right) \), the following inequality holds:

\[
R^{TT}(B, s_l, S_h) < R^{TT}(B, s_l, S_l)
\]

Then, at \( \theta = \hat{\theta}^{TT} \) we have that

\[
M^{TT} = R^{TT}(B, s_l, S_h) < R^{TT}(B, s_l, S_l)
\]

Since \( R^{TT}(B, s_l, S_l) \) is monotonically decreasing in \( \theta \), the equality

\[
M^{TT} = R^{TT}(B, s_l, S_l)
\]
is satisfied only for $\theta > \theta^{TT}$. ■

8 Appendix 3

8.1 Proof of Proposition 5

Proof. Consider the putative (partial pooling) equilibrium in which the bad IB disregards its private information and always provides a negative evaluation, the good IB truthtells and the firm accepts after $s_h$ and refuses after $s_l$. In this equilibrium, ex-post reputation $\hat{\alpha}$ assumes the following values:\(^{21}\)

$$
\hat{\alpha}_{0,R} = \alpha \\
\hat{\alpha}_{1,R} = 0 \\
\hat{\alpha}_{1,s_h} = 1
$$

Notice that the event $(F = 0, s_h)$ has zero probability to occur in the putative equilibrium that we are considering. Indeed, high evaluations are sent in equilibrium only by good IBs, which in turn have complete information about the true value of the firm and thus do not make mistakes. Accordingly $\hat{\alpha}_{0,s_h}$ cannot be computed via Bayes rule. However, it is reasonable to assume that when the market observes the outcome $(F = 0, s_h)$ it believes that the evaluation has been reported by a bad IB. Thus, we assume:

$$
\hat{\alpha}_{0,s_h} = 0
$$

As of the possible values that the firm, let $V_{L}^{PP}(s_l, \alpha)$ and $V_{L}^{PP}(s_h, \alpha)$ respectively denote the value of the firm when evaluations $s_l$ and $s_h$ reach the market.

\(^{21}\)See appendix 1 (remembering that here we are assuming $p = 1$).
In this equilibrium, the firm refuses after $s_l$. Accordingly:

$$d = V_L^{PP}(s_l, \alpha) = \frac{\theta(1 - \alpha)}{\theta(1 - \alpha) + (1 - \theta)}$$

Indeed, when the firm refuses, the market infers that $s_l$ has been proposed and set $d \equiv \Pr(F = 1 \mid R) = \Pr(F = 1 \mid s_l)$. Given the equilibrium strategies of IBs, $\Pr(F = 1 \mid s_l) = V_L^{PP}(s_l, \alpha)$.

Analogously, since in equilibrium the firm accepts after $s_h$, and $s_h$ is sent only by good IB (that have perfect information), then $V_L^{PP}(s_h, \alpha) = 1$.

Finally, accepting after $s_l$ is an out of equilibrium action by the firm. In this case, the IB’s low evaluation would reach the market. Notice that given the equilibrium strategies of the IBs, the value of the firm can be computed via Bayes and set equal to $V_L^{PP}(s_l, \alpha)$.

**Good IBs’ problem.** Given the equilibrium strategies of the firm and bad IBs, a good IB truth tells if:

$$kV_L^{PP}(s_h, \alpha) + E(\hat{\alpha}_{F,s_h} \mid G, S_h) \geq E(\hat{\alpha}_{F,R} \mid G, S_h)$$

$$E(\hat{\alpha}_{F,R} \mid G, S_l) \geq kV_L^{PP}(s_h, \alpha) + E(\hat{\alpha}_{F,s_h} \mid G, S_l)$$

where:

$$E(\hat{\alpha}_{F,s_h} \mid G, S_h) = \hat{\alpha}_{1,s_h} \Pr(F = 1 \mid G, S_h) + \hat{\alpha}_{0,s_h} \Pr(F = 0 \mid G, S_h) = 1$$

$$E(\hat{\alpha}_{F,R} \mid G, S_h) = \hat{\alpha}_{1,R} \Pr(F = 1 \mid G, S_h) + \hat{\alpha}_{0,R} \Pr(F = 0 \mid G, S_h) = \alpha \Pr(F = 0 \mid G, S_h) = 0$$

$$E(\hat{\alpha}_{F,s_h} \mid G, S_l) = \hat{\alpha}_{1,s_h} \Pr(F = 1 \mid G, S_l) + \hat{\alpha}_{0,s_h} \Pr(F = 0 \mid G, S_l) = 0$$

$$E(\hat{\alpha}_{F,R} \mid G, S_l) = \hat{\alpha}_{1,R} \Pr(F = 1 \mid G, S_l) + \hat{\alpha}_{0,R} \Pr(F = 0 \mid G, S_l) = \alpha \Pr(F = 0 \mid G, S_l) = \alpha$$

Therefore, considering also that $V_L^{PP}(s_h, \alpha) = 1$, our conditions for the good
IB can be written as follows:

\[ k + 1 \geq 0 \quad \rightarrow \quad \text{always satisfied} \]
\[ \alpha \geq k \quad \rightarrow \quad \alpha \geq k \]

**Bad IBs’ problem.** Given the equilibrium strategies of the firm and good IBs, a bad IB pools around \( s_l \) if:

\[
E(\hat{\alpha}_{F,R} \mid B, S_h) \geq kV^P_L(s_h, \alpha) + E(\hat{\alpha}_{F,s_h} \mid B, S_h)
\]
\[
E(\hat{\alpha}_{F,R} \mid B, S_l) \geq kV^P_L(s_l, \alpha) + E(\hat{\alpha}_{F,s_h} \mid B, S_l)
\]

Notice that since the bad IB’s signal is informative, it is sufficient to show that the first inequality is satisfied. Let us write the first inequality as follows:

\[
E(\hat{\alpha}_{F,R} - \hat{\alpha}_{F,s_h} \mid B, S_h) \geq kV^P_L(s_h, \alpha) \quad (21)
\]

After a bit of algebra, it is possible to show that the previous condition can be written as\(^{22}\):

\[
\frac{\alpha(1 - \theta)(1 - z)}{\theta z + (1 - \theta)(1 - z)} \geq k
\]

At \( \theta = 0 \), the LHS=\( \alpha \) and the RHS=\( k \). At \( \theta = 1 \), the LHS=\(-1 \) and RHS=\( k \). Notice that for \( \theta \in (0, 1) \), both the LHS is continuous and strictly decreasing in \( \theta \). Therefore, provided that \( \alpha > k \), there always exists a unique value of \( \theta \equiv \theta^p_L \in (0, 1) \) such that LHS=RHS and such that for every \( \theta \in (0, \theta^p_L) \) the

---

\(^{22}\)Notice that \( V^P_L(s_h, \alpha) = 1 \), while for expected reputations we have that:

\[
E(\hat{\alpha}_{F,s_h} \mid B, S_h) = \hat{\alpha}_{1,s_h} \Pr(F = 1 \mid B, S_h) + \hat{\alpha}_{0,s_h} \Pr(F = 0 \mid B, S_h) = \frac{\theta z}{\theta z + (1 - \theta)(1 - z)}
\]
\[
E(\hat{\alpha}_{F,R} \mid B, S_h) = \hat{\alpha}_{1,R} \Pr(F = 1 \mid B, S_h) + \hat{\alpha}_{0,R} \Pr(F = 0 \mid B, S_h) = \alpha \frac{(1 - \theta)(1 - z)}{\theta z + (1 - \theta)(1 - z)}
\]

After a bit of algebra, it is possible to show that the previous condition can be written as\(^{22}\):

\[
\frac{(1 - \theta)(1 - z)}{\theta z + (1 - \theta)(1 - z)} \geq k
\]

At \( \theta = 0 \), the LHS=\( \alpha \) and the RHS=\( k \). At \( \theta = 1 \), the LHS=\(-1 \) and RHS=\( k \). Notice that for \( \theta \in (0, 1) \), both the LHS is continuous and strictly decreasing in \( \theta \). Therefore, provided that \( \alpha > k \), there always exists a unique value of \( \theta \equiv \theta^p_L \in (0, 1) \) such that LHS=RHS and such that for every \( \theta \in (0, \theta^p_L) \) the
previous condition is always satisfied. Thus, we can conclude that given the posited equilibrium strategy of the firm, the good IB truth-tells and the bad IB pools around $s_l$ as long as $\alpha > k$ and $\theta \in (0, \theta^p_L)$.

**The firm’s problem.** We then have to check whether it is optimal for the firm to follow its equilibrium strategy (accept after $s_h$ and refuse after $s_l$). Accepting after $s_h$ gives the firm a payoff equal to $(1 - k) V^{PP}_L(s_h, \alpha)$. If the firm deviates and refuses, it is believed that $s_l$ has been proposed and the firm gets $d = V^{PP}_L(s_l, \alpha)$. Thus accepting after $s_h$ is optimal as long as $1 - k \geq V^{PP}_L(s_l, \alpha)$, or equivalently:

$$k \leq 1 - V^{PP}_L(s_l, \alpha) \equiv \bar{k}^{PP} \quad (22)$$

Refusing after $s_l$ gives the firm a payoff equal to $d = V^{PP}_L(s_l, \alpha)$. If the firm deviates and accepts after $s_l$, then it gets $(1 - k) V^{PP}_L(s_l, \alpha)$. Thus, for the firm is always optimal to refuse after $s_l$. ■

**References**


